# CHAPTER 12
# Sources of Magnetic Fields


**Figure 12.1** An external hard drive attached to a computer works by magnetically encoding information that can be stored or retrieved quickly. A key idea in the development of digital devices is the ability to produce and use magnetic fields in this way. (credit: modification of work by "Miss Karen"/Flickr)

## Chapter Outline



**INTRODUCTION** In the preceding chapter, we saw that a moving charged particle produces a magnetic field. This connection between electricity and magnetism is exploited in electromagnetic devices, such as a computer hard drive. In fact, it is the underlying principle behind most of the technology in modern society, including telephones, television, computers, and the internet.

In this chapter, we examine how magnetic fields are created by arbitrary distributions of electric current, using the Biot-Savart law. Then we look at how current-carrying wires create magnetic fields and deduce the forces that arise between two current-carrying wires due to these magnetic fields. We also study the torques produced by the magnetic fields of current loops. We then generalize these results to an important law of electromagnetism, called Ampère's law.

We examine some devices that produce magnetic fields from currents in geometries based on loops, known as solenoids and toroids. Finally, we look at how materials behave in magnetic fields and categorize materials based on their responses to magnetic fields.

# 12.1 The Biot-Savart Law

### Learning Objectives

*By the end of this section, you will be able to:*

- Explain how to derive a magnetic field from an arbitrary current in a line segment
- Calculate magnetic field from the Biot-Savart law in specific geometries, such as a current in a line and a current in a circular arc

We have seen that mass produces a gravitational field and also interacts with that field. Charge produces an electric field and also interacts with that field. Since moving charge (that is, current) interacts with a magnetic field, we might expect that it also creates that field—and it does.

The equation used to calculate the magnetic field produced by a current is known as the Biot-Savart law. It is an empirical law named in honor of two scientists who investigated the interaction between a straight, current-carrying wire and a permanent magnet. This law enables us to calculate the magnitude and direction of the magnetic field produced by a current in a wire. The **Biot-Savart law** states that at any point $P$ (Figure 12.2), the magnetic field $d\vec{\mathbf{B}}$ due to an element $d\vec{\mathbf{l}}$ of a current-carrying wire is given by

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}. \tag{12.1}$$

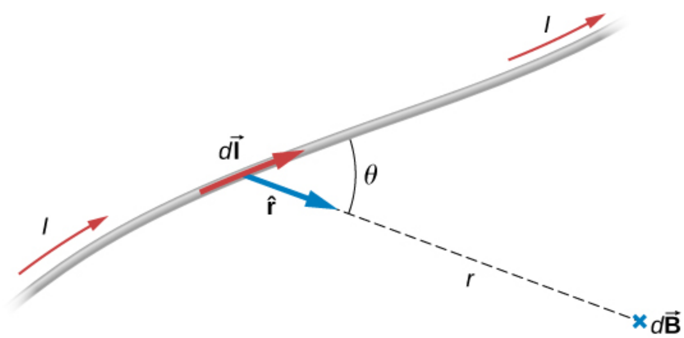


Figure 12.2 A current element $Id\vec{\mathbf{l}}$ produces a magnetic field at point $P$ given by the Biot-Savart law.

The constant $\mu_0$ is known as the **permeability of free space** and is exactly

$$\mu_0 = 4\pi \times 10^{-7} \,\mathrm{T \cdot m/A} \tag{12.2}$$

in the SI system. The infinitesimal wire segment $d\vec{\mathbf{l}}$ is in the same direction as the current $I$ (assumed positive), $r$ is the distance from $d\vec{\mathbf{l}}$ to $P$ and $\hat{\mathbf{r}}$ is a unit vector that points from $d\vec{\mathbf{l}}$ to $P$, as shown in the figure.

The direction of $d\vec{\mathbf{B}}$ is determined by applying the right-hand rule to the vector product $d\vec{\mathbf{l}} \times \hat{\mathbf{r}}$. The magnitude of $d\vec{\mathbf{B}}$ is

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl\,\sin\theta}{r^2} \tag{12.3}$$

where $\theta$ is the angle between $d\vec{\mathbf{l}}$ and $\hat{\mathbf{r}}$. Notice that if $\theta = 0$, then $d\vec{\mathbf{B}} = \vec{\mathbf{0}}$. The field produced by a current element $Id\vec{\mathbf{l}}$ has no component parallel to $d\vec{\mathbf{l}}$.

The magnetic field due to a finite length of current-carrying wire is found by integrating Equation 12.3 along the wire, giving us the usual form of the Biot-Savart law.

### Biot-Savart law

The magnetic field $\vec{\mathbf{B}}$ due to an element $d\vec{\mathbf{l}}$ of a current-carrying wire is given by

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I\, d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}. \tag{12.4}$$

Since this is a vector integral, contributions from different current elements may not point in the same direction. Consequently, the integral is often difficult to evaluate, even for fairly simple geometries. The following strategy may be helpful.

## PROBLEM-SOLVING STRATEGY

### Solving Biot-Savart Problems

To solve Biot-Savart law problems, the following steps are helpful:

1. Identify that the Biot-Savart law is the chosen method to solve the given problem. If there is symmetry in the problem comparing $\vec{\mathbf{B}}$ and $\mathbf{d}\vec{\mathbf{l}}$, Ampère's law may be the preferred method to solve the question.
2. Draw the current element length $d\vec{\mathbf{l}}$ and the unit vector $\hat{\mathbf{r}}$, noting that $d\vec{\mathbf{l}}$ points in the direction of the current and $\hat{\mathbf{r}}$ points from the current element toward the point where the field is desired.
3. Calculate the cross product $d\vec{\mathbf{l}} \times \hat{\mathbf{r}}$. The resultant vector gives the direction of the magnetic field according to the Biot-Savart law.
4. Use Equation 12.4 and substitute all given quantities into the expression to solve for the magnetic field. Note all variables that remain constant over the entire length of the wire may be factored out of the integration.
5. Use the right-hand rule to verify the direction of the magnetic field produced from the current or to write down the direction of the magnetic field if only the magnitude was solved for in the previous part.

## EXAMPLE 12.1

### Calculating Magnetic Fields of Short Current Segments

A short wire of length 1.0 cm carries a current of 2.0 A in the vertical direction (Figure 12.3). The rest of the wire is shielded so it does not add to the magnetic field produced by the wire. Calculate the magnetic field at point *P*, which is 1 meter from the wire in the *x*-direction.

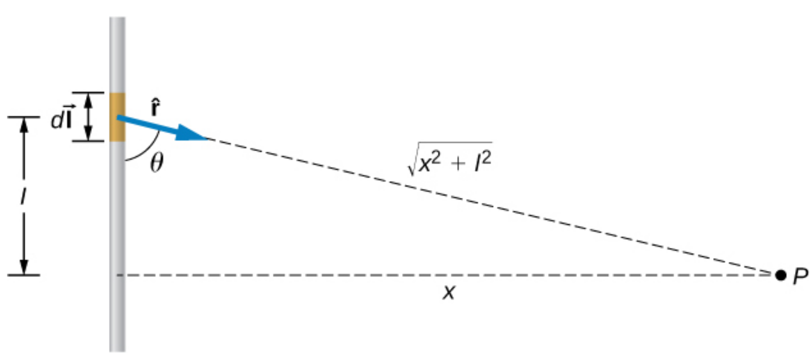


Figure 12.3 A small line segment carries a current *I* in the vertical direction. What is the magnetic field at a distance *x* from the segment?

#### Strategy

We can determine the magnetic field at point $P$ using the Biot-Savart law. Since the current segment is much smaller than the distance $x$, we can drop the integral from the expression. The integration is converted back into a summation, but only for small $dl$, which we now write as $\Delta l$. Another way to think about it is that each of the radius values is nearly the same, no matter where the current element is on the line segment, if $\Delta l$ is small compared to $x$. The angle $\theta$ is calculated using a tangent function. Using the numbers given, we can calculate the magnetic field at $P$.

#### Solution

The angle between $\Delta\vec{\mathbf{l}}$ and $\hat{\mathbf{r}}$ is calculated from trigonometry, knowing the distances $l$ and $x$ from the problem:

$$\theta = \tan^{-1}\left(\frac{1\text{ m}}{0.01\text{ m}}\right) = 89.4°.$$

The magnetic field at point $P$ is calculated by the Biot-Savart law:

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta l\sin\theta}{r^2} = (1\times 10^{-7}\text{T}\cdot\text{m/A})\left(\frac{2\text{ A}(0.01\text{ m})\sin(89.4°)}{(1\text{ m})^2}\right) = 2.0\times 10^{-9}\text{T}.$$

From the right-hand rule and the Biot-Savart law, the field is directed into the page.

#### Significance

This approximation is only good if the length of the line segment is very small compared to the distance from the current element to the point. If not, the integral form of the Biot-Savart law must be used over the entire line segment to calculate the magnetic field.

### CHECK YOUR UNDERSTANDING 12.1

Using Example 12.1, at what distance would $P$ have to be to measure a magnetic field half of the given answer?

### EXAMPLE 12.2

### Calculating Magnetic Field of a Circular Arc of Wire

A wire carries a current $I$ in a circular arc with radius $R$ swept through an arbitrary angle $\theta$ (Figure 12.4). Calculate the magnetic field at the center of this arc at point $P$.

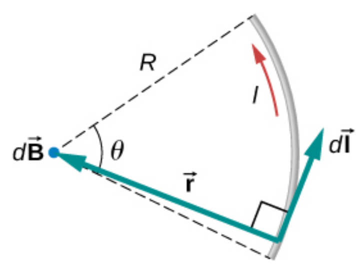


Figure 12.4 A wire segment carrying a current $I$. The path $d\vec{\mathbf{l}}$ and radial direction $\hat{\mathbf{r}}$ are indicated.

#### Strategy

We can determine the magnetic field at point $P$ using the Biot-Savart law. The radial and path length directions are always at a right angle, so the cross product turns into multiplication. We also know that the distance along the path $dl$ is related to the radius times the angle $\theta$ (in radians). Then we can pull all constants out of the integration and solve for the magnetic field.

#### Solution

The Biot-Savart law starts with the following equation:

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}.$$

As we integrate along the arc, all the contributions to the magnetic field are in the same direction (out of the page), so we can work with the magnitude of the field. The cross product turns into multiplication because the path $dl$ and the radial direction are perpendicular. We can also substitute the arc length formula, $dl = r d\theta$:

$$B = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I r \, d\theta}{r^2}.$$

The current and radius can be pulled out of the integral because they are the same regardless of where we are on the path. This leaves only the integral over the angle,

$$B = \frac{\mu_0 I}{4\pi r} \int_{\text{wire}} d\theta.$$

The angle varies on the wire from 0 to $\theta$; hence, the result is

$$B = \frac{\mu_0 I \theta}{4\pi r}.$$

**Significance**

The direction of the magnetic field at point $P$ is determined by the right-hand rule, as shown in the previous chapter. If there are other wires in the diagram along with the arc, and you are asked to find the net magnetic field, find each contribution from a wire or arc and add the results by superposition of vectors. Make sure to pay attention to the direction of each contribution. Also note that in a symmetric situation, like a straight or circular wire, contributions from opposite sides of point $P$ cancel each other.

**CHECK YOUR UNDERSTANDING 12.2**

The wire loop forms a full circle of radius $R$ and current $I$. What is the magnitude of the magnetic field at the center?

# 12.2 Magnetic Field Due to a Thin Straight Wire

## Learning Objectives

*By the end of this section, you will be able to:*

- Explain how the Biot-Savart law is used to determine the magnetic field due to a thin, straight wire.
- Determine the dependence of the magnetic field from a thin, straight wire based on the distance from it and the current flowing in the wire.
- Sketch the magnetic field created from a thin, straight wire by using the second right-hand rule.

How much current is needed to produce a significant magnetic field, perhaps as strong as Earth's field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted in Chapter 28 that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire? We can use the Biot-Savart law to answer all of these questions, including determining the magnetic field of a long straight wire.

Figure 12.5 shows a section of an infinitely long, straight wire that carries a current $I$. What is the magnetic field at a point $P$, located a distance $R$ from the wire?

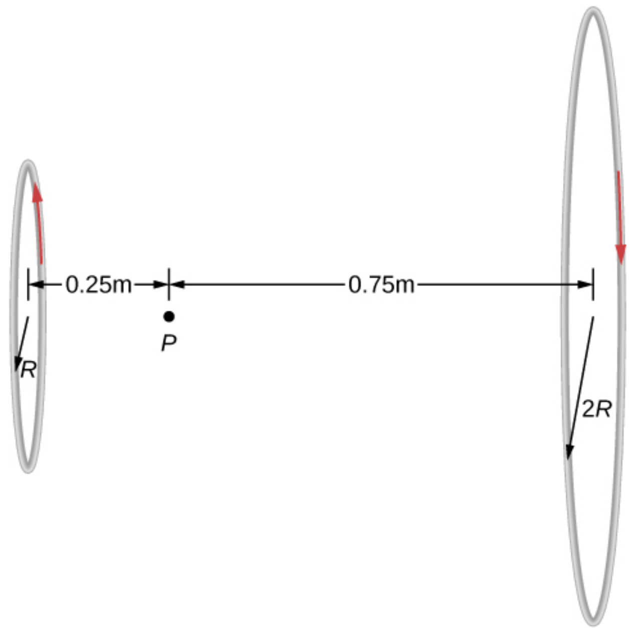


Figure 12.13 Two loops of different radii have the same current but flowing in opposite directions. The magnetic field at point $P$ is measured to be zero.

**Strategy**

The magnetic field at point $P$ has been determined in Equation 12.15. Since the currents are flowing in opposite directions, the net magnetic field is the difference between the two fields generated by the coils. Using the given quantities in the problem, the net magnetic field is then calculated.

**Solution**

Solving for the net magnetic field using Equation 12.15 and the given quantities in the problem yields

$$
\begin{aligned}
B &= \frac{\mu_0 I R_1{}^2}{2\left(y_1{}^2 + R_1{}^2\right)^{3/2}} - \frac{\mu_0 I R_2{}^2}{2\left(y_2{}^2 + R_2{}^2\right)^{3/2}} \\
B &= \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(0.010\ \mathrm{A})(0.5\ \mathrm{m})^2}{2((0.25\ \mathrm{m})^2 + (0.5\ \mathrm{m})^2)^{3/2}} - \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(0.010\ \mathrm{A})(1.0\ \mathrm{m})^2}{2((0.75\ \mathrm{m})^2 + (1.0\ \mathrm{m})^2)^{3/2}} \\
B &= 5.77 \times 10^{-9}\,\mathrm{T}\ \text{to the right.}
\end{aligned}
$$

**Significance**

Helmholtz coils typically have loops with equal radii with current flowing in the same direction to have a strong uniform field at the midpoint between the loops. A similar application of the magnetic field distribution created by Helmholtz coils is found in a magnetic bottle that can temporarily trap charged particles. See Magnetic Forces and Fields for a discussion on this.

### CHECK YOUR UNDERSTANDING 12.5

Using Example 12.5, at what distance would you have to move the first coil to have zero measurable magnetic field at point $P$?

## 12.5 Ampère's Law

### Learning Objectives

*By the end of this section, you will be able to:*

- Explain how Ampère's law relates the magnetic field produced by a current to the value of the current
- Calculate the magnetic field from a long straight wire, either thin or thick, by Ampère's law

A fundamental property of a static magnetic field is that, unlike an electrostatic field, it is not conservative. A conservative vector field is one whose line integral between two end points is the same regardless of the path chosen. Magnetic fields do not have such a property. Instead, there is a relationship between the magnetic field and its source, electric current. It is expressed in terms of the line integral of $\vec{\mathbf{B}}$ and is known as **Ampère's law**. This law can also be derived directly from the Biot-Savart law. We now consider that derivation for the special case of an infinite, straight wire.

Figure 12.14 shows an arbitrary plane perpendicular to an infinite, straight wire whose current $I$ is directed out of the page. The magnetic field lines are circles directed counterclockwise and centered on the wire. To begin, let's consider $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ over the closed paths $M$ and $N$. Notice that one path ($M$) encloses the wire, whereas the other ($N$) does not. Since the field lines are circular, $\vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ is the product of $B$ and the projection of $dl$ onto the circle passing through $d\vec{\mathbf{l}}$. If the radius of this particular circle is $r$, the projection is $rd\theta$, and

$$\vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = Br\, d\theta.$$

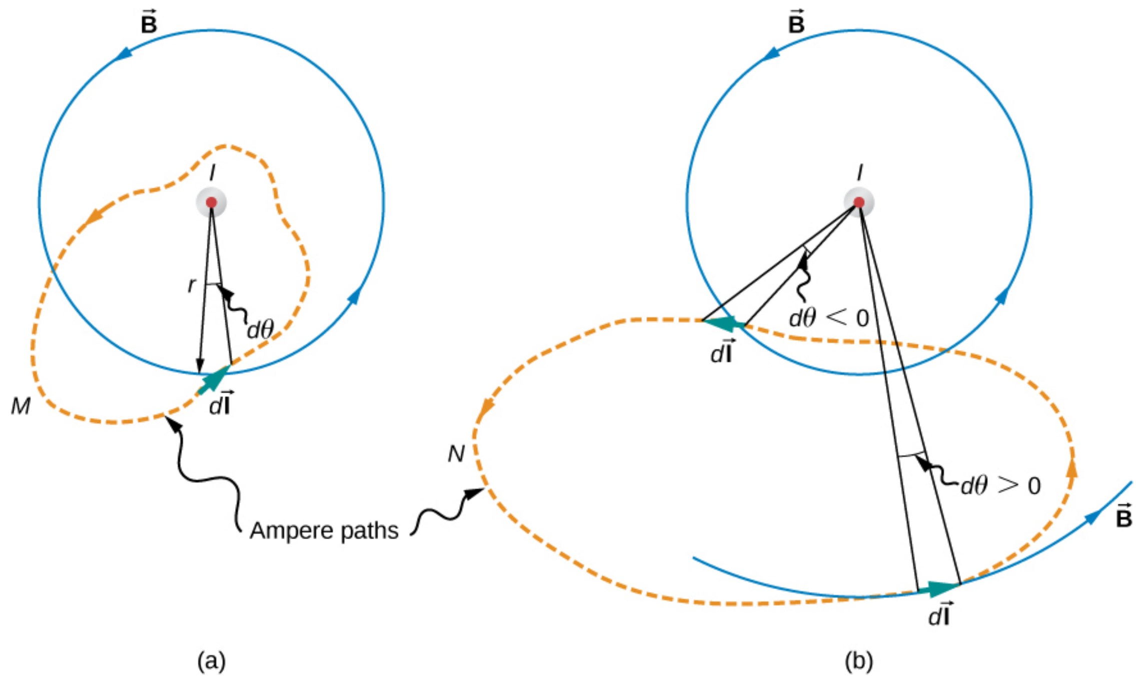


**Figure 12.14** The current $I$ of a long, straight wire is directed out of the page. The integral $\oint d\theta$ equals $2\pi$ and 0, respectively, for paths $M$ and $N$.

With $\vec{\mathbf{B}}$ given by Equation 12.9,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \oint \left(\frac{\mu_0 I}{2\pi r}\right) r\, d\theta = \frac{\mu_0 I}{2\pi} \oint d\theta. \quad \textbf{12.20}$$

For path $M$, which circulates around the wire, $\oint_M d\theta = 2\pi$ and

$$\oint_M \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I. \quad \textbf{12.21}$$

Path $N$, on the other hand, circulates through both positive (counterclockwise) and negative (clockwise) $d\theta$ (see Figure 12.14), and since it is closed, $\oint_N d\theta = 0$. Thus for path $N$,

$$\oint_N \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = 0. \quad \textbf{12.22}$$

The extension of this result to the general case is Ampère's law.

### Ampère's law

Over an arbitrary closed path,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I \tag{12.23}$$

where $I$ is the total current passing through any open surface $S$ whose perimeter is the path of integration. Only currents inside the path of integration need be considered.

To determine whether a specific current $I$ is positive or negative, curl the fingers of your right hand in the direction of the path of integration, as shown in Figure 12.14. If $I$ passes through $S$ in the same direction as your extended thumb, $I$ is positive; if $I$ passes through $S$ in the direction opposite to your extended thumb, it is negative.

## PROBLEM-SOLVING STRATEGY

### Ampère's Law

To calculate the magnetic field created from current in wire(s), use the following steps:

1. Identify the symmetry of the current in the wire(s). If there is no symmetry, use the Biot-Savart law to determine the magnetic field.
2. Determine the direction of the magnetic field created by the wire(s) by right-hand rule 2.
3. Chose a path loop where the magnetic field is either constant or zero.
4. Calculate the current inside the loop.
5. Calculate the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ around the closed loop.
6. Equate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ with $\mu_0 I_{\text{enc}}$ and solve for $\vec{\mathbf{B}}$.

## EXAMPLE 12.6

### Using Ampère's Law to Calculate the Magnetic Field Due to a Wire

Use Ampère's law to calculate the magnetic field due to a steady current $I$ in an infinitely long, thin, straight wire as shown in Figure 12.15.

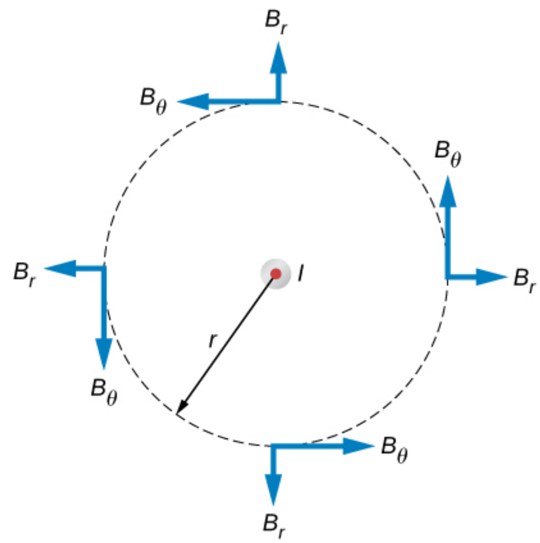


Figure 12.15 The possible components of the magnetic field $B$ due to a current $I$, which is directed out of the page. The radial component is zero because the angle between the magnetic field and the path is at a right angle.

**Strategy**

Consider an arbitrary plane perpendicular to the wire, with the current directed out of the page. The possible magnetic field components in this plane, $B_r$ and $B_\theta$, are shown at arbitrary points on a circle of radius $r$ centered on the wire. Since the field is cylindrically symmetric, neither $B_r$ nor $B_\theta$ varies with the position on this circle. Also from symmetry, the radial lines, if they exist, must be directed either all inward or all outward from the wire. This means, however, that there must be a net magnetic flux across an arbitrary cylinder concentric with the wire. The radial component of the magnetic field must be zero because $\vec{\mathbf{B}}_r \cdot d\vec{\mathbf{l}} = 0$. Therefore, we can apply Ampère's law to the circular path as shown.

**Solution**

Over this path $\vec{\mathbf{B}}$ is constant and parallel to $d\vec{\mathbf{l}}$, so

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = B_\theta \oint dl = B_\theta(2\pi r).$$

Thus Ampère's law reduces to

$$B_\theta(2\pi r) = \mu_0 I.$$

Finally, since $B_\theta$ is the only component of $\vec{\mathbf{B}}$, we can drop the subscript and write

$$B = \frac{\mu_0 I}{2\pi r}.$$

This agrees with the Biot-Savart calculation above.

**Significance**

Ampère's law works well if you have a path to integrate over which $\vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ has results that are easy to simplify. For the infinite wire, this works easily with a path that is circular around the wire so that the magnetic field factors out of the integration. If the path dependence looks complicated, you can always go back to the Biot-Savart law and use that to find the magnetic field.

## EXAMPLE 12.7

### Calculating the Magnetic Field of a Thick Wire with Ampère's Law

The radius of the long, straight wire of Figure 12.16 is *a*, and the wire carries a current $I_0$ that is distributed uniformly over its cross-section. Find the magnetic field both inside and outside the wire.

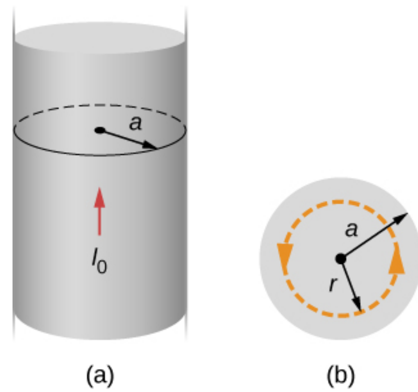


Figure 12.16 (a) A model of a current-carrying wire of radius *a* and current $I_0$. (b) A cross-section of the same wire showing the radius *a* and the Ampère's loop of radius *r*.

#### Strategy

This problem has the same geometry as Example 12.6, but the enclosed current changes as we move the integration path from outside the wire to inside the wire, where it doesn't capture the entire current enclosed (see Figure 12.16).

#### Solution

For any circular path of radius *r* that is centered on the wire,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \oint Bdl = B\oint dl = B(2\pi r).$$

From Ampère's law, this equals the total current passing through any surface bounded by the path of integration.

Consider first a circular path that is inside the wire $(r \le a)$ such as that shown in part (a) of Figure 12.16. We need the current *I* passing through the area enclosed by the path. It's equal to the current density *J* times the area enclosed. Since the current is uniform, the current density inside the path equals the current density in the whole wire, which is $I_0/\pi a^2$. Therefore the current *I* passing through the area enclosed by the path is

$$I = \frac{\pi r^2}{\pi a^2} I_0 = \frac{r^2}{a^2} I_0.$$

We can consider this ratio because the current density *J* is constant over the area of the wire. Therefore, the current density of a part of the wire is equal to the current density in the whole area. Using Ampère's law, we obtain

$$B(2\pi r) = \mu_0 \left(\frac{r^2}{a^2}\right) I_0,$$

and the magnetic field inside the wire is

$$B = \frac{\mu_0 I_0}{2\pi} \frac{r}{a^2} \ (r \le a).$$

Outside the wire, the situation is identical to that of the infinite thin wire of the previous example; that is,

$$B = \frac{\mu_0 I_0}{2\pi r} \; (r \geq a).$$

The variation of $B$ with $r$ is shown in Figure 12.17.

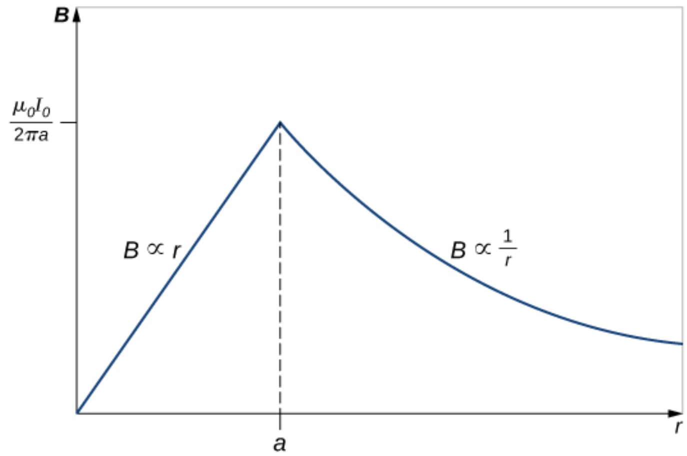


**Figure 12.17** Variation of the magnetic field produced by a current $I_0$ in a long, straight wire of radius *a*.

**Significance**

The results show that as the radial distance increases inside the thick wire, the magnetic field increases from zero to a familiar value of the magnetic field of a thin wire. Outside the wire, the field drops off regardless of whether it was a thick or thin wire.

This result is similar to how Gauss's law for electrical charges behaves inside a uniform charge distribution, except that Gauss's law for electrical charges has a uniform volume distribution of charge, whereas Ampère's law here has a uniform area of current distribution. Also, the drop-off outside the thick wire is similar to how an electric field drops off outside of a linear charge distribution, since the two cases have the same geometry and neither case depends on the configuration of charges or currents once the loop is outside the distribution.

## EXAMPLE 12.8

### Using Ampère's Law with Arbitrary Paths

Use Ampère's law to evaluate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$ for the current configurations and paths in Figure 12.18.

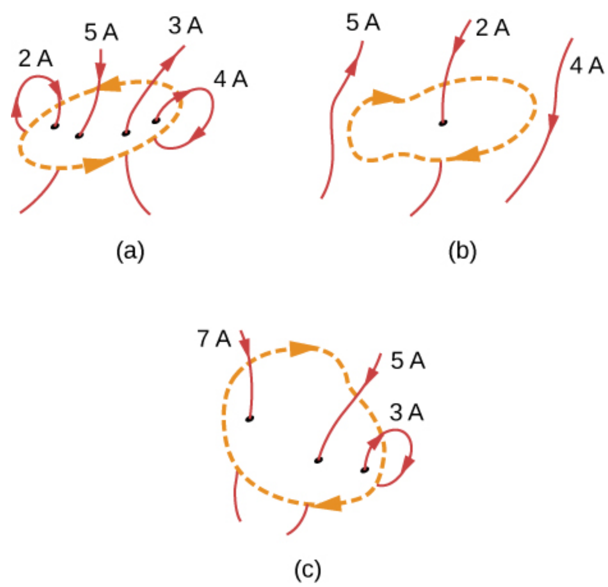


**Figure 12.18** Current configurations and paths for Example 12.8.

**Strategy**

Ampère's law states that $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I$ where $I$ is the total current passing through the enclosed loop. The quickest way to evaluate the integral is to calculate $\mu_0 I$ by finding the net current through the loop. Positive currents flow with your right-hand thumb if your fingers wrap around in the direction of the loop. This will tell us the sign of the answer.

**Solution**

(a) The current going downward through the loop equals the current going out of the loop, so the net current is zero. Thus, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = 0$.

(b) The only current to consider in this problem is 2A because it is the only current inside the loop. The right-hand rule shows us the current going downward through the loop is in the positive direction. Therefore, the answer is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 (2\text{ A}) = 2.51 \times 10^{-6} \text{T} \cdot \text{m}$.

(c) The right-hand rule shows us the current going downward through the loop is in the positive direction. There are $7\text{A} + 5\text{A} = 12\text{A}$ of current going downward and –3 A going upward. Therefore, the total current is 9 A and $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 (9\text{ A}) = 1.13 \times 10^{-5} \text{T} \cdot \text{m}$.

**Significance**

If the currents all wrapped around so that the same current went into the loop and out of the loop, the net current would be zero and no magnetic field would be present. This is why wires are very close to each other in an electrical cord. The currents flowing toward a device and away from a device in a wire equal zero total current flow through an Ampère loop around these wires. Therefore, no stray magnetic fields can be present from cords carrying current.

## CHECK YOUR UNDERSTANDING 12.6

Consider using Ampère's law to calculate the magnetic fields of a finite straight wire and of a circular loop of wire. Why is it not useful for these calculations?

$$\vec{\mathbf{B}} = \mu_0 n I \hat{\mathbf{j}}, \tag{12.28}$$

where $n$ is the number of turns per unit length. You can find the direction of $\vec{\mathbf{B}}$ with a right-hand rule: Curl your fingers in the direction of the current, and your thumb points along the magnetic field in the interior of the solenoid.

We now use these properties, along with Ampère's law, to calculate the magnitude of the magnetic field at any location inside the infinite solenoid. Consider the closed path of Figure 12.20. Along segment 1, $\vec{\mathbf{B}}$ is uniform and parallel to the path. Along segments 2 and 4, $\vec{\mathbf{B}}$ is perpendicular to part of the path and vanishes over the rest of it. Therefore, segments 2 and 4 do not contribute to the line integral in Ampère's law. Along segment 3, $\vec{\mathbf{B}} = 0$ because the magnetic field is zero outside the solenoid. If you consider an Ampère's law loop outside of the solenoid, the current flows in opposite directions on different segments of wire. Therefore, there is no enclosed current and no magnetic field according to Ampère's law. Thus, there is no contribution to the line integral from segment 3. As a result, we find

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \int_1 \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = Bl. \tag{12.29}$$

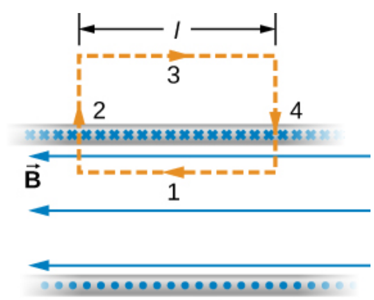


Figure 12.20 The path of integration used in Ampère's law to evaluate the magnetic field of an infinite solenoid.

The solenoid has $n$ turns per unit length, so the current that passes through the surface enclosed by the path is $nlI$. Therefore, from Ampère's law,

$$Bl = \mu_0 n l I$$

and

$$B = \mu_0 n I \tag{12.30}$$

within the solenoid. This agrees with what we found earlier for $B$ on the central axis of the solenoid. Here, however, the location of segment 1 is arbitrary, so we have found that this equation gives the magnetic field everywhere inside the infinite solenoid.

When a patient undergoes a magnetic resonance imaging (MRI) scan, the person lies down on a table that is moved into the center of a large solenoid that can generate very large magnetic fields. The solenoid is capable of these high fields from high currents flowing through superconducting wires. The large magnetic field is used to change the spin of protons in the patient's body. The time it takes for the spins to align or relax (return to original orientation) is a signature of different tissues that can be analyzed to see if the structures of the tissues is normal (Figure 12.21).

**Figure 12.21** In an MRI machine, a large magnetic field is generated by the cylindrical solenoid surrounding the patient. (credit: Liz West)

## EXAMPLE 12.9

### Magnetic Field Inside a Solenoid

A solenoid has 300 turns wound around a cylinder of diameter 1.20 cm and length 14.0 cm. If the current through the coils is 0.410 A, what is the magnitude of the magnetic field inside and near the middle of the solenoid?

**Strategy**

We are given the number of turns and the length of the solenoid so we can find the number of turns per unit length. Therefore, the magnetic field inside and near the middle of the solenoid is given by Equation 12.30. Outside the solenoid, the magnetic field is zero.

**Solution**

The number of turns per unit length is

$$n = \frac{300 \text{ turns}}{0.140 \text{ m}} = 2.14 \times 10^3 \text{turns/m}.$$

The magnetic field produced inside the solenoid is

$$\begin{aligned} B &= \mu_0 nI = (4\pi \times 10^{-7} \text{T} \cdot \text{m/A})(2.14 \times 10^3 \text{turns/m})(0.410 \text{ A}) \\ B &= 1.10 \times 10^{-3} \text{T}. \end{aligned}$$

**Significance**

This solution is valid only if the length of the solenoid is reasonably large compared with its diameter. This example is a case where this is valid.

## CHECK YOUR UNDERSTANDING 12.7

What is the ratio of the magnetic field produced from using a finite formula over the infinite approximation for an angle $\theta$ of (a) 85°? (b) 89°? The solenoid has 1000 turns in 50 cm with a current of 1.0 A flowing through the coils

## Toroids

A toroid is a donut-shaped coil closely wound with one continuous wire, as illustrated in part (a) of Figure 12.22. If the toroid has $N$ windings and the current in the wire is $I$, what is the magnetic field both inside and outside the toroid?

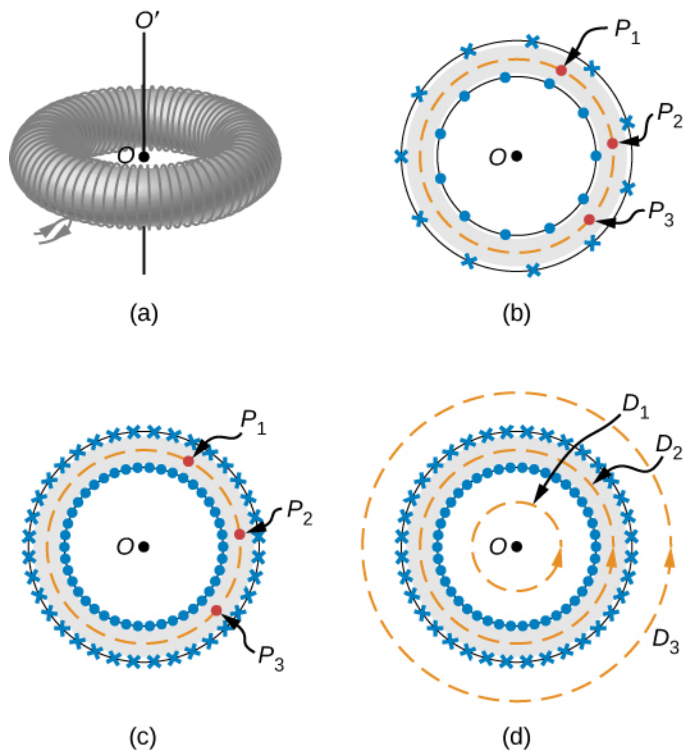


**Figure 12.22** (a) A toroid is a coil wound into a donut-shaped object. (b) A loosely wound toroid does not have cylindrical symmetry. (c) In a tightly wound toroid, cylindrical symmetry is a very good approximation. (d) Several paths of integration for Ampère's law.

We begin by assuming cylindrical symmetry around the axis *OO*′. Actually, this assumption is not precisely correct, for as part (b) of Figure 12.22 shows, the view of the toroidal coil varies from point to point (for example, $P_1$, $P_2$, and $P_3$) on a circular path centered around *OO*′. However, if the toroid is tightly wound, all points on the circle become essentially equivalent [part (c) of Figure 12.22], and cylindrical symmetry is an accurate approximation.

With this symmetry, the magnetic field must be tangent to and constant in magnitude along any circular path centered on *OO*′. This allows us to write for each of the paths $D_1$, $D_2$, and $D_3$ shown in part (d) of Figure 12.22,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = B(2\pi r). \tag{12.31}$$

Ampère's law relates this integral to the net current passing through any surface bounded by the path of integration. For a path that is external to the toroid, either no current passes through the enclosing surface (path $D_1$), or the current passing through the surface in one direction is exactly balanced by the current passing through it in the opposite direction (path $D_3$). In either case, there is no net current passing through the surface, so

$$\oint B(2\pi r) = 0$$

and

$$B = 0 \quad \text{(outside the toroid).} \tag{12.32}$$

The turns of a toroid form a helix, rather than circular loops. As a result, there is a small field external to the coil; however, the derivation above holds if the coils were circular.

For a circular path within the toroid (path $D_2$), the current in the wire cuts the surface *N* times, resulting in a net current *NI* through the surface. We now find with Ampère's law,

$$B(2\pi r) = \mu_0 N I$$

and

$$B = \frac{\mu_0 N I}{2\pi r} \quad \text{(within the toroid).} \tag{12.33}$$

The magnetic field is directed in the counterclockwise direction for the windings shown. When the current in the coils is reversed, the direction of the magnetic field also reverses.

The magnetic field inside a toroid is not uniform, as it varies inversely with the distance $r$ from the axis $OO'$. However, if the central radius $R$ (the radius midway between the inner and outer radii of the toroid) is much larger than the cross-sectional diameter of the coils $r$, the variation is fairly small, and the magnitude of the magnetic field may be calculated by Equation 12.33 where $r = R$.

# 12.7 Magnetism in Matter

## Learning Objectives

*By the end of this section, you will be able to:*

- Classify magnetic materials as paramagnetic, diamagnetic, or ferromagnetic, based on their response to a magnetic field
- Sketch how magnetic dipoles align with the magnetic field in each type of substance
- Define hysteresis and magnetic susceptibility, which determines the type of magnetic material

Why are certain materials magnetic and others not? And why do certain substances become magnetized by a field, whereas others are unaffected? To answer such questions, we need an understanding of magnetism on a microscopic level.

Within an atom, every electron travels in an orbit and spins on an internal axis. Both types of motion produce current loops and therefore magnetic dipoles. For a particular atom, the net magnetic dipole moment is the vector sum of the magnetic dipole moments. Values of $\mu$ for several types of atoms are given in Table 12.1. Notice that some atoms have a zero net dipole moment and that the magnitudes of the nonvanishing moments are typically $10^{-23}\,\text{A}\cdot\text{m}^2$.

| Atom | Magnetic Moment $\left(10^{-24}\,\text{A}\cdot\text{m}^2\right)$ |
|---|---|
| H | 9.27 |
| He | 0 |
| Li | 9.27 |
| O | 13.9 |
| Na | 9.27 |
| S | 13.9 |

**Table 12.1** Magnetic Moments of Some Atoms

A handful of matter has approximately $10^{26}$ atoms and ions, each with its magnetic dipole moment. If no external magnetic field is present, the magnetic dipoles are randomly oriented—as many are pointed up as down, as many are pointed east as west, and so on. Consequently, the net magnetic dipole moment of the sample is zero. However, if the sample is placed in a magnetic field, these dipoles tend to align with the field (see Equation 12.14), and this alignment determines how the sample responds to the field. On the basis of this response, a material is said to be either paramagnetic, ferromagnetic, or diamagnetic.

In a **paramagnetic material**, only a small fraction (roughly one-third) of the magnetic dipoles are aligned with

# CHAPTER REVIEW

## Key Terms

**Ampère's law** physical law that states that the line integral of the magnetic field around an electric current is proportional to the current

**Biot-Savart law** an equation giving the magnetic field at a point produced by a current-carrying wire

**diamagnetic materials** their magnetic dipoles align oppositely to an applied magnetic field; when the field is removed, the material is unmagnetized

**ferromagnetic materials** contain groups of dipoles, called domains, that align with the applied magnetic field; when this field is removed, the material is still magnetized

**hysteresis** property of ferromagnets that is seen when a material's magnetic field is examined versus the applied magnetic field; a loop is created resulting from sweeping the applied field forward and reverse

**magnetic domains** groups of magnetic dipoles that are all aligned in the same direction and are coupled together quantum mechanically

**magnetic susceptibility** ratio of the magnetic field in the material over the applied field at that time; positive susceptibilities are either paramagnetic or ferromagnetic (aligned with the field) and negative susceptibilities are diamagnetic (aligned oppositely with the field)

**paramagnetic materials** their magnetic dipoles align partially in the same direction as the applied magnetic field; when this field is removed, the material is unmagnetized

**permeability of free space** $\mu_0$, measure of the ability of a material, in this case free space, to support a magnetic field

**solenoid** thin wire wound into a coil that produces a magnetic field when an electric current is passed through it

**toroid** donut-shaped coil closely wound around that is one continuous wire

## Key Equations

| | |
|---|---|
| Permeability of free space | $\mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A}$ |
| Contribution to magnetic field from a current element | $dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}$ |
| Biot–Savart law | $\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}$ |
| Magnetic field due to a long straight wire | $B = \frac{\mu_0 I}{2\pi R}$ |
| Force between two parallel currents | $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$ |
| Magnetic field of a current loop | $B = \frac{\mu_0 I}{2R}$ (at center of loop) |
| Ampère's law | $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I$ |
| Magnetic field strength inside a solenoid | $B = \mu_0 n I$ |
| Magnetic field strength inside a toroid | $B = \frac{\mu_o N I}{2\pi r}$ |

| | |
|---|---|
| Magnetic permeability | $\mu = (1 + \chi)\mu_0$ |
| Magnetic field of a solenoid filled with paramagnetic material | $B = \mu n I$ |

## Summary

### 12.1 The Biot-Savart Law

- The magnetic field created by a current-carrying wire is found by the Biot-Savart law.
- The current element $Id\vec{\mathbf{l}}$ produces a magnetic field a distance $r$ away.

### 12.2 Magnetic Field Due to a Thin Straight Wire

- The strength of the magnetic field created by current in a long straight wire is given by $B = \frac{\mu_0 I}{2\pi R}$ (long straight wire) where $I$ is the current, $R$ is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7}\ \text{T} \cdot \text{m/s}$ is the permeability of free space.
- The direction of the magnetic field created by a long straight wire is given by right-hand rule 2 (RHR-2): Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops created by it.

### 12.3 Magnetic Force between Two Parallel Currents

- The force between two parallel currents $I_1$ and $I_2$, separated by a distance $r$, has a magnitude per unit length given by $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$.
- The force is attractive if the currents are in the same direction, repulsive if they are in opposite directions.

### 12.4 Magnetic Field of a Current Loop

- The magnetic field strength at the center of a circular loop is given by $B = \frac{\mu_0 I}{2R}$ (at center of loop), where $R$ is the radius of the loop. RHR-2 gives the direction of the field about the loop.

### 12.5 Ampère's Law

- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampère's law.
- Ampère's law can be used to determine the magnetic field from a thin wire or thick wire by a geometrically convenient path of integration. The results are consistent with the Biot-Savart law.

### 12.6 Solenoids and Toroids

- The magnetic field strength inside a solenoid is
$$B = \mu_0 n I \quad \text{(inside a solenoid)}$$
where $n$ is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.
- The magnetic field strength inside a toroid is
$$B = \frac{\mu_o N I}{2\pi r} \quad \text{(within the toroid)}$$
where $N$ is the number of windings. The field inside a toroid is not uniform and varies with the distance as $1/r$.

### 12.7 Magnetism in Matter

- Materials are classified as paramagnetic, diamagnetic, or ferromagnetic, depending on how they behave in an applied magnetic field.
- Paramagnetic materials have partial alignment of their magnetic dipoles with an applied magnetic field. This is a positive magnetic susceptibility. Only a surface current remains, creating a solenoid-like magnetic field.
- Diamagnetic materials exhibit induced dipoles opposite to an applied magnetic field. This is a negative magnetic susceptibility.
- Ferromagnetic materials have groups of dipoles, called domains, which align with the applied magnetic field. However, when the field is removed, the ferromagnetic material remains magnetized, unlike paramagnetic materials. This magnetization of the material versus the applied field effect is called hysteresis.