CHAPTER 11
Magnetic Forces and Fields

Figure 11.1 An industrial electromagnet is capable of lifting thousands of pounds of metallic waste. (credit: modification of work by "BedfordAl"/Flickr)

Chapter Outline

INTRODUCTION For the past few chapters, we have been studying electrostatic forces and fields, which are caused by electric charges at rest. These electric fields can move other free charges, such as producing a current in a circuit; however, the electrostatic forces and fields themselves come from other static charges. In this chapter, we see that when an electric charge moves, it generates other forces and fields. These additional forces and fields are what we commonly call magnetism.

Before we examine the origins of magnetism, we first describe what it is and how magnetic fields behave. Once we are more familiar with magnetic effects, we can explain how they arise from the behavior of atoms and molecules, and how magnetism is related to electricity. The connection between electricity and magnetism is fascinating from a theoretical point of view, but it is also immensely practical, as shown by an industrial electromagnet that can lift thousands of pounds of metal.

11.1 Magnetism and Its Historical Discoveries

Learning Objectives

By the end of this section, you will be able to:

- Explain attraction and repulsion by magnets
- Describe the historical and contemporary applications of magnetism

Magnetism has been known since the time of the ancient Greeks, but it has always been a bit mysterious. You can see electricity in the flash of a lightning bolt, but when a compass needle points to magnetic north, you can't see any force causing it to rotate. People learned about magnetic properties gradually, over many years, before several physicists of the nineteenth century connected magnetism with electricity. In this section, we review the basic ideas of magnetism and describe how they fit into the picture of a magnetic field.

Brief History of Magnetism

Magnets are commonly found in everyday objects, such as toys, hangers, elevators, doorbells, and computer devices. Experimentation on these magnets shows that all magnets have two poles: One is labeled north (N) and the other is labeled south (S). Magnetic poles repel if they are alike (both N or both S), they attract if they are opposite (one N and the other S), and both poles of a magnet attract unmagnetized pieces of iron. An important point to note here is that you cannot isolate an individual magnetic pole. Every piece of a magnet, no matter how small, which contains a north pole must also contain a south pole.

INTERACTIVE

Visit this website (https://openstax.org/l/21magnetcompass) for an interactive demonstration of magnetic north and south poles.

An example of a magnet is a compass needle. It is simply a thin bar magnet suspended at its center, so it is free to rotate in a horizontal plane. Earth itself also acts like a very large bar magnet, with its south-seeking pole near the geographic North Pole (Figure 11.2). The north pole of a compass is attracted toward Earth's geographic North Pole because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole. Confusion arises because the geographic term "North Pole" has come to be used (incorrectly) for the magnetic pole that is near the North Pole. Thus, "**north magnetic pole**" is actually a misnomer—it should be called the **south magnetic pole**. [Note that the orientation of Earth's magnetic field is not permanent but changes ("flips") after long time intervals. Eventually, Earth's north magnetic pole may be located near its geographic North Pole.]

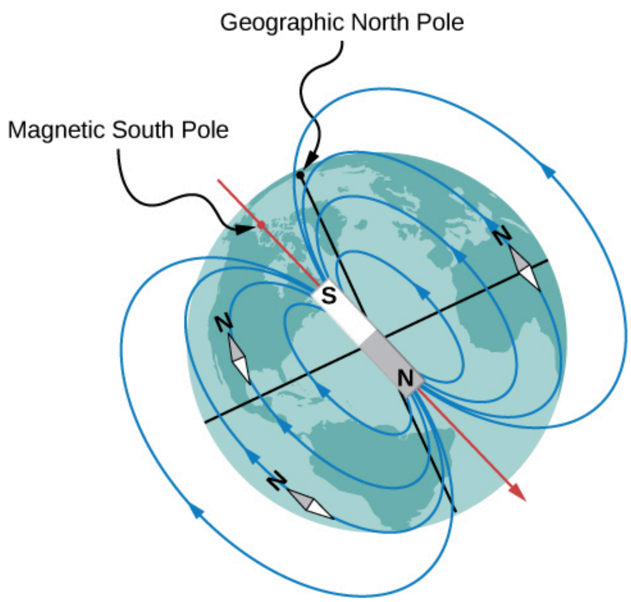

Figure 11.2 The north pole of a compass needle points toward the south pole of a magnet, which is how today's magnetic field is oriented from inside Earth. It also points toward Earth's geographic North Pole because the geographic North Pole is near the magnetic south pole.

Back in 1819, the Danish physicist Hans Oersted was performing a lecture demonstration for some students and noticed that a compass needle moved whenever current flowed in a nearby wire. Further investigation of this phenomenon convinced Oersted that an electric current could somehow cause a magnetic force. He reported this finding to an 1820 meeting of the French Academy of Science.

Soon after this report, Oersted's investigations were repeated and expanded upon by other scientists. Among those whose work was especially important were Jean-Baptiste Biot and Felix Savart, who investigated the forces exerted on magnets by currents; André Marie Ampère, who studied the forces exerted by one current on another; François Arago, who found that iron could be magnetized by a current; and Humphry Davy, who discovered that a magnet exerts a force on a wire carrying an electric current. Within 10 years of Oersted's discovery, Michael Faraday found that the relative motion of a magnet and a metallic wire induced current in the wire. This finding showed not only that a current has a magnetic effect, but that a magnet can generate electric current. You will see later that the names of Biot, Savart, Ampère, and Faraday are linked to some of the fundamental laws of electromagnetism.

The evidence from these various experiments led Ampère to propose that electric current is the source of all magnetic phenomena. To explain permanent magnets, he suggested that matter contains microscopic current loops that are somehow aligned when a material is magnetized. Today, we know that permanent magnets are actually created by the alignment of spinning electrons, a situation quite similar to that proposed by Ampère. This model of permanent magnets was developed by Ampère almost a century before the atomic nature of matter was understood. (For a full quantum mechanical treatment of magnetic spins, see Quantum Mechanics and Atomic Structure.)

Contemporary Applications of Magnetism

Today, magnetism plays many important roles in our lives. Physicists' understanding of magnetism has enabled the development of technologies that affect both individuals and society. The electronic tablet in your purse or backpack, for example, wouldn't have been possible without the applications of magnetism and electricity on a small scale (Figure 11.3). Weak changes in a magnetic field in a thin film of iron and chromium were discovered to bring about much larger changes in resistance, called giant magnetoresistance. Information can then be recorded magnetically based on the direction in which the iron layer is magnetized. As a result of the discovery of giant magnetoresistance and its applications to digital storage, the 2007 Nobel Prize in Physics was awarded to Albert Fert from France and Peter Grunberg from Germany.

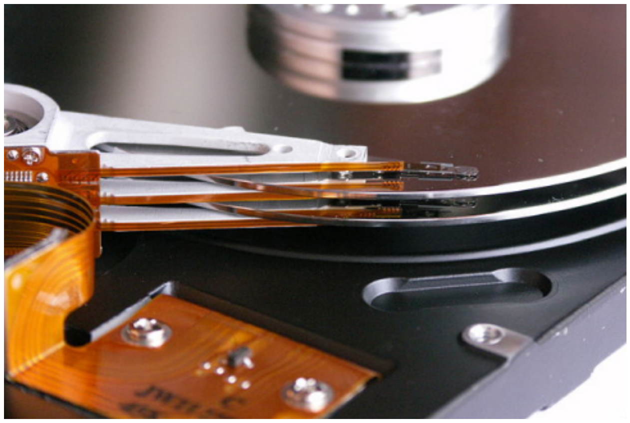

Figure 11.3 Engineering technology like computer storage would not be possible without a deep understanding of magnetism. (credit: Klaus Eifert)

All electric motors—with uses as diverse as powering refrigerators, starting cars, and moving elevators—contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Research into using magnetic containment of fusion as a future energy source has been continuing for several years. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is involved in the structure of atomic energy levels, as well as the motion of cosmic rays and charged particles trapped in the Van Allen belts around Earth. Once again, we see that all these disparate phenomena are linked by a small number of underlying physical principles.

11.2 Magnetic Fields and Lines

Learning Objectives

By the end of this section, you will be able to:

- Define the magnetic field based on a moving charge experiencing a force
- Apply the right-hand rule to determine the direction of a magnetic force based on the motion of a charge in a magnetic field
- Sketch magnetic field lines to understand which way the magnetic field points and how strong it is in a region of space

We have outlined the properties of magnets, described how they behave, and listed some of the applications of magnetic properties. Even though there are no such things as isolated magnetic charges, we can still define the attraction and repulsion of magnets as based on a field. In this section, we define the magnetic field, determine its direction based on the right-hand rule, and discuss how to draw magnetic field lines.

Defining the Magnetic Field

A magnetic field is defined by the force that a charged particle experiences moving in this field, after we account for the gravitational and any additional electric forces possible on the charge. The magnitude of this force is proportional to the amount of charge q, the speed of the charged particle v, and the magnitude of the applied magnetic field. The direction of this force is perpendicular to both the direction of the moving charged particle and the direction of the applied magnetic field. Based on these observations, we define the magnetic field strength B based on the **magnetic force** $\vec{\mathbf{F}}$ on a charge q moving at velocity $\vec{\mathbf{v}}$ as the cross product of the velocity and magnetic field, that is,

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}. \qquad 11.1$$

In fact, this is how we define the magnetic field $\vec{\mathbf{B}}$—in terms of the force on a charged particle moving in a magnetic field. The magnitude of the force is determined from the definition of the cross product as it relates to the magnitudes of each of the vectors. In other words, the magnitude of the force satisfies

$$F = qvB\sin\theta \tag{11.2}$$

where θ is the angle between the velocity and the magnetic field.

The SI unit for magnetic field strength B is called the **tesla** (T) after the eccentric but brilliant inventor Nikola Tesla (1856–1943), where

$$1\,\mathrm{T} = \frac{1\,\mathrm{N}}{\mathrm{A}\cdot\mathrm{m}}. \tag{11.3}$$

A smaller unit, called the **gauss** (G), where $1\,\mathrm{G} = 10^{-4}\,\mathrm{T}$, is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. Earth's magnetic field on its surface is only about $5 \times 10^{-5}\,\mathrm{T}$, or 0.5 G.

PROBLEM-SOLVING STRATEGY

Direction of the Magnetic Field by the Right-Hand Rule

The direction of the magnetic force $\vec{\mathbf{F}}$ is perpendicular to the plane formed by $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$, as determined by the **right-hand rule-1** (or RHR-1), which is illustrated in Figure 11.4.

1. Orient your right hand so that your fingers curl in the plane defined by the velocity and magnetic field vectors.
2. Using your right hand, sweep from the velocity toward the magnetic field with your fingers through the smallest angle possible.
3. The magnetic force is directed where your thumb is pointing.
4. If the charge was negative, reverse the direction found by these steps.

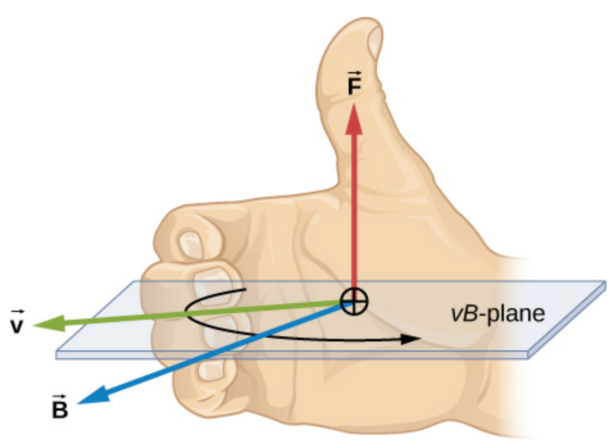

Figure 11.4 Magnetic fields exert forces on moving charges. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$ and follows the right-hand rule-1 (RHR-1) as shown. The magnitude of the force is proportional to q, v, B, and the sine of the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$.

INTERACTIVE

Visit this website (https://openstax.org/l/21magfields) for additional practice with the direction of magnetic fields.

There is no magnetic force on static charges. However, there is a magnetic force on charges moving at an angle to a magnetic field. When charges are stationary, their electric fields do not affect magnets. However, when charges move, they produce magnetic fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic forces emerges—each affects the other.

EXAMPLE 11.1

An Alpha-Particle Moving in a Magnetic Field

An alpha-particle $\left(q = 3.2 \times 10^{-19}\,\text{C}\right)$ moves through a uniform magnetic field whose magnitude is 1.5 T. The field is directly parallel to the positive z-axis of the rectangular coordinate system of Figure 11.5. What is the magnetic force on the alpha-particle when it is moving (a) in the positive x-direction with a speed of $5.0 \times 10^4\,\text{m/s}$? (b) in the negative y-direction with a speed of $5.0 \times 10^4\,\text{m/s}$? (c) in the positive z-direction with a speed of $5.0 \times 10^4\,\text{m/s}$? (d) with a velocity $\vec{\mathbf{v}} = \left(2.0\hat{\mathbf{i}} - 3.0\hat{\mathbf{j}} + 1.0\hat{\mathbf{k}}\right) \times 10^4\,\text{m/s}$?

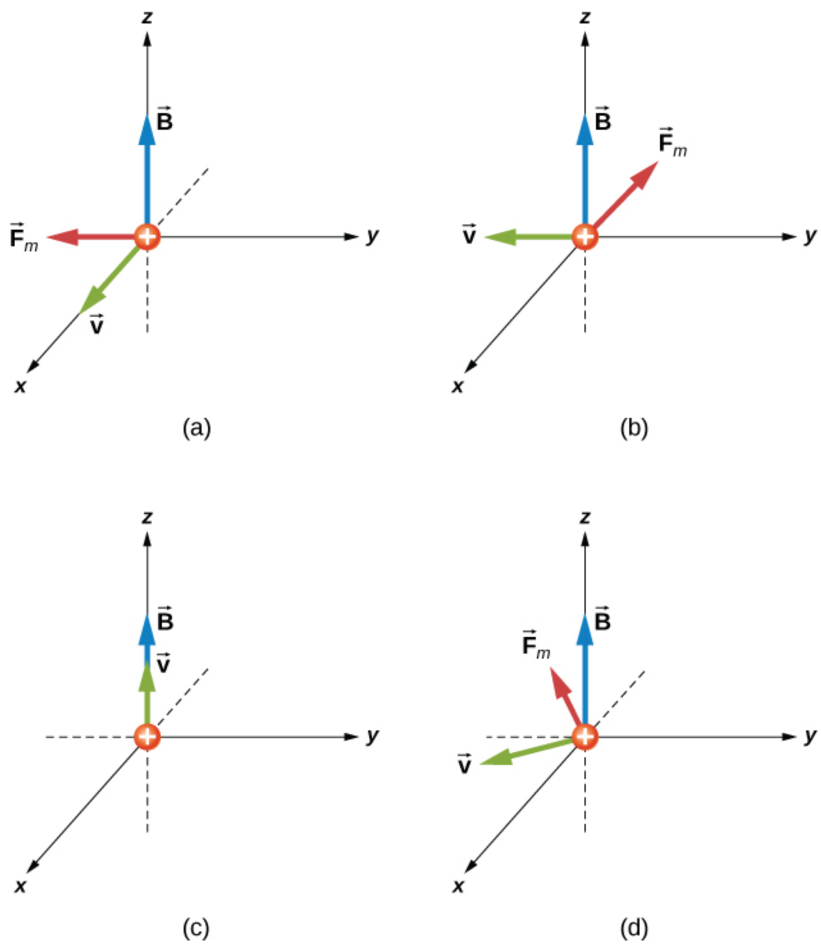

Figure 11.5 The magnetic forces on an alpha-particle moving in a uniform magnetic field. The field is the same in each drawing, but the velocity is different.

Strategy

We are given the charge, its velocity, and the magnetic field strength and direction. We can thus use the equation $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ or $F = qvB\sin\theta$ to calculate the force. The direction of the force is determined by RHR-1.

Solution

a. First, to determine the direction, start with your fingers pointing in the positive x-direction. Sweep your fingers upward in the direction of magnetic field. Your thumb should point in the negative y-direction. This should match the mathematical answer. To calculate the force, we use the given charge, velocity, and magnetic field and the definition of the magnetic force in cross-product form to calculate:

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \left(3.2 \times 10^{-19}\,\mathrm{C}\right)\left(5.0 \times 10^{4}\,\mathrm{m/s}\,\hat{\mathbf{i}}\right) \times \left(1.5\,\mathrm{T}\,\hat{\mathbf{k}}\right) = -2.4 \times 10^{-14}\,\mathrm{N}\,\hat{\mathbf{j}}.$$

b. First, to determine the directionality, start with your fingers pointing in the negative y-direction. Sweep your fingers upward in the direction of magnetic field as in the previous problem. Your thumb should be open in the negative x-direction. This should match the mathematical answer. To calculate the force, we use the given charge, velocity, and magnetic field and the definition of the magnetic force in cross-product form to calculate:

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \left(3.2 \times 10^{-19}\,\mathrm{C}\right)\left(-5.0 \times 10^{4}\,\mathrm{m/s}\,\hat{\mathbf{j}}\right) \times \left(1.5\,\mathrm{T}\,\hat{\mathbf{k}}\right) = -2.4 \times 10^{-14}\,\mathrm{N}\,\hat{\mathbf{i}}.$$

An alternative approach is to use Equation 11.2 to find the magnitude of the force. This applies for both parts (a) and (b). Since the velocity is perpendicular to the magnetic field, the angle between them is 90 degrees. Therefore, the magnitude of the force is:

$$F = qvB\sin\theta = \left(3.2 \times 10^{-19}\,\mathrm{C}\right)\left(5.0 \times 10^{4}\,\mathrm{m/s}\right)(1.5\,\mathrm{T})\sin(90°) = 2.4 \times 10^{-14}\,\mathrm{N}.$$

c. Since the velocity and magnetic field are parallel to each other, there is no orientation of your hand that will result in a force direction. Therefore, the force on this moving charge is zero. This is confirmed by the cross product. When you cross two vectors pointing in the same direction, the result is equal to zero.

d. First, to determine the direction, your fingers could point in any orientation; however, you must sweep your fingers upward in the direction of the magnetic field. As you rotate your hand, notice that the thumb can point in any x- or y-direction possible, but not in the z-direction. This should match the mathematical answer. To calculate the force, we use the given charge, velocity, and magnetic field and the definition of the magnetic force in cross-product form to calculate:

$$\begin{aligned}\vec{\mathbf{F}} &= q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \left(3.2 \times 10^{-19}\,\mathrm{C}\right)\left(\left(2.0\hat{\mathbf{i}} - 3.0\hat{\mathbf{j}} + 1.0\hat{\mathbf{k}}\right) \times 10^{4}\,\mathrm{m/s}\right) \times \left(1.5\,\mathrm{T}\,\hat{\mathbf{k}}\right)\\ &= \left(-14.4\hat{\mathbf{i}} - 9.6\hat{\mathbf{j}}\right) \times 10^{-15}\,\mathrm{N}.\end{aligned}$$

This solution can be rewritten in terms of a magnitude and angle in the xy-plane:

$$\begin{aligned}\left|\vec{\mathbf{F}}\right| &= \sqrt{F_x^2 + F_y^2} = \sqrt{(-14.4)^2 + (-9.6)^2} \times 10^{-15}\,\mathrm{N} = 1.7 \times 10^{-14}\,\mathrm{N}\\ \theta &= \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-9.6 \times 10^{-15}\,\mathrm{N}}{-14.4 \times 10^{-15}\,\mathrm{N}}\right) = 34°.\end{aligned}$$

The magnitude of the force can also be calculated using Equation 11.2. The velocity in this question, however, has three components. The z-component of the velocity can be neglected, because it is parallel to the magnetic field and therefore generates no force. The magnitude of the velocity is calculated from the x- and y-components. The angle between the velocity in the xy-plane and the magnetic field in the z-plane is 90 degrees. Therefore, the force is calculated to be:

$$\begin{aligned}\left|\vec{\mathbf{v}}\right| &= \sqrt{(2)^2 + (-3)^2} \times 10^{4}\,\tfrac{\mathrm{m}}{\mathrm{s}} = 3.6 \times 10^{4}\,\tfrac{\mathrm{m}}{\mathrm{s}}\\ F &= qvB\sin\theta = (3.2 \times 10^{-19}\,\mathrm{C})(3.6 \times 10^{4}\,\mathrm{m/s})(1.5\,\mathrm{T})\sin(90°) = 1.7 \times 10^{-14}\,\mathrm{N}.\end{aligned}$$

This is the same magnitude of force calculated by unit vectors.

Significance

The cross product in this formula results in a third vector that must be perpendicular to the other two. Other physical quantities, such as angular momentum, also have three vectors that are related by the cross product. Note that typical force values in magnetic force problems are much larger than the gravitational force. Therefore, for an isolated charge, the magnetic force is the dominant force governing the charge's motion.

CHECK YOUR UNDERSTANDING 11.1

Repeat the previous problem with the magnetic field in the x-direction rather than in the z-direction. Check your answers with RHR-1.

Representing Magnetic Fields

The representation of magnetic fields by **magnetic field lines** is very useful in visualizing the strength and direction of the magnetic field. As shown in Figure 11.6, each of these lines forms a closed loop, even if not shown by the constraints of the space available for the figure. The field lines emerge from the north pole (N), loop around to the south pole (S), and continue through the bar magnet back to the north pole.

Magnetic field lines have several hard-and-fast rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without a beginning or end. They are directed from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which generally begin on positive charges and end on negative charges or at infinity. If isolated magnetic charges (referred to as magnetic monopoles) existed, then magnetic field lines would begin and end on them.

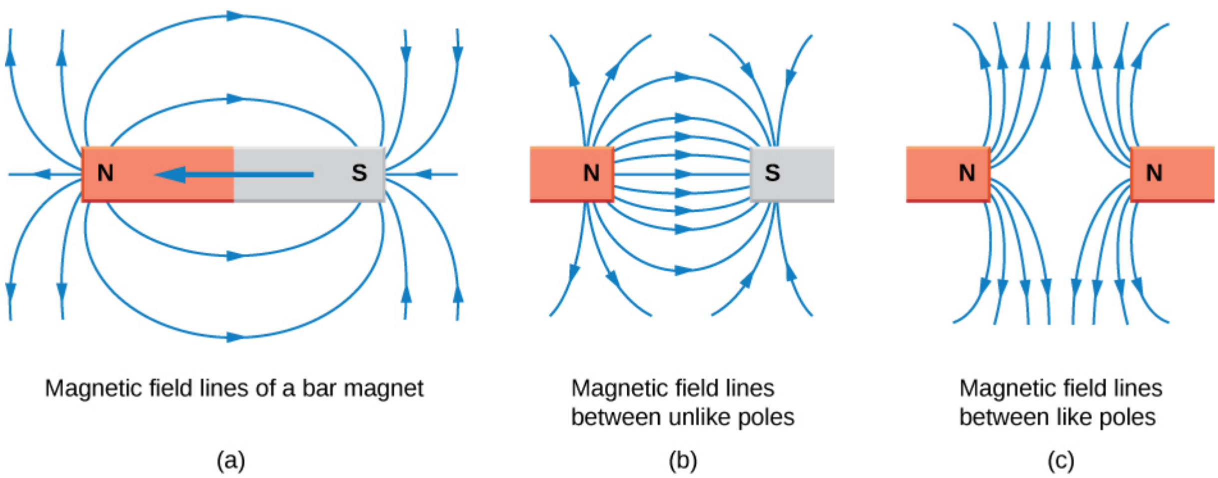

Figure 11.6 Magnetic field lines are defined to have the direction in which a small compass points when placed at a location in the field. The strength of the field is proportional to the closeness (or density) of the lines. If the interior of the magnet could be probed, the field lines would be found to form continuous, closed loops. To fit in a reasonable space, some of these drawings may not show the closing of the loops; however, if enough space were provided, the loops would be closed.

11.3 Motion of a Charged Particle in a Magnetic Field

Learning Objectives

By the end of this section, you will be able to:

- Explain how a charged particle in an external magnetic field undergoes circular motion
- Describe how to determine the radius of the circular motion of a charged particle in a magnetic field

A charged particle experiences a force when moving through a magnetic field. What happens if this field is uniform over the motion of the charged particle? What path does the particle follow? In this section, we discuss the circular motion of the charged particle as well as other motion that results from a charged particle entering a magnetic field.

The simplest case occurs when a charged particle moves perpendicular to a uniform *B*-field (Figure 11.7). If the field is in a vacuum, the magnetic field is the dominant factor determining the motion. Since the magnetic force is perpendicular to the direction of travel, a charged particle follows a curved path in a magnetic field. The particle continues to follow this curved path until it forms a complete circle. Another way to look at this is that the magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected but not the speed.

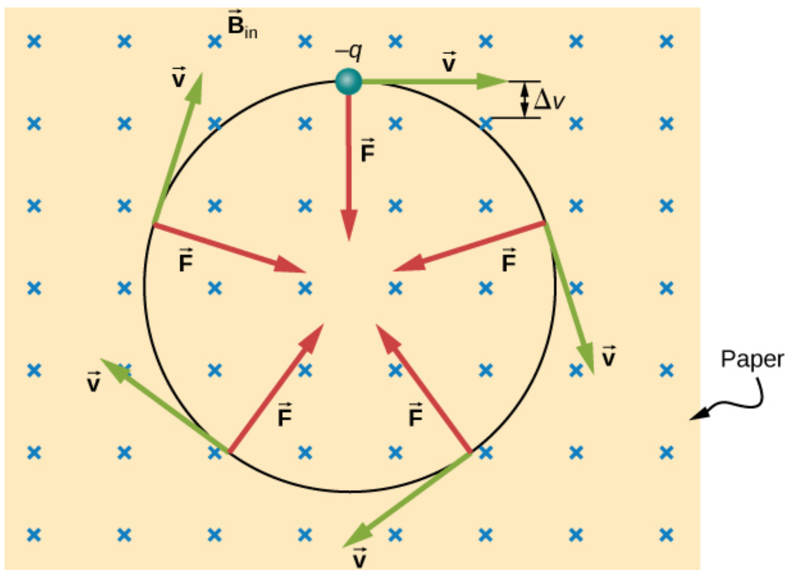

Figure 11.7 A negatively charged particle moves in the plane of the paper in a region where the magnetic field is perpendicular to the paper (represented by the small × 's—like the tails of arrows). The magnetic force is perpendicular to the velocity, so velocity changes in direction but not magnitude. The result is uniform circular motion. (Note that because the charge is negative, the force is opposite in direction to the prediction of the right-hand rule.)

In this situation, the magnetic force supplies the centripetal force $F_c = \frac{mv^2}{r}$. Noting that the velocity is perpendicular to the magnetic field, the magnitude of the magnetic force is reduced to $F = qvB$. Because the magnetic force F supplies the centripetal force F_c, we have

$$qvB = \frac{mv^2}{r}. \quad (11.4)$$

Solving for r yields

$$r = \frac{mv}{qB}. \quad (11.5)$$

Here, r is the radius of curvature of the path of a charged particle with mass m and charge q, moving at a speed v that is perpendicular to a magnetic field of strength B. The time for the charged particle to go around the circular path is defined as the period, which is the same as the distance traveled (the circumference) divided by the speed. Based on this and Equation 11.4, we can derive the period of motion as

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}. \quad (11.6)$$

If the velocity is not perpendicular to the magnetic field, then we can compare each component of the velocity separately with the magnetic field. The component of the velocity perpendicular to the magnetic field produces a magnetic force perpendicular to both this velocity and the field:

$$v_{perp} = v\sin\theta, \quad v_{para} = v\cos\theta. \quad (11.7)$$

where θ is the angle between v and B. The component parallel to the magnetic field creates constant motion along the same direction as the magnetic field, also shown in Equation 11.7. The parallel motion determines the *pitch* p of the helix, which is the distance between adjacent turns. This distance equals the parallel component of the velocity times the period:

$$p = v_{\text{para}} T. \tag{11.8}$$

The result is a **helical motion**, as shown in the following figure.

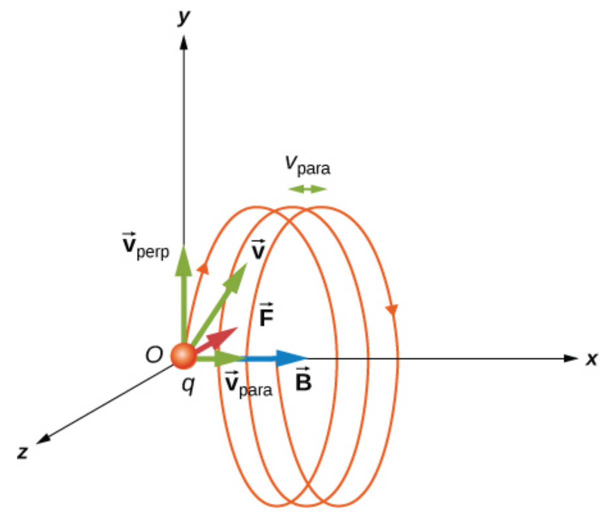

Figure 11.8 A charged particle moving with a velocity not in the same direction as the magnetic field. The velocity component perpendicular to the magnetic field creates circular motion, whereas the component of the velocity parallel to the field moves the particle along a straight line. The pitch is the horizontal distance between two consecutive circles. The resulting motion is helical.

While the charged particle travels in a helical path, it may enter a region where the magnetic field is not uniform. In particular, suppose a particle travels from a region of strong magnetic field to a region of weaker field, then back to a region of stronger field. The particle may reflect back before entering the stronger magnetic field region. This is similar to a wave on a string traveling from a very light, thin string to a hard wall and reflecting backward. If the reflection happens at both ends, the particle is trapped in a so-called magnetic bottle.

Trapped particles in magnetic fields are found in the Van Allen radiation belts around Earth, which are part of Earth's magnetic field. These belts were discovered by James Van Allen while trying to measure the flux of **cosmic rays** on Earth (high-energy particles that come from outside the solar system) to see whether this was similar to the flux measured on Earth. Van Allen found that due to the contribution of particles trapped in Earth's magnetic field, the flux was much higher on Earth than in outer space. Aurorae, like the famous aurora borealis (northern lights) in the Northern Hemisphere (Figure 11.9), are beautiful displays of light emitted as ions recombine with electrons entering the atmosphere as they spiral along magnetic field lines. (The ions are primarily oxygen and nitrogen atoms that are initially ionized by collisions with energetic particles in Earth's atmosphere.) Aurorae have also been observed on other planets, such as Jupiter and Saturn.

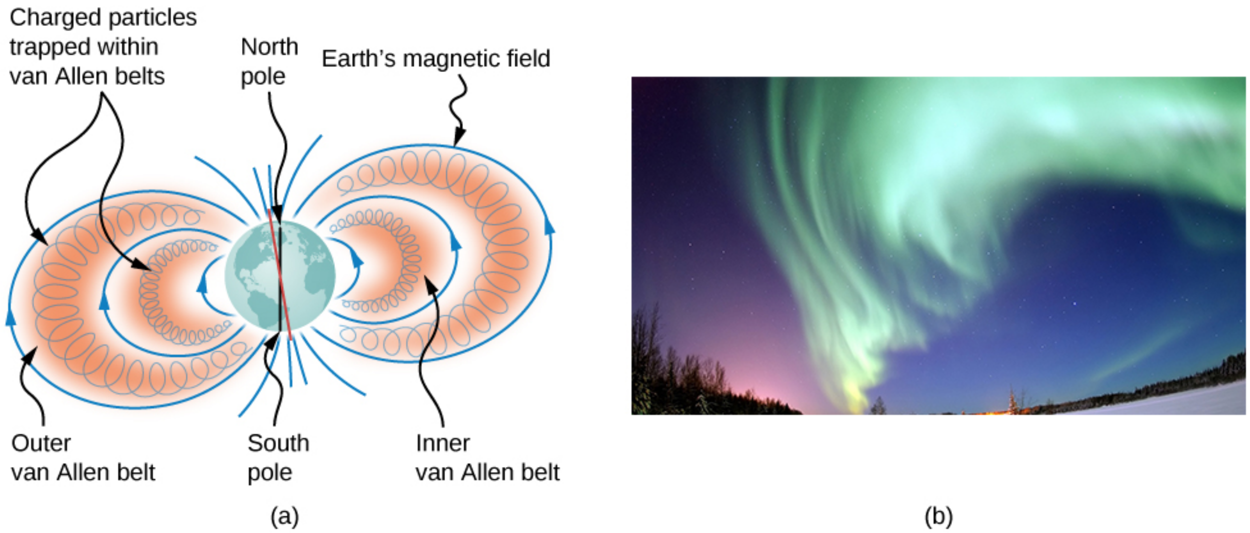

Figure 11.9 (a) The Van Allen radiation belts around Earth trap ions produced by cosmic rays striking Earth's atmosphere. (b) The magnificent spectacle of the aurora borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by Earth's magnetic field, this light is produced by glowing molecules and ions of oxygen and nitrogen. (credit b: modification of work by USAF Senior Airman Joshua Strang)

EXAMPLE 11.2

Beam Deflector

A research group is investigating short-lived radioactive isotopes. They need to design a way to transport alpha-particles (helium nuclei) from where they are made to a place where they will collide with another material to form an isotope. The beam of alpha-particles $\left(m = 6.64 \times 10^{-27}\,\text{kg},\, q = 3.2 \times 10^{-19}\,\text{C}\right)$ bends through a 90-degree region with a uniform magnetic field of 0.050 T (Figure 11.10). (a) In what direction should the magnetic field be applied? (b) How much time does it take the alpha-particles to traverse the uniform magnetic field region?

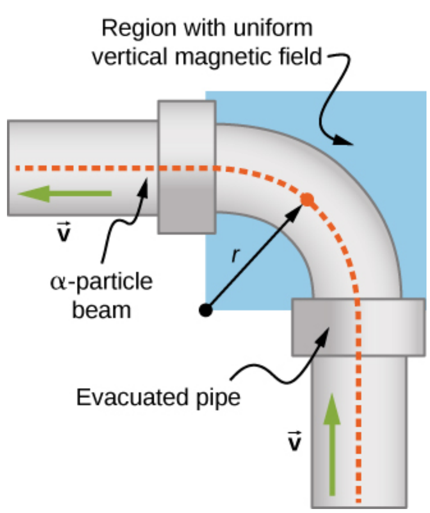

Figure 11.10 Top view of the beam deflector setup.

Strategy

a. The direction of the magnetic field is shown by the RHR-1. Your fingers point in the direction of v, and your thumb needs to point in the direction of the force, to the left. Therefore, since the alpha-particles are positively charged, the magnetic field must point down.

b. The period of the alpha-particle going around the circle is

$$T = \frac{2\pi m}{qB}. \quad 11.9$$

Because the particle is only going around a quarter of a circle, we can take 0.25 times the period to find the time it takes to go around this path.

Solution

a. Let's start by focusing on the alpha-particle entering the field near the bottom of the picture. First, point your thumb up the page. In order for your palm to open to the left where the centripetal force (and hence the magnetic force) points, your fingers need to change orientation until they point into the page. This is the direction of the applied magnetic field.
b. The period of the charged particle going around a circle is calculated by using the given mass, charge, and magnetic field in the problem. This works out to be
$$T = \frac{2\pi m}{qB} = \frac{2\pi\left(6.64 \times 10^{-27}\,\text{kg}\right)}{\left(3.2 \times 10^{-19}\,\text{C}\right)(0.050\,\text{T})} = 2.6 \times 10^{-6}\,\text{s}.$$
However, for the given problem, the alpha-particle goes around a quarter of the circle, so the time it takes would be
$$t = 0.25 \times 2.61 \times 10^{-6}\,\text{s} = 6.5 \times 10^{-7}\,\text{s}.$$

Significance

This time may be quick enough to get to the material we would like to bombard, depending on how short-lived the radioactive isotope is and continues to emit alpha-particles. If we could increase the magnetic field applied in the region, this would shorten the time even more. The path the particles need to take could be shortened, but this may not be economical given the experimental setup.

CHECK YOUR UNDERSTANDING 11.2

A uniform magnetic field of magnitude 1.5 T is directed horizontally from west to east. (a) What is the magnetic force on a proton at the instant when it is moving vertically downward in the field with a speed of 4×10^7 m/s? (b) Compare this force with the weight w of a proton.

EXAMPLE 11.3

Helical Motion in a Magnetic Field

A proton enters a uniform magnetic field of $1.0 \times 10^{-4}\,\text{T}$ with a speed of 5×10^5 m/s. At what angle must the magnetic field be from the velocity so that the pitch of the resulting helical motion is equal to the radius of the helix?

Strategy

The pitch of the motion relates to the parallel velocity times the period of the circular motion, whereas the radius relates to the perpendicular velocity component. After setting the radius and the pitch equal to each other, solve for the angle between the magnetic field and velocity or θ.

Solution

The pitch is given by Equation 11.8, the period is given by Equation 11.6, and the radius of circular motion is given by Equation 11.5. Note that the velocity in the radius equation is related to only the perpendicular velocity, which is where the circular motion occurs. Therefore, we substitute the sine component of the overall velocity into the radius equation to equate the pitch and radius:

$$\begin{aligned} p &= r \\ v_{\parallel} T &= \frac{m v_{\perp}}{qB} \\ v \cos\theta \frac{2\pi m}{qB} &= \frac{m v \sin\theta}{qB} \\ 2\pi &= \tan\theta \\ \theta &= 81.0°. \end{aligned}$$

Significance

If this angle were $0°$, only parallel velocity would occur and the helix would not form, because there would be no circular motion in the perpendicular plane. If this angle were $90°$, only circular motion would occur and there would be no movement of the circles perpendicular to the motion. That is what creates the helical motion.

11.4 Magnetic Force on a Current-Carrying Conductor

Learning Objectives

By the end of this section, you will be able to:

- Determine the direction in which a current-carrying wire experiences a force in an external magnetic field
- Calculate the force on a current-carrying wire in an external magnetic field

Moving charges experience a force in a magnetic field. If these moving charges are in a wire—that is, if the wire is carrying a current—the wire should also experience a force. However, before we discuss the force exerted on a current by a magnetic field, we first examine the magnetic field generated by an electric current. We are studying two separate effects here that interact closely: A current-carrying wire generates a magnetic field and the magnetic field exerts a force on the current-carrying wire.

Magnetic Fields Produced by Electrical Currents

When discussing historical discoveries in magnetism, we mentioned Oersted's finding that a wire carrying an electrical current caused a nearby compass to deflect. A connection was established that electrical currents produce magnetic fields. (This connection between electricity and magnetism is discussed in more detail in Sources of Magnetic Fields.)

The compass needle near the wire experiences a force that aligns the needle tangent to a circle around the wire. Therefore, a current-carrying wire produces circular loops of magnetic field. To determine the direction of the magnetic field generated from a wire, we use a second right-hand rule. In RHR-2, your thumb points in the direction of the current while your fingers wrap around the wire, pointing in the direction of the magnetic field produced (Figure 11.11). If the magnetic field were coming at you or out of the page, we represent this with a dot. If the magnetic field were going into the page, we represent this with an $\times$. These symbols come from considering a vector arrow: An arrow pointed toward you, from your perspective, would look like a dot or the tip of an arrow. An arrow pointed away from you, from your perspective, would look like a cross or an $\times$. A composite sketch of the magnetic circles is shown in Figure 11.11, where the field strength is shown to decrease as you get farther from the wire by loops that are farther separated.

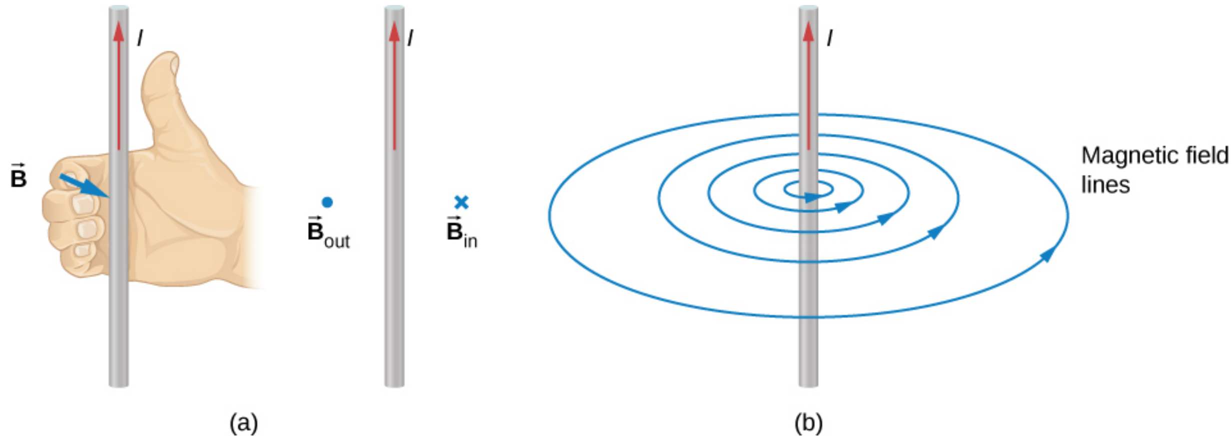

Figure 11.11 (a) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow). (b) A long and straight wire creates a field with magnetic field lines forming circular loops.

Calculating the Magnetic Force

Electric current is an ordered movement of charge. A current-carrying wire in a magnetic field must therefore experience a force due to the field. To investigate this force, let's consider the infinitesimal section of wire as shown in Figure 11.12. The length and cross-sectional area of the section are dl and A, respectively, so its volume is $V = A \cdot dl$. The wire is formed from material that contains n charge carriers per unit volume, so the number of charge carriers in the section is $nA \cdot dl$. If the charge carriers move with drift velocity $\vec{\mathbf{v}}_{\mathrm{d}}$, the current I in the wire is (from Current and Resistance)

$$I = neAv_d.$$

The magnetic force on any single charge carrier is $e\vec{\mathbf{v}}_{\mathrm{d}} \times \vec{\mathbf{B}}$, so the total magnetic force $d\vec{\mathbf{F}}$ on the $nA \cdot dl$ charge carriers in the section of wire is

$$d\vec{\mathbf{F}} = (nA \cdot dl)e\vec{\mathbf{v}}_{\mathrm{d}} \times \vec{\mathbf{B}}. \qquad 11.10$$

We can define dl to be a vector of length dl pointing along $\vec{\mathbf{v}}_{\mathrm{d}}$, which allows us to rewrite this equation as

$$d\vec{\mathbf{F}} = neAv_{\mathrm{d}}\vec{\mathbf{dl}} \times \vec{\mathbf{B}}, \qquad 11.11$$

or

$$d\vec{\mathbf{F}} = I\vec{\mathbf{dl}} \times \vec{\mathbf{B}}. \qquad 11.12$$

This is the magnetic force on the section of wire. Note that it is actually the net force exerted by the field on the charge carriers themselves. The direction of this force is given by RHR-1, where you point your fingers in the direction of the current and curl them toward the field. Your thumb then points in the direction of the force.

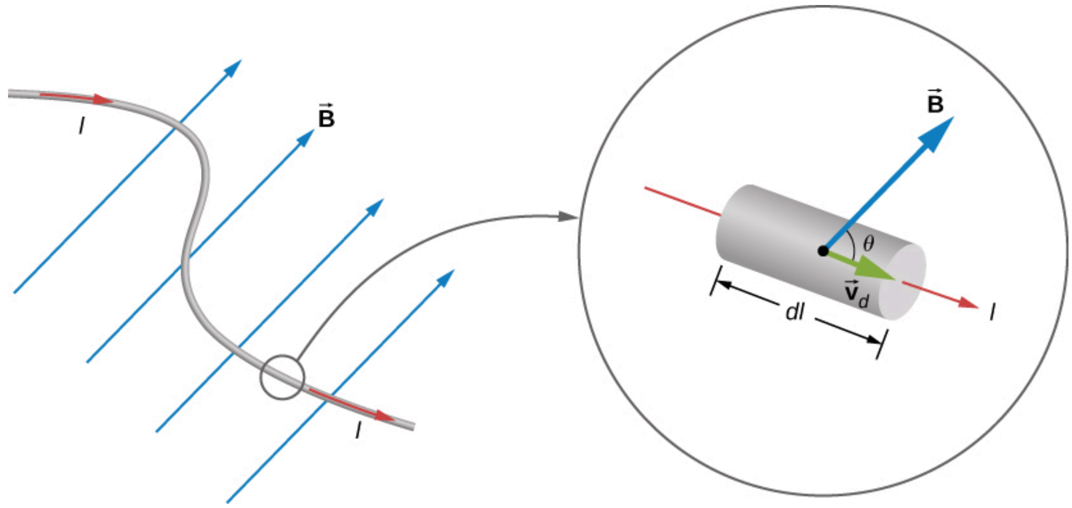

Figure 11.12 An infinitesimal section of current-carrying wire in a magnetic field.

To determine the magnetic force $\vec{\mathbf{F}}$ on a wire of arbitrary length and shape, we must integrate Equation 11.12 over the entire wire. If the wire section happens to be straight and B is uniform, the equation differentials become absolute quantities, giving us

$$\vec{\mathbf{F}} = I\vec{\mathbf{l}} \times \vec{\mathbf{B}}. \tag{11.13}$$

This is the force on a straight, current-carrying wire in a uniform magnetic field.

EXAMPLE 11.4

Balancing the Gravitational and Magnetic Forces on a Current-Carrying Wire

A wire of length 50 cm and mass 10 g is suspended in a horizontal plane by a pair of flexible leads (Figure 11.13). The wire is then subjected to a constant magnetic field of magnitude 0.50 T, which is directed as shown. What are the magnitude and direction of the current in the wire needed to remove the tension in the supporting leads?

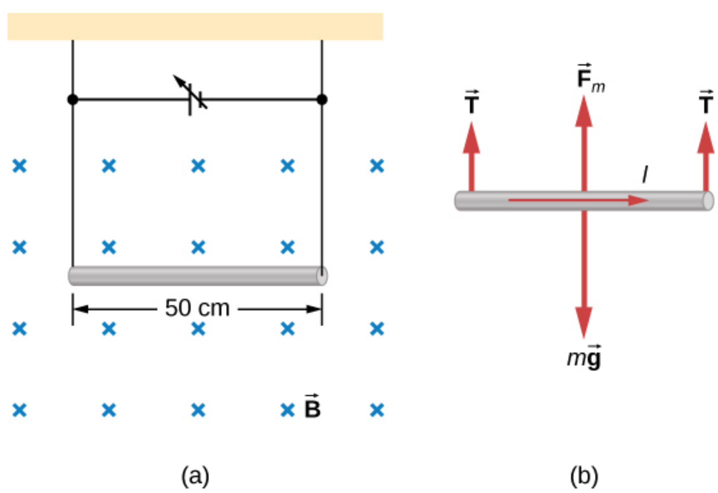

Figure 11.13 (a) A wire suspended in a magnetic field. (b) The free-body diagram for the wire.

Strategy

From the free-body diagram in the figure, the tensions in the supporting leads go to zero when the gravitational and magnetic forces balance each other. Using the RHR-1, we find that the magnetic force points up. We can then determine the current I by equating the two forces.

Solution

Equate the two forces of weight and magnetic force on the wire:

$$mg = IlB.$$

Thus,

$$I = \frac{mg}{lB} = \frac{(0.010\text{ kg})(9.8\text{ m/s}^2)}{(0.50\text{ m})(0.50\text{ T})} = 0.39\text{ A}.$$

Significance

This large magnetic field creates a significant force on a length of wire to counteract the weight of the wire.

EXAMPLE 11.5

Calculating Magnetic Force on a Current-Carrying Wire

A long, rigid wire lying along the y-axis carries a 5.0-A current flowing in the positive y-direction. (a) If a constant magnetic field of magnitude 0.30 T is directed along the positive x-axis, what is the magnetic force per unit length on the wire? (b) If a constant magnetic field of 0.30 T is directed 30 degrees from the $+x$-axis towards the $+y$-axis, what is the magnetic force per unit length on the wire?

Strategy

The magnetic force on a current-carrying wire in a magnetic field is given by $\vec{\mathbf{F}} = I\vec{\mathbf{l}} \times \vec{\mathbf{B}}$. For part a, since the current and magnetic field are perpendicular in this problem, we can simplify the formula to give us the magnitude and find the direction through the RHR-1. The angle θ is 90 degrees, which means $\sin\theta = 1$. Also, the length can be divided over to the left-hand side to find the force per unit length. For part b, the current times length is written in unit vector notation, as well as the magnetic field. After the cross product is taken, the directionality is evident by the resulting unit vector.

Solution

a. We start with the general formula for the magnetic force on a wire. We are looking for the force per unit length, so we divide by the length to bring it to the left-hand side. We also set $\sin\theta = 1$. The solution therefore is

$$\begin{aligned} F &= IlB\sin\theta \\ \frac{F}{l} &= (5.0\text{ A})(0.30\text{ T}) \\ \frac{F}{l} &= 1.5\text{ N/m}. \end{aligned}$$

Directionality: Point your fingers in the positive y-direction and curl your fingers in the positive x-direction. Your thumb will point in the $-\vec{\mathbf{k}}$ direction. Therefore, with directionality, the solution is

$$\frac{\vec{\mathbf{F}}}{l} = -1.5\vec{\mathbf{k}}\text{ N/m}.$$

b. The current times length and the magnetic field are written in unit vector notation. Then, we take the cross product to find the force:

$$\begin{aligned} \vec{\mathbf{F}} &= I\vec{\mathbf{l}} \times \vec{\mathbf{B}} = (5.0A)\,l\hat{\mathbf{j}} \times \left(0.30T\cos(30°)\,\hat{\mathbf{i}} + 0.30T\sin(30°)\,\hat{\mathbf{j}}\right) \\ \vec{\mathbf{F}}/l &= -1.30\hat{\mathbf{k}}\text{ N/m}. \end{aligned}$$

Significance

This large magnetic field creates a significant force on a small length of wire. As the angle of the magnetic field becomes more closely aligned to the current in the wire, there is less of a force on it, as seen from comparing parts a and b.

CHECK YOUR UNDERSTANDING 11.3

A straight, flexible length of copper wire is immersed in a magnetic field that is directed into the page. (a) If the wire's current runs in the $+x$-direction, which way will the wire bend? (b) Which way will the wire bend if the current runs in the $-x$-direction?

EXAMPLE 11.6

Force on a Circular Wire

A circular current loop of radius R carrying a current I is placed in the xy-plane. A constant uniform magnetic field cuts through the loop parallel to the y-axis (Figure 11.14). Find the magnetic force on the upper half of the loop, the lower half of the loop, and the total force on the loop.

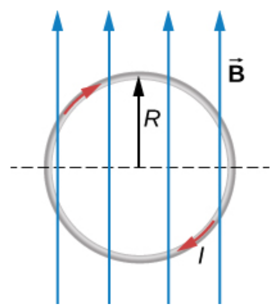

Figure 11.14 A loop of wire carrying a current in a magnetic field.

Strategy

The magnetic force on the upper loop should be written in terms of the differential force acting on each segment of the loop. If we integrate over each differential piece, we solve for the overall force on that section of the loop. The force on the lower loop is found in a similar manner, and the total force is the addition of these two forces.

Solution

A differential force on an arbitrary piece of wire located on the upper ring is:

$$dF = IB\sin\theta\, dl.$$

where θ is the angle between the magnetic field direction ($+y$) and the segment of wire. A differential segment is located at the same radius, so using an arc-length formula, we have:

$$\begin{aligned} dl &= R\,d\theta \\ dF &= IBR\sin\theta\, d\theta. \end{aligned}$$

In order to find the force on a segment, we integrate over the upper half of the circle, from 0 to π. This results in:

$$F = IBR\int_0^{\pi} \sin\theta\, d\theta = IBR(-\cos\pi + \cos 0) = 2IBR.$$

The lower half of the loop is integrated from π to zero, giving us:

$$F = IBR\int_{\pi}^{0} \sin\theta\, d\theta = IBR(-\cos 0 + \cos\pi) = -2IBR.$$

The net force is the sum of these forces, which is zero.

Significance

The total force on any closed loop in a uniform magnetic field is zero. Even though each piece of the loop has a force acting on it, the net force on the system is zero. (Note that there is a net torque on the loop, which we consider in the next section.)

11.5 Force and Torque on a Current Loop

Learning Objectives

By the end of this section, you will be able to:

- Evaluate the net force on a current loop in an external magnetic field
- Evaluate the net torque on a current loop in an external magnetic field
- Define the magnetic dipole moment of a current loop

Motors are the most common application of magnetic force on current-carrying wires. Motors contain loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted into mechanical work in the process. Once the loop's surface area is aligned with the magnetic field, the direction of current is reversed, so there is a continual torque on the loop (Figure 11.15). This reversal of the current is done with commutators and brushes. The commutator is set to reverse the current flow at set points to keep continual motion in the motor. A basic commutator has three contact areas to avoid and dead spots where the loop would have zero instantaneous torque at that point. The brushes press against the commutator, creating electrical contact between parts of the commutator during the spinning motion.

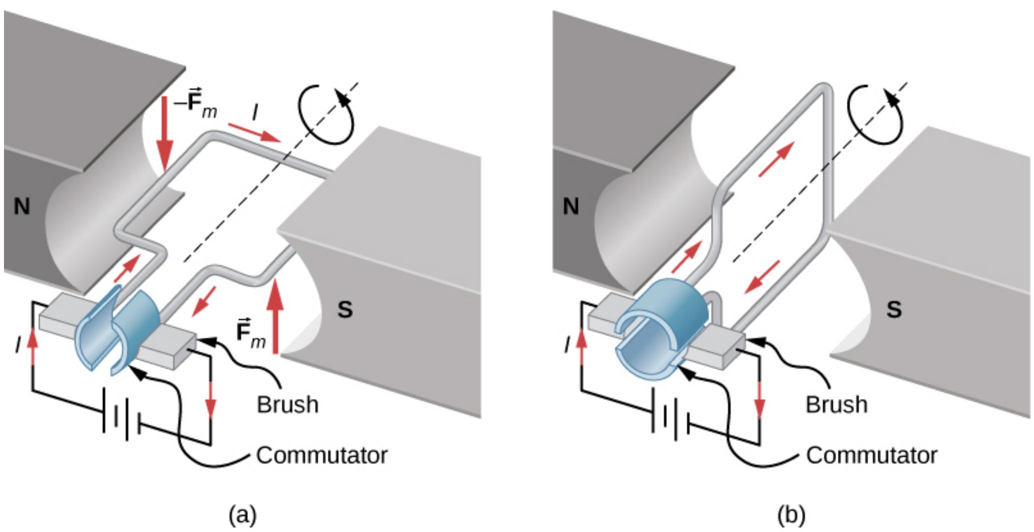

Figure 11.15 A simplified version of a dc electric motor. (a) The rectangular wire loop is placed in a magnetic field. The forces on the wires closest to the magnetic poles (N and S) are opposite in direction as determined by the right-hand rule-1. Therefore, the loop has a net torque and rotates to the position shown in (b). (b) The brushes now touch the commutator segments so that no current flows through the loop. No torque acts on the loop, but the loop continues to spin from the initial velocity given to it in part (a). By the time the loop flips over, current flows through the wires again but now in the opposite direction, and the process repeats as in part (a). This causes continual rotation of the loop.

In a uniform magnetic field, a current-carrying loop of wire, such as a loop in a motor, experiences both forces and torques on the loop. Figure 11.16 shows a rectangular loop of wire that carries a current I and has sides of lengths *a* and *b*. The loop is in a uniform magnetic field: $\vec{\mathbf{B}} = B\hat{\mathbf{j}}$. The magnetic force on a straight current-carrying wire of length l is given by $I\vec{\mathbf{l}} \times \vec{\mathbf{B}}$. To find the net force on the loop, we have to apply this equation to each of the four sides. The force on side 1 is

$$\vec{\mathbf{F}}_1 = IaB\sin(90° - \theta)\hat{\mathbf{i}} = IaB\cos\theta\hat{\mathbf{i}} \quad \mathbf{11.14}$$

where the direction has been determined with the RHR-1. The current in side 3 flows in the opposite direction to that of side 1, so

$$\vec{\mathbf{F}}_3 = -IaB\sin(90° + \theta)\hat{\mathbf{i}} = -IaB\cos\theta\hat{\mathbf{i}}. \quad \text{11.15}$$

The currents in sides 2 and 4 are perpendicular to $\vec{\mathbf{B}}$ and the forces on these sides are

$$\vec{\mathbf{F}}_2 = IbB\hat{\mathbf{k}}, \quad \vec{\mathbf{F}}_4 = -IbB\hat{\mathbf{k}}. \quad \text{11.16}$$

We can now find the net force on the loop:

$$\sum \vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \vec{\mathbf{F}}_4 = 0. \quad \text{11.17}$$

Although this result ($\Sigma F = 0$) has been obtained for a rectangular loop, it is far more general and holds for current-carrying loops of arbitrary shapes; that is, there is no net force on a current loop in a uniform magnetic field.

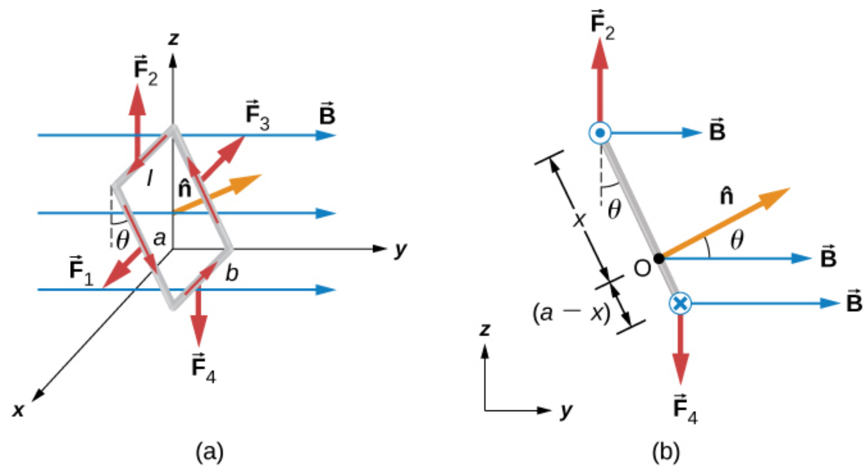

Figure 11.16 (a) A rectangular current loop in a uniform magnetic field is subjected to a net torque but not a net force. (b) A side view of the coil.

To find the net torque on the current loop shown in Figure 11.16, we first consider F_1 and F_3. Since they have the same line of action and are equal and opposite, the sum of their torques about any axis is zero (see Fixed-Axis Rotation). Thus, if there is any torque on the loop, it must be furnished by F_2 and F_4. Let's calculate the torques around the axis that passes through point O of Figure 11.16 (a side view of the coil) and is perpendicular to the plane of the page. The point O is a distance x from side 2 and a distance $(a - x)$ from side 4 of the loop. The moment arms of F_2 and F_4 are $x \sin\theta$ and $(a - x)\sin\theta$, respectively, so the net torque on the loop is

$$\begin{aligned}\sum \vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\tau}}_1 + \vec{\boldsymbol{\tau}}_2 + \vec{\boldsymbol{\tau}}_3 + \vec{\boldsymbol{\tau}}_4 &= F_2 x\sin\theta\hat{\mathbf{i}} - F_4(a - x)\sin(\theta)\hat{\mathbf{i}} \\ &= -IbBx\sin\theta\hat{\mathbf{i}} - IbB(a - x)\sin\theta\hat{\mathbf{i}}.\end{aligned} \quad \text{11.18}$$

This simplifies to

$$\vec{\boldsymbol{\tau}} = -IAB\sin\theta\hat{\mathbf{i}} \quad \text{11.19}$$

where $A = ab$ is the area of the loop.

Notice that this torque is independent of x; it is therefore independent of where point O is located in the plane of the current loop. Consequently, the loop experiences the same torque from the magnetic field about any axis in the plane of the loop and parallel to the x-axis.

A closed-current loop is commonly referred to as a **magnetic dipole** and the term IA is known as its **magnetic dipole moment** μ. Actually, the magnetic dipole moment is a vector that is defined as

$$\vec{\boldsymbol{\mu}} = IA\hat{\mathbf{n}} \quad \text{11.20}$$

where $\hat{\mathbf{n}}$ is a unit vector directed perpendicular to the plane of the loop (see Figure 11.16). The direction of $\hat{\mathbf{n}}$ is obtained with the RHR-2—if you curl the fingers of your right hand in the direction of current flow in the loop, then your thumb points along $\hat{\mathbf{n}}$. If the loop contains N turns of wire, then its magnetic dipole moment is given by

$$\vec{\mathbf{\mu}} = NIA\hat{\mathbf{n}}. \tag{11.21}$$

In terms of the magnetic dipole moment, the torque on a current loop due to a uniform magnetic field can be written simply as

$$\vec{\tau} = \vec{\mathbf{\mu}} \times \vec{\mathbf{B}}. \tag{11.22}$$

This equation holds for a current loop in a two-dimensional plane of arbitrary shape.

Using a calculation analogous to that found in Capacitance for an electric dipole, the potential energy of a magnetic dipole is

$$U = -\vec{\mathbf{\mu}} \cdot \vec{\mathbf{B}}. \tag{11.23}$$

EXAMPLE 11.7

Forces and Torques on Current-Carrying Loops

A circular current loop of radius 2.0 cm carries a current of 2.0 mA. (a) What is the magnitude of its magnetic dipole moment? (b) If the dipole is oriented at 30 degrees to a uniform magnetic field of magnitude 0.50 T, what is the magnitude of the torque it experiences and what is its potential energy?

Strategy

The dipole moment is defined by the current times the area of the loop. The area of the loop can be calculated from the area of the circle. The torque on the loop and potential energy are calculated from identifying the magnetic moment, magnetic field, and angle oriented in the field.

Solution

a. The magnetic moment μ is calculated by the current times the area of the loop or πr^2.

$$\mu = IA = (2.0 \times 10^{-3}\ \text{A})(\pi(0.02\ \text{m})^2) = 2.5 \times 10^{-6}\ \text{A} \cdot \text{m}^2$$

b. The torque and potential energy are calculated by identifying the magnetic moment, magnetic field, and the angle between these two vectors. The calculations of these quantities are:

$$\begin{aligned} \tau &= \vec{\mathbf{\mu}} \times \vec{\mathbf{B}} = \mu B \sin\theta = \left(2.5 \times 10^{-6}\text{A} \cdot \text{m}^2\right)(0.50\text{T})\sin(30°) = 6.3 \times 10^{-7}\text{N} \cdot \text{m} \\ U &= -\vec{\mathbf{\mu}} \cdot \vec{\mathbf{B}} = -\mu B \cos\theta = -\left(2.5 \times 10^{-6}\text{A} \cdot \text{m}^2\right)(0.50\text{T})\cos(30°) = -1.1 \times 10^{-6}\text{J}. \end{aligned}$$

Significance

The concept of magnetic moment at the atomic level is discussed in the next chapter. The concept of aligning the magnetic moment with the magnetic field is the functionality of devices like magnetic motors, whereby switching the external magnetic field results in a constant spinning of the loop as it tries to align with the field to minimize its potential energy.

CHECK YOUR UNDERSTANDING 11.4

In what orientation would a magnetic dipole have to be to produce (a) a maximum torque in a magnetic field? (b) A maximum energy of the dipole?

that the magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected but not the speed.

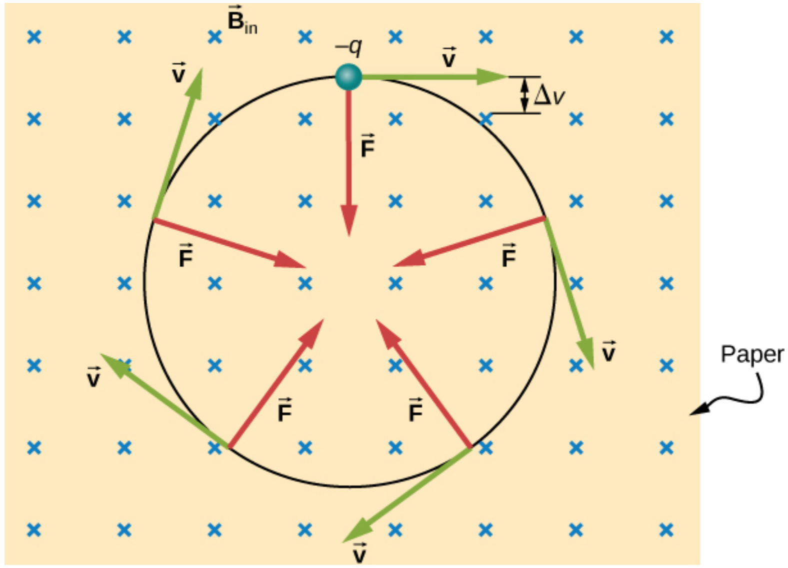

Figure 11.7 A negatively charged particle moves in the plane of the paper in a region where the magnetic field is perpendicular to the paper (represented by the small × 's—like the tails of arrows). The magnetic force is perpendicular to the velocity, so velocity changes in direction but not magnitude. The result is uniform circular motion. (Note that because the charge is negative, the force is opposite in direction to the prediction of the right-hand rule.)

In this situation, the magnetic force supplies the centripetal force $F_c = \frac{mv^2}{r}$. Noting that the velocity is perpendicular to the magnetic field, the magnitude of the magnetic force is reduced to $F = qvB$. Because the magnetic force F supplies the centripetal force F_c, we have

$$qvB = \frac{mv^2}{r}. \quad 11.4$$

Solving for r yields

$$r = \frac{mv}{qB}. \quad 11.5$$

Here, r is the radius of curvature of the path of a charged particle with mass m and charge q, moving at a speed v that is perpendicular to a magnetic field of strength B. The time for the charged particle to go around the circular path is defined as the period, which is the same as the distance traveled (the circumference) divided by the speed. Based on this and Equation 11.4, we can derive the period of motion as

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}. \quad 11.6$$

If the velocity is not perpendicular to the magnetic field, then we can compare each component of the velocity separately with the magnetic field. The component of the velocity perpendicular to the magnetic field produces a magnetic force perpendicular to both this velocity and the field:

$$v_{\text{perp}} = v\sin\theta, \quad v_{\text{para}} = v\cos\theta. \quad 11.7$$

where θ is the angle between v and B. The component parallel to the magnetic field creates constant motion along the same direction as the magnetic field, also shown in Equation 11.7. The parallel motion determines the *pitch* p of the helix, which is the distance between adjacent turns. This distance equals the parallel component of the velocity times the period:

$$p = v_{\text{para}}T. \tag{11.8}$$

The result is a **helical motion**, as shown in the following figure.

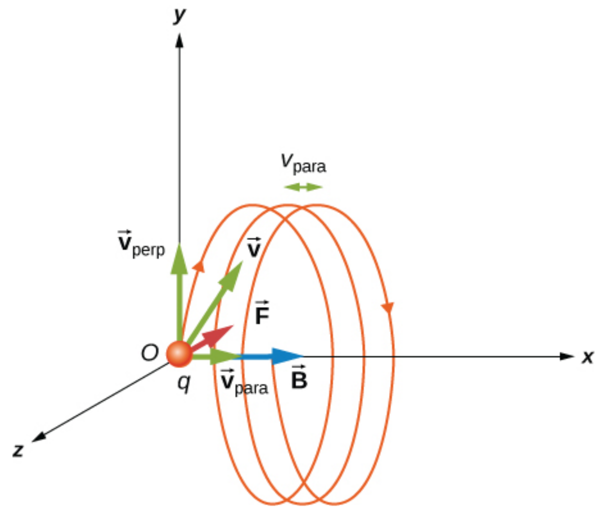

Figure 11.8 A charged particle moving with a velocity not in the same direction as the magnetic field. The velocity component perpendicular to the magnetic field creates circular motion, whereas the component of the velocity parallel to the field moves the particle along a straight line. The pitch is the horizontal distance between two consecutive circles. The resulting motion is helical.

While the charged particle travels in a helical path, it may enter a region where the magnetic field is not uniform. In particular, suppose a particle travels from a region of strong magnetic field to a region of weaker field, then back to a region of stronger field. The particle may reflect back before entering the stronger magnetic field region. This is similar to a wave on a string traveling from a very light, thin string to a hard wall and reflecting backward. If the reflection happens at both ends, the particle is trapped in a so-called magnetic bottle.

Trapped particles in magnetic fields are found in the Van Allen radiation belts around Earth, which are part of Earth's magnetic field. These belts were discovered by James Van Allen while trying to measure the flux of **cosmic rays** on Earth (high-energy particles that come from outside the solar system) to see whether this was similar to the flux measured on Earth. Van Allen found that due to the contribution of particles trapped in Earth's magnetic field, the flux was much higher on Earth than in outer space. Aurorae, like the famous aurora borealis (northern lights) in the Northern Hemisphere (Figure 11.9), are beautiful displays of light emitted as ions recombine with electrons entering the atmosphere as they spiral along magnetic field lines. (The ions are primarily oxygen and nitrogen atoms that are initially ionized by collisions with energetic particles in Earth's atmosphere.) Aurorae have also been observed on other planets, such as Jupiter and Saturn.

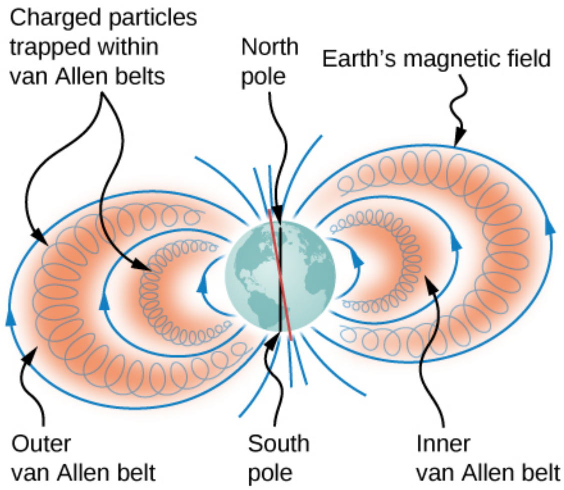

Figure 11.9 (a) The Van Allen radiation belts around Earth trap ions produced by cosmic rays striking Earth's atmosphere. (b) The magnificent spectacle of the aurora borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by Earth's magnetic field, this light is produced by glowing molecules and ions of oxygen and nitrogen. (credit b: modification of work by USAF Senior Airman Joshua Strang)

EXAMPLE 11.2

Beam Deflector

A research group is investigating short-lived radioactive isotopes. They need to design a way to transport alpha-particles (helium nuclei) from where they are made to a place where they will collide with another material to form an isotope. The beam of alpha-particles $\left(m = 6.64 \times 10^{-27}\,\text{kg}, q = 3.2 \times 10^{-19}\,\text{C}\right)$ bends through a 90-degree region with a uniform magnetic field of 0.050 T (Figure 11.10). (a) In what direction should the magnetic field be applied? (b) How much time does it take the alpha-particles to traverse the uniform magnetic field region?

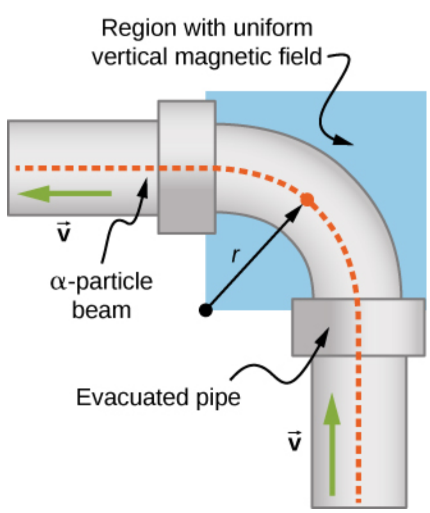

Figure 11.10 Top view of the beam deflector setup.

Strategy

a. The direction of the magnetic field is shown by the RHR-1. Your fingers point in the direction of *v*, and your thumb needs to point in the direction of the force, to the left. Therefore, since the alpha-particles are positively charged, the magnetic field must point down.

b. The period of the alpha-particle going around the circle is

$$T = \frac{2\pi m}{qB}. \tag{11.9}$$

Because the particle is only going around a quarter of a circle, we can take 0.25 times the period to find the time it takes to go around this path.

Solution

a. Let's start by focusing on the alpha-particle entering the field near the bottom of the picture. First, point your thumb up the page. In order for your palm to open to the left where the centripetal force (and hence the magnetic force) points, your fingers need to change orientation until they point into the page. This is the direction of the applied magnetic field.
b. The period of the charged particle going around a circle is calculated by using the given mass, charge, and magnetic field in the problem. This works out to be

$$T = \frac{2\pi m}{qB} = \frac{2\pi \left(6.64 \times 10^{-27}\,\text{kg}\right)}{\left(3.2 \times 10^{-19}\,\text{C}\right)(0.050\,\text{T})} = 2.6 \times 10^{-6}\,\text{s}.$$

However, for the given problem, the alpha-particle goes around a quarter of the circle, so the time it takes would be

$$t = 0.25 \times 2.61 \times 10^{-6}\,\text{s} = 6.5 \times 10^{-7}\,\text{s}.$$

Significance

This time may be quick enough to get to the material we would like to bombard, depending on how short-lived the radioactive isotope is and continues to emit alpha-particles. If we could increase the magnetic field applied in the region, this would shorten the time even more. The path the particles need to take could be shortened, but this may not be economical given the experimental setup.

CHECK YOUR UNDERSTANDING 11.2

A uniform magnetic field of magnitude 1.5 T is directed horizontally from west to east. (a) What is the magnetic force on a proton at the instant when it is moving vertically downward in the field with a speed of 4×10^7 m/s? (b) Compare this force with the weight w of a proton.

EXAMPLE 11.3

Helical Motion in a Magnetic Field

A proton enters a uniform magnetic field of $1.0 \times 10^{-4}\,\text{T}$ with a speed of 5×10^5 m/s. At what angle must the magnetic field be from the velocity so that the pitch of the resulting helical motion is equal to the radius of the helix?

Strategy

The pitch of the motion relates to the parallel velocity times the period of the circular motion, whereas the radius relates to the perpendicular velocity component. After setting the radius and the pitch equal to each other, solve for the angle between the magnetic field and velocity or θ.

Solution

The pitch is given by Equation 11.8, the period is given by Equation 11.6, and the radius of circular motion is given by Equation 11.5. Note that the velocity in the radius equation is related to only the perpendicular velocity, which is where the circular motion occurs. Therefore, we substitute the sine component of the overall velocity into the radius equation to equate the pitch and radius:

$$\begin{aligned} p &= r \\ v_{\parallel} T &= \frac{m v_{\perp}}{qB} \\ v \cos\theta \frac{2\pi m}{qB} &= \frac{m v \sin\theta}{qB} \\ 2\pi &= \tan\theta \\ \theta &= 81.0^{\circ}. \end{aligned}$$

Significance

If this angle were $0°$, only parallel velocity would occur and the helix would not form, because there would be no circular motion in the perpendicular plane. If this angle were $90°$, only circular motion would occur and there would be no movement of the circles perpendicular to the motion. That is what creates the helical motion.

11.4 Magnetic Force on a Current-Carrying Conductor

Learning Objectives

By the end of this section, you will be able to:

- Determine the direction in which a current-carrying wire experiences a force in an external magnetic field
- Calculate the force on a current-carrying wire in an external magnetic field

Moving charges experience a force in a magnetic field. If these moving charges are in a wire—that is, if the wire is carrying a current—the wire should also experience a force. However, before we discuss the force exerted on a current by a magnetic field, we first examine the magnetic field generated by an electric current. We are studying two separate effects here that interact closely: A current-carrying wire generates a magnetic field and the magnetic field exerts a force on the current-carrying wire.

Magnetic Fields Produced by Electrical Currents

When discussing historical discoveries in magnetism, we mentioned Oersted's finding that a wire carrying an electrical current caused a nearby compass to deflect. A connection was established that electrical currents produce magnetic fields. (This connection between electricity and magnetism is discussed in more detail in Sources of Magnetic Fields.)

The compass needle near the wire experiences a force that aligns the needle tangent to a circle around the wire. Therefore, a current-carrying wire produces circular loops of magnetic field. To determine the direction of the magnetic field generated from a wire, we use a second right-hand rule. In RHR-2, your thumb points in the direction of the current while your fingers wrap around the wire, pointing in the direction of the magnetic field produced (Figure 11.11). If the magnetic field were coming at you or out of the page, we represent this with a dot. If the magnetic field were going into the page, we represent this with an $\times$. These symbols come from considering a vector arrow: An arrow pointed toward you, from your perspective, would look like a dot or the tip of an arrow. An arrow pointed away from you, from your perspective, would look like a cross or an $\times$. A composite sketch of the magnetic circles is shown in Figure 11.11, where the field strength is shown to decrease as you get farther from the wire by loops that are farther separated.

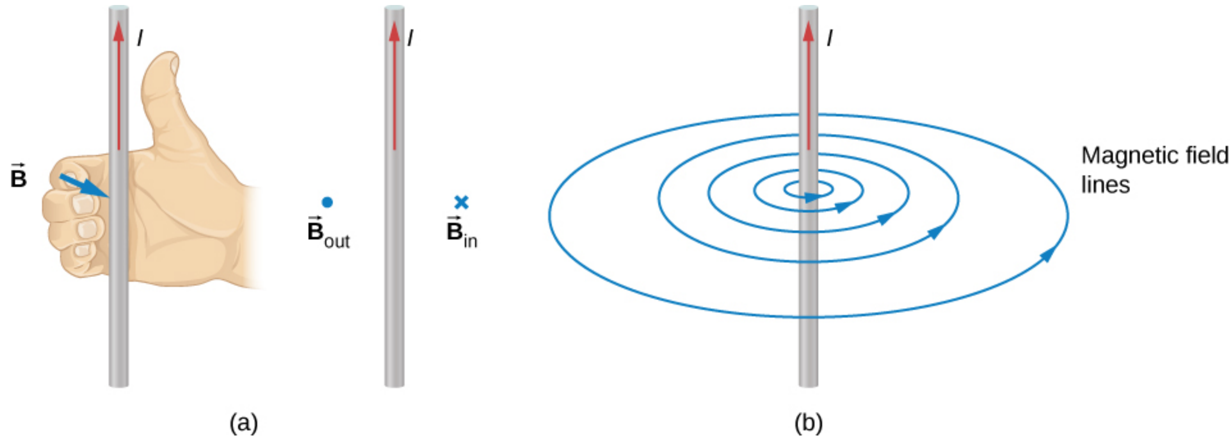

Figure 11.11 (a) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow). (b) A long and straight wire creates a field with magnetic field lines forming circular loops.

Calculating the Magnetic Force

Electric current is an ordered movement of charge. A current-carrying wire in a magnetic field must therefore experience a force due to the field. To investigate this force, let's consider the infinitesimal section of wire as shown in Figure 11.12. The length and cross-sectional area of the section are dl and A, respectively, so its volume is $V = A \cdot dl$. The wire is formed from material that contains n charge carriers per unit volume, so the number of charge carriers in the section is $nA \cdot dl$. If the charge carriers move with drift velocity $\vec{\mathbf{v}}_{\mathrm{d}}$, the current I in the wire is (from Current and Resistance)

$$I = neAv_d.$$

The magnetic force on any single charge carrier is $e\vec{\mathbf{v}}_{\mathrm{d}} \times \vec{\mathbf{B}}$, so the total magnetic force $d\vec{\mathbf{F}}$ on the $nA \cdot dl$ charge carriers in the section of wire is

$$d\vec{\mathbf{F}} = (nA \cdot dl)e\vec{\mathbf{v}}_{\mathrm{d}} \times \vec{\mathbf{B}}. \tag{11.10}$$

We can define dl to be a vector of length dl pointing along $\vec{\mathbf{v}}_{\mathrm{d}}$, which allows us to rewrite this equation as

$$d\vec{\mathbf{F}} = neAv_{\mathrm{d}}\vec{\mathbf{dl}} \times \vec{\mathbf{B}}, \tag{11.11}$$

or

$$d\vec{\mathbf{F}} = I\vec{\mathbf{dl}} \times \vec{\mathbf{B}}. \tag{11.12}$$

This is the magnetic force on the section of wire. Note that it is actually the net force exerted by the field on the charge carriers themselves. The direction of this force is given by RHR-1, where you point your fingers in the direction of the current and curl them toward the field. Your thumb then points in the direction of the force.

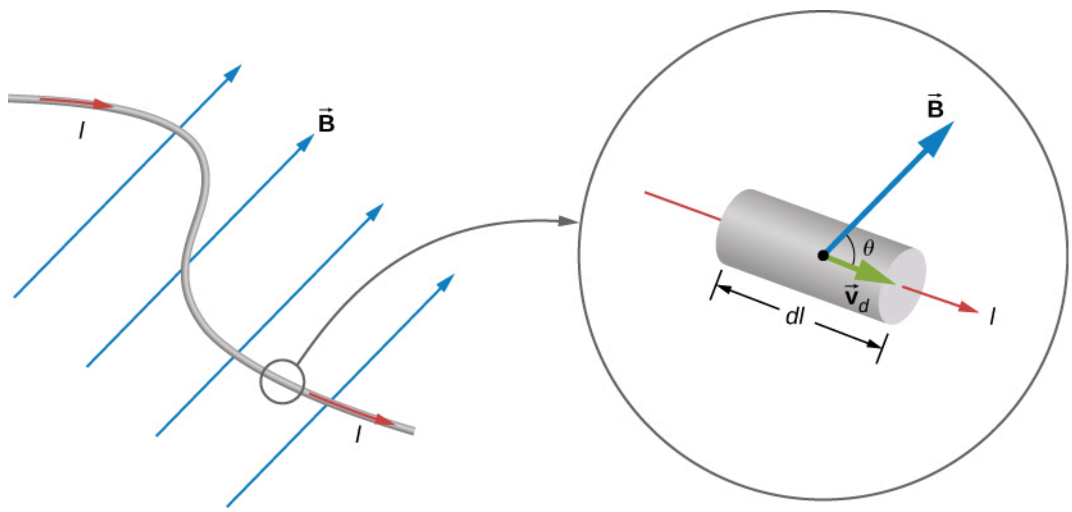

Figure 11.12 An infinitesimal section of current-carrying wire in a magnetic field.

To determine the magnetic force $\vec{F}$ on a wire of arbitrary length and shape, we must integrate Equation 11.12 over the entire wire. If the wire section happens to be straight and B is uniform, the equation differentials become absolute quantities, giving us

$$\vec{F} = I\vec{l} \times \vec{B}. \tag{11.13}$$

This is the force on a straight, current-carrying wire in a uniform magnetic field.

EXAMPLE 11.4

Balancing the Gravitational and Magnetic Forces on a Current-Carrying Wire

A wire of length 50 cm and mass 10 g is suspended in a horizontal plane by a pair of flexible leads (Figure 11.13). The wire is then subjected to a constant magnetic field of magnitude 0.50 T, which is directed as shown. What are the magnitude and direction of the current in the wire needed to remove the tension in the supporting leads?

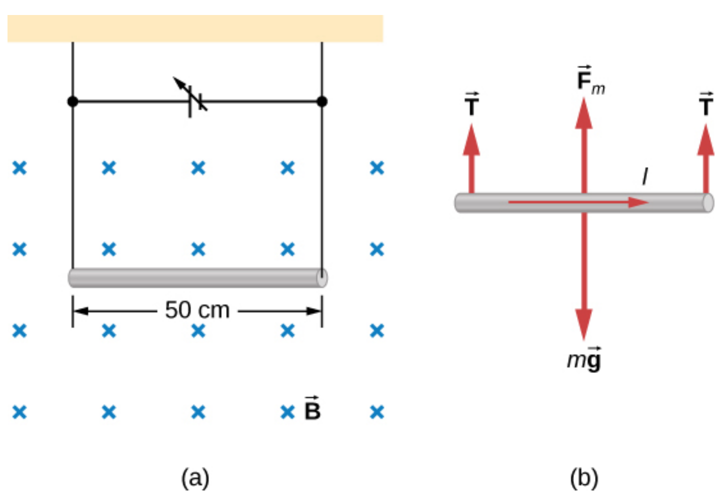

Figure 11.13 (a) A wire suspended in a magnetic field. (b) The free-body diagram for the wire.

Strategy

From the free-body diagram in the figure, the tensions in the supporting leads go to zero when the gravitational and magnetic forces balance each other. Using the RHR-1, we find that the magnetic force points up. We can then determine the current I by equating the two forces.

Solution

Equate the two forces of weight and magnetic force on the wire:

$$mg = IlB.$$

Thus,

$$I = \frac{mg}{lB} = \frac{(0.010\text{ kg})(9.8\text{ m/s}^2)}{(0.50\text{ m})(0.50\text{ T})} = 0.39\text{ A}.$$

Significance

This large magnetic field creates a significant force on a length of wire to counteract the weight of the wire.

EXAMPLE 11.5

Calculating Magnetic Force on a Current-Carrying Wire

A long, rigid wire lying along the y-axis carries a 5.0-A current flowing in the positive y-direction. (a) If a constant magnetic field of magnitude 0.30 T is directed along the positive x-axis, what is the magnetic force per unit length on the wire? (b) If a constant magnetic field of 0.30 T is directed 30 degrees from the $+x$-axis towards the $+y$-axis, what is the magnetic force per unit length on the wire?

Strategy

The magnetic force on a current-carrying wire in a magnetic field is given by $\vec{\mathbf{F}} = I\,\vec{\mathbf{l}} \times \vec{\mathbf{B}}$. For part a, since the current and magnetic field are perpendicular in this problem, we can simplify the formula to give us the magnitude and find the direction through the RHR-1. The angle θ is 90 degrees, which means $\sin\theta = 1$. Also, the length can be divided over to the left-hand side to find the force per unit length. For part b, the current times length is written in unit vector notation, as well as the magnetic field. After the cross product is taken, the directionality is evident by the resulting unit vector.

Solution

a. We start with the general formula for the magnetic force on a wire. We are looking for the force per unit length, so we divide by the length to bring it to the left-hand side. We also set $\sin\theta = 1$. The solution therefore is

$$\begin{aligned} F &= IlB\sin\theta \\ \tfrac{F}{l} &= (5.0\text{ A})(0.30\text{ T}) \\ \tfrac{F}{l} &= 1.5\text{ N/m}. \end{aligned}$$

Directionality: Point your fingers in the positive y-direction and curl your fingers in the positive x-direction. Your thumb will point in the $-\vec{\mathbf{k}}$ direction. Therefore, with directionality, the solution is

$$\frac{\vec{\mathbf{F}}}{l} = -1.5\vec{\mathbf{k}}\text{ N/m}.$$

b. The current times length and the magnetic field are written in unit vector notation. Then, we take the cross product to find the force:

$$\begin{aligned} \vec{\mathbf{F}} &= I\vec{\mathbf{l}} \times \vec{\mathbf{B}} = (5.0A)\,l\hat{\mathbf{j}} \times \left(0.30T\cos(30°)\,\hat{\mathbf{i}} + 0.30T\sin(30°)\,\hat{\mathbf{j}}\right) \\ \vec{\mathbf{F}}/l &= -1.30\hat{\mathbf{k}}\text{ N/m}. \end{aligned}$$

Significance

This large magnetic field creates a significant force on a small length of wire. As the angle of the magnetic field becomes more closely aligned to the current in the wire, there is less of a force on it, as seen from comparing parts a and b.

CHECK YOUR UNDERSTANDING 11.3

A straight, flexible length of copper wire is immersed in a magnetic field that is directed into the page. (a) If the wire's current runs in the $+x$-direction, which way will the wire bend? (b) Which way will the wire bend if the current runs in the $-x$-direction?

EXAMPLE 11.6

Force on a Circular Wire

A circular current loop of radius R carrying a current I is placed in the xy-plane. A constant uniform magnetic field cuts through the loop parallel to the y-axis (Figure 11.14). Find the magnetic force on the upper half of the loop, the lower half of the loop, and the total force on the loop.

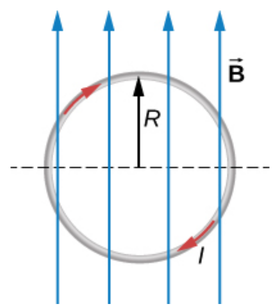

Figure 11.14 A loop of wire carrying a current in a magnetic field.

Strategy

The magnetic force on the upper loop should be written in terms of the differential force acting on each segment of the loop. If we integrate over each differential piece, we solve for the overall force on that section of the loop. The force on the lower loop is found in a similar manner, and the total force is the addition of these two forces.

Solution

A differential force on an arbitrary piece of wire located on the upper ring is:

$$dF = IB\sin\theta\, dl.$$

where θ is the angle between the magnetic field direction ($+y$) and the segment of wire. A differential segment is located at the same radius, so using an arc-length formula, we have:

$$\begin{aligned} dl &= R\,d\theta \\ dF &= IBR\sin\theta\, d\theta. \end{aligned}$$

In order to find the force on a segment, we integrate over the upper half of the circle, from 0 to π. This results in:

$$F = IBR\int_0^\pi \sin\theta\, d\theta = IBR(-\cos\pi + \cos 0) = 2IBR.$$

The lower half of the loop is integrated from π to zero, giving us:

$$F = IBR\int_\pi^0 \sin\theta\, d\theta = IBR(-\cos 0 + \cos\pi) = -2IBR.$$

The net force is the sum of these forces, which is zero.

Significance

The total force on any closed loop in a uniform magnetic field is zero. Even though each piece of the loop has a force acting on it, the net force on the system is zero. (Note that there is a net torque on the loop, which we consider in the next section.)

11.5 Force and Torque on a Current Loop

Learning Objectives

By the end of this section, you will be able to:

- Evaluate the net force on a current loop in an external magnetic field
- Evaluate the net torque on a current loop in an external magnetic field
- Define the magnetic dipole moment of a current loop

Motors are the most common application of magnetic force on current-carrying wires. Motors contain loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted into mechanical work in the process. Once the loop's surface area is aligned with the magnetic field, the direction of current is reversed, so there is a continual torque on the loop (Figure 11.15). This reversal of the current is done with commutators and brushes. The commutator is set to reverse the current flow at set points to keep continual motion in the motor. A basic commutator has three contact areas to avoid and dead spots where the loop would have zero instantaneous torque at that point. The brushes press against the commutator, creating electrical contact between parts of the commutator during the spinning motion.

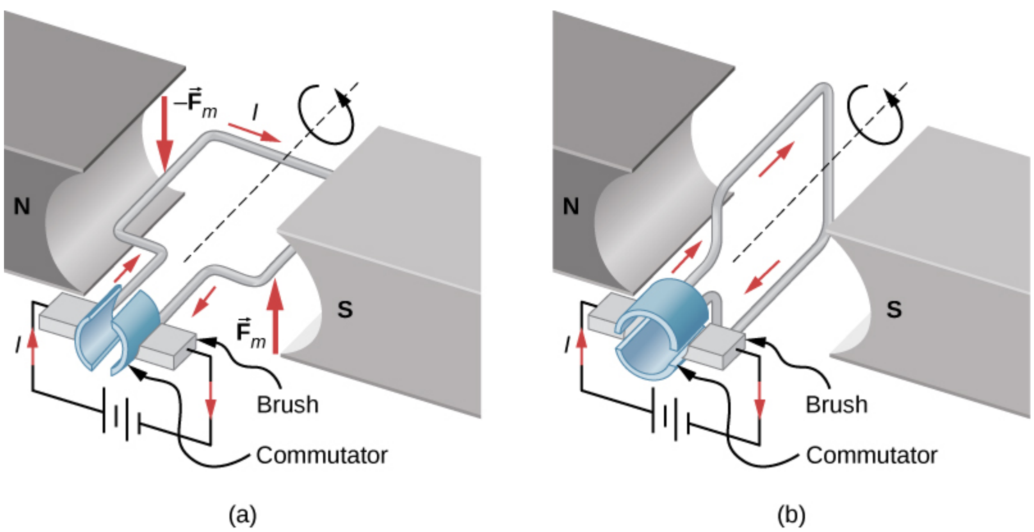

Figure 11.15 A simplified version of a dc electric motor. (a) The rectangular wire loop is placed in a magnetic field. The forces on the wires closest to the magnetic poles (N and S) are opposite in direction as determined by the right-hand rule-1. Therefore, the loop has a net torque and rotates to the position shown in (b). (b) The brushes now touch the commutator segments so that no current flows through the loop. No torque acts on the loop, but the loop continues to spin from the initial velocity given to it in part (a). By the time the loop flips over, current flows through the wires again but now in the opposite direction, and the process repeats as in part (a). This causes continual rotation of the loop.

In a uniform magnetic field, a current-carrying loop of wire, such as a loop in a motor, experiences both forces and torques on the loop. Figure 11.16 shows a rectangular loop of wire that carries a current I and has sides of lengths *a* and *b*. The loop is in a uniform magnetic field: $\vec{\mathbf{B}} = B\hat{\mathbf{j}}$. The magnetic force on a straight current-carrying wire of length l is given by $I\vec{\mathbf{l}} \times \vec{\mathbf{B}}$. To find the net force on the loop, we have to apply this equation to each of the four sides. The force on side 1 is

$$\vec{\mathbf{F}}_1 = IaB\sin(90° - \theta)\hat{\mathbf{i}} = IaB\cos\theta\hat{\mathbf{i}} \qquad 11.14$$

where the direction has been determined with the RHR-1. The current in side 3 flows in the opposite direction to that of side 1, so

$$\vec{\mathbf{F}}_3 = -IaB\sin(90° + \theta)\hat{\mathbf{i}} = -IaB\cos\theta\hat{\mathbf{i}}. \tag{11.15}$$

The currents in sides 2 and 4 are perpendicular to $\vec{\mathbf{B}}$ and the forces on these sides are

$$\vec{\mathbf{F}}_2 = IbB\hat{\mathbf{k}}, \quad \vec{\mathbf{F}}_4 = -IbB\hat{\mathbf{k}}. \tag{11.16}$$

We can now find the net force on the loop:

$$\sum \vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \vec{\mathbf{F}}_4 = 0. \tag{11.17}$$

Although this result ($\Sigma F = 0$) has been obtained for a rectangular loop, it is far more general and holds for current-carrying loops of arbitrary shapes; that is, there is no net force on a current loop in a uniform magnetic field.

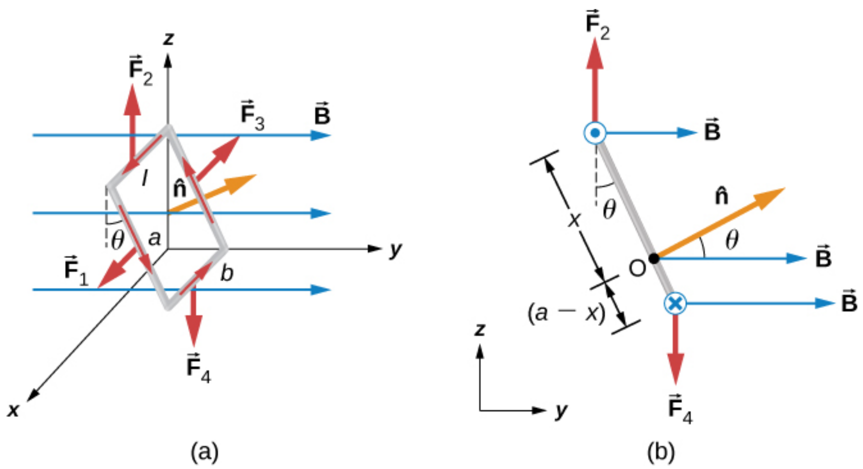

Figure 11.16 (a) A rectangular current loop in a uniform magnetic field is subjected to a net torque but not a net force. (b) A side view of the coil.

To find the net torque on the current loop shown in Figure 11.16, we first consider F_1 and F_3. Since they have the same line of action and are equal and opposite, the sum of their torques about any axis is zero (see Fixed-Axis Rotation). Thus, if there is any torque on the loop, it must be furnished by F_2 and F_4. Let's calculate the torques around the axis that passes through point O of Figure 11.16 (a side view of the coil) and is perpendicular to the plane of the page. The point O is a distance x from side 2 and a distance $(a - x)$ from side 4 of the loop. The moment arms of F_2 and F_4 are $x\sin\theta$ and $(a - x)\sin\theta$, respectively, so the net torque on the loop is

$$\begin{aligned}\sum \vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\tau}}_1 + \vec{\boldsymbol{\tau}}_2 + \vec{\boldsymbol{\tau}}_3 + \vec{\boldsymbol{\tau}}_4 &= F_2 x\sin\theta\hat{\mathbf{i}} - F_4(a - x)\sin(\theta)\hat{\mathbf{i}} \\ &= -IbBx\sin\theta\hat{\mathbf{i}} - IbB(a - x)\sin\theta\hat{\mathbf{i}}.\end{aligned} \tag{11.18}$$

This simplifies to

$$\vec{\boldsymbol{\tau}} = -IAB\sin\theta\hat{\mathbf{i}} \tag{11.19}$$

where $A = ab$ is the area of the loop.

Notice that this torque is independent of x; it is therefore independent of where point O is located in the plane of the current loop. Consequently, the loop experiences the same torque from the magnetic field about any axis in the plane of the loop and parallel to the x-axis.

A closed-current loop is commonly referred to as a **magnetic dipole** and the term IA is known as its **magnetic dipole moment** μ. Actually, the magnetic dipole moment is a vector that is defined as

$$\vec{\boldsymbol{\mu}} = IA\hat{\mathbf{n}} \tag{11.20}$$

where $\hat{\mathbf{n}}$ is a unit vector directed perpendicular to the plane of the loop (see Figure 11.16). The direction of $\hat{\mathbf{n}}$ is obtained with the RHR-2—if you curl the fingers of your right hand in the direction of current flow in the loop, then your thumb points along $\hat{\mathbf{n}}$. If the loop contains N turns of wire, then its magnetic dipole moment is given by

$$\vec{\boldsymbol{\mu}} = NIA\hat{\mathbf{n}}. \tag{11.21}$$

In terms of the magnetic dipole moment, the torque on a current loop due to a uniform magnetic field can be written simply as

$$\vec{\tau} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}. \tag{11.22}$$

This equation holds for a current loop in a two-dimensional plane of arbitrary shape.

Using a calculation analogous to that found in Capacitance for an electric dipole, the potential energy of a magnetic dipole is

$$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}. \tag{11.23}$$

EXAMPLE 11.7

Forces and Torques on Current-Carrying Loops

A circular current loop of radius 2.0 cm carries a current of 2.0 mA. (a) What is the magnitude of its magnetic dipole moment? (b) If the dipole is oriented at 30 degrees to a uniform magnetic field of magnitude 0.50 T, what is the magnitude of the torque it experiences and what is its potential energy?

Strategy

The dipole moment is defined by the current times the area of the loop. The area of the loop can be calculated from the area of the circle. The torque on the loop and potential energy are calculated from identifying the magnetic moment, magnetic field, and angle oriented in the field.

Solution

a. The magnetic moment μ is calculated by the current times the area of the loop or πr^2.
$$\mu = IA = (2.0 \times 10^{-3}\text{ A})(\pi(0.02\text{ m})^2) = 2.5 \times 10^{-6}\text{ A} \cdot \text{m}^2$$

b. The torque and potential energy are calculated by identifying the magnetic moment, magnetic field, and the angle between these two vectors. The calculations of these quantities are:
$$\begin{aligned} \tau &= \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}} = \mu B \sin\theta = \left(2.5 \times 10^{-6}\text{A} \cdot \text{m}^2\right)(0.50\text{T})\sin(30°) = 6.3 \times 10^{-7}\text{N} \cdot \text{m} \\ U &= -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}} = -\mu B \cos\theta = -\left(2.5 \times 10^{-6}\text{A} \cdot \text{m}^2\right)(0.50\text{T})\cos(30°) = -1.1 \times 10^{-6}\text{J}. \end{aligned}$$

Significance

The concept of magnetic moment at the atomic level is discussed in the next chapter. The concept of aligning the magnetic moment with the magnetic field is the functionality of devices like magnetic motors, whereby switching the external magnetic field results in a constant spinning of the loop as it tries to align with the field to minimize its potential energy.

CHECK YOUR UNDERSTANDING 11.4

In what orientation would a magnetic dipole have to be to produce (a) a maximum torque in a magnetic field? (b) A maximum energy of the dipole?

11.6 The Hall Effect

Learning Objectives

By the end of this section, you will be able to:

- Explain a scenario where the magnetic and electric fields are crossed and their forces balance each other as a charged particle moves through a velocity selector
- Compare how charge carriers move in a conductive material and explain how this relates to the Hall effect

In 1879, E.H. Hall devised an experiment that can be used to identify the sign of the predominant charge carriers in a conducting material. From a historical perspective, this experiment was the first to demonstrate that the charge carriers in most metals are negative.

INTERACTIVE

Visit this website (https://openstax.org/l/21halleffect) to find more information about the Hall effect.

We investigate the **Hall effect** by studying the motion of the free electrons along a metallic strip of width l in a constant magnetic field (Figure 11.17). The electrons are moving from left to right, so the magnetic force they experience pushes them to the bottom edge of the strip. This leaves an excess of positive charge at the top edge of the strip, resulting in an electric field E directed from top to bottom. The charge concentration at both edges builds up until the electric force on the electrons in one direction is balanced by the magnetic force on them in the opposite direction. Equilibrium is reached when:

$$eE = ev_d B \tag{11.24}$$

where e is the magnitude of the electron charge, v_d is the drift speed of the electrons, and E is the magnitude of the electric field created by the separated charge. Solving this for the drift speed results in

$$v_d = \frac{E}{B}. \tag{11.25}$$

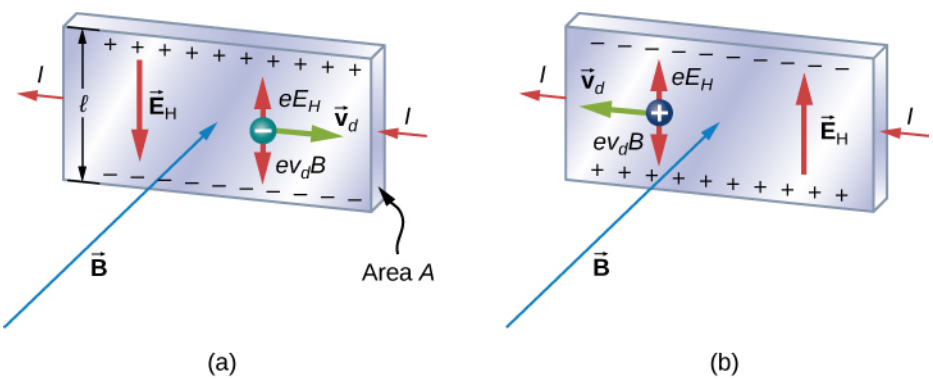

Figure 11.17 In the Hall effect, a potential difference between the top and bottom edges of the metal strip is produced when moving charge carriers are deflected by the magnetic field. (a) Hall effect for negative charge carriers; (b) Hall effect for positive charge carriers.

A scenario where the electric and magnetic fields are perpendicular to one another is called a crossed-field situation. If these fields produce equal and opposite forces on a charged particle with the velocity that equates the forces, these particles are able to pass through an apparatus, called a **velocity selector**, undeflected. This velocity is represented in Equation 11.26. Any other velocity of a charged particle sent into the same fields would be deflected by the magnetic force or electric force.

Going back to the Hall effect, if the current in the strip is I, then from Current and Resistance, we know that

$$I = nev_d A \tag{11.26}$$

where n is the number of charge carriers per volume and A is the cross-sectional area of the strip. Combining the equations for v_d and I results in

$$I = ne\left(\frac{E}{B}\right)A. \tag{11.27}$$

The field E is related to the potential difference V between the edges of the strip by

$$E = \frac{V}{l}. \tag{11.28}$$

The quantity V is called the Hall potential and can be measured with a voltmeter. Finally, combining the equations for I and E gives us

$$V = \frac{IBl}{neA} \tag{11.29}$$

where the upper edge of the strip in Figure 11.17 is positive with respect to the lower edge.

We can also combine Equation 11.24 and Equation 11.28 to get an expression for the Hall voltage in terms of the magnetic field:

$$V = Blv_d. \tag{11.30}$$

What if the charge carriers are positive, as in Figure 11.17? For the same current I, the magnitude of V is still given by Equation 11.29. However, the upper edge is now negative with respect to the lower edge. Therefore, by simply measuring the sign of V, we can determine the sign of the majority charge carriers in a metal.

Hall potential measurements show that electrons are the dominant charge carriers in most metals. However, Hall potentials indicate that for a few metals, such as tungsten, beryllium, and many semiconductors, the majority of charge carriers are positive. It turns out that conduction by positive charge is caused by the migration of missing electron sites (called holes) on ions. Conduction by holes is studied later in Condensed Matter Physics.

The Hall effect can be used to measure magnetic fields. If a material with a known density of charge carriers n is placed in a magnetic field and V is measured, then the field can be determined from Equation 11.29. In research laboratories where the fields of electromagnets used for precise measurements have to be extremely steady, a "Hall probe" is commonly used as part of an electronic circuit that regulates the field.

EXAMPLE 11.8

Velocity Selector

An electron beam enters a crossed-field velocity selector with magnetic and electric fields of 2.0 mT and 6.0×10^3 N/C, respectively. (a) What must the velocity of the electron beam be to traverse the crossed fields undeflected? If the electric field is turned off, (b) what is the acceleration of the electron beam and (c) what is the radius of the circular motion that results?

Strategy

The electron beam is not deflected by either of the magnetic or electric fields if these forces are balanced. Based on these balanced forces, we calculate the velocity of the beam. Without the electric field, only the magnetic force is used in Newton's second law to find the acceleration. Lastly, the radius of the path is based on the resulting circular motion from the magnetic force.

Solution

a. The velocity of the unperturbed beam of electrons with crossed fields is calculated by Equation 11.25:

$$v_d = \frac{E}{B} = \frac{6 \times 10^3 \text{ N/C}}{2 \times 10^{-3} \text{ T}} = 3 \times 10^6 \text{m/s}.$$

b. The acceleration is calculated from the net force from the magnetic field, equal to mass times acceleration.

CHAPTER REVIEW

Key Terms

cosmic rays comprised of particles that originate mainly from outside the solar system and reach Earth

cyclotron device used to accelerate charged particles to large kinetic energies

dees large metal containers used in cyclotrons that serve contain a stream of charged particles as their speed is increased

gauss G, unit of the magnetic field strength; $1\ \text{G} = 10^{-4}\,\text{T}$

Hall effect creation of voltage across a current-carrying conductor by a magnetic field

helical motion superposition of circular motion with a straight-line motion that is followed by a charged particle moving in a region of magnetic field at an angle to the field

magnetic dipole closed-current loop

magnetic dipole moment term *IA* of the magnetic dipole, also called μ

magnetic field lines continuous curves that show the direction of a magnetic field; these lines point in the same direction as a compass points, toward the magnetic south pole of a bar magnet

magnetic force force applied to a charged particle moving through a magnetic field

mass spectrometer device that separates ions according to their charge-to-mass ratios

motor (dc) loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft; electrical energy is converted into mechanical work in the process

north magnetic pole currently where a compass points to north, near the geographic North Pole; this is the effective south pole of a bar magnet but has flipped between the effective north and south poles of a bar magnet multiple times over the age of Earth

right-hand rule-1 using your right hand to determine the direction of either the magnetic force, velocity of a charged particle, or magnetic field

south magnetic pole currently where a compass points to the south, near the geographic South Pole; this is the effective north pole of a bar magnet but has flipped just like the north magnetic pole

tesla SI unit for magnetic field: 1 T = 1 N/A-m

velocity selector apparatus where the crossed electric and magnetic fields produce equal and opposite forces on a charged particle moving with a specific velocity; this particle moves through the velocity selector not affected by either field while particles moving with different velocities are deflected by the apparatus

Key Equations

Force on a charge in a magnetic field	$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$
Magnitude of magnetic force	$F = qvB\sin\theta$
Radius of a particle's path in a magnetic field	$r = \frac{mv}{qB}$
Period of a particle's motion in a magnetic field	$T = \frac{2\pi m}{qB}$
Force on a current-carrying wire in a uniform magnetic field	$\vec{\mathbf{F}} = I\,\vec{\mathbf{l}} \times \vec{\mathbf{B}}$
Magnetic dipole moment	$\vec{\boldsymbol{\mu}} = NIA\hat{\mathbf{n}}$
Torque on a current loop	$\vec{\tau} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$
Energy of a magnetic dipole	$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$

Drift velocity in crossed electric and magnetic fields	$v_d = \frac{E}{B}$
Hall potential	$V = \frac{IBl}{neA}$
Hall potential in terms of drift velocity	$V = Blv_d$
Charge-to-mass ratio in a mass spectrometer	$\frac{q}{m} = \frac{E}{BB_0 R}$
Maximum speed of a particle in a cyclotron	$v_{\max} = \frac{qBR}{m}$

Summary

11.1 Magnetism and Its Historical Discoveries

- Magnets have two types of magnetic poles, called the north magnetic pole and the south magnetic pole. North magnetic poles are those that are attracted toward Earth's geographic North Pole.
- Like poles repel and unlike poles attract.
- Discoveries of how magnets respond to currents by Oersted and others created a framework that led to the invention of modern electronic devices, electric motors, and magnetic imaging technology.

11.2 Magnetic Fields and Lines

- Charges moving across a magnetic field experience a force determined by $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. The force is perpendicular to the plane formed by $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$.
- The direction of the force on a moving charge is given by the right hand rule 1 (RHR-1): Sweep your fingers in a velocity, magnetic field plane. Start by pointing them in the direction of velocity and sweep towards the magnetic field. Your thumb points in the direction of the magnetic force for positive charges.
- Magnetic fields can be pictorially represented by magnetic field lines, which have the following properties:
  1. The field is tangent to the magnetic field line.
  2. Field strength is proportional to the line density.
  3. Field lines cannot cross.
  4. Field lines form continuous, closed loops.
- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

11.3 Motion of a Charged Particle in a Magnetic Field

- A magnetic force can supply centripetal force and cause a charged particle to move in a circular path of radius $r = \frac{mv}{qB}$.
- The period of circular motion for a charged particle moving in a magnetic field perpendicular to the plane of motion is $T = \frac{2\pi m}{qB}$.
- Helical motion results if the velocity of the charged particle has a component parallel to the magnetic field as well as a component perpendicular to the magnetic field.

11.4 Magnetic Force on a Current-Carrying Conductor

- An electrical current produces a magnetic field around the wire.
- The directionality of the magnetic field produced is determined by the right hand rule-2, where your thumb points in the direction of the current and your fingers wrap around the wire in the direction of the magnetic field.
- The magnetic force on current-carrying conductors is given by $\vec{\mathbf{F}} = I\,\vec{\mathbf{l}} \times \vec{\mathbf{B}}$ where I is the current and l is the length of a wire in a uniform magnetic field B.

11.5 Force and Torque on a Current Loop

- The net force on a current-carrying loop of any plane shape in a uniform magnetic field is zero.
- The net torque τ on a current-carrying loop of any shape in a uniform magnetic field is calculated using $\tau = \vec{\mathbf{\mu}} \times \vec{\mathbf{B}}$ where $\vec{\mu}$ is the magnetic dipole moment and $\vec{\mathbf{B}}$ is the magnetic field strength.
- The magnetic dipole moment μ is the product of

the number of turns of wire N, the current in the loop I, and the area of the loop A or $\vec{\mathbf{\mu}} = N I A \hat{\mathbf{n}}$.

11.6 The Hall Effect

- Perpendicular electric and magnetic fields exert equal and opposite forces for a specific velocity of entering particles, thereby acting as a velocity selector. The velocity that passes through undeflected is calculated by $v = \frac{E}{B}$.
- The Hall effect can be used to measure the sign of the majority of charge carriers for metals. It can also be used to measure a magnetic field.

11.7 Applications of Magnetic Forces and Fields

- A mass spectrometer is a device that separates ions according to their charge-to-mass ratios by first sending them through a velocity selector, then a uniform magnetic field.
- Cyclotrons are used to accelerate charged particles to large kinetic energies through applied electric and magnetic fields.

Conceptual Questions

11.2 Magnetic Fields and Lines

1. Discuss the similarities and differences between the electrical force on a charge and the magnetic force on a charge.

2. (a) Is it possible for the magnetic force on a charge moving in a magnetic field to be zero? (b) Is it possible for the electric force on a charge moving in an electric field to be zero? (c) Is it possible for the resultant of the electric and magnetic forces on a charge moving simultaneously through both fields to be zero?

11.3 Motion of a Charged Particle in a Magnetic Field

3. At a given instant, an electron and a proton are moving with the same velocity in a constant magnetic field. Compare the magnetic forces on these particles. Compare their accelerations.

4. Does increasing the magnitude of a uniform magnetic field through which a charge is traveling necessarily mean increasing the magnetic force on the charge? Does changing the direction of the field necessarily mean a change in the force on the charge?

5. An electron passes through a magnetic field without being deflected. What do you conclude about the magnetic field?

6. If a charged particle moves in a straight line, can you conclude that there is no magnetic field present?

7. How could you determine which pole of an electromagnet is north and which pole is south?

11.4 Magnetic Force on a Current-Carrying Conductor

8. Describe the error that results from accidently using your left rather than your right hand when determining the direction of a magnetic force.

9. Considering the magnetic force law, are the velocity and magnetic field always perpendicular? Are the force and velocity always perpendicular? What about the force and magnetic field?

10. Why can a nearby magnet distort a cathode ray tube television picture?

11. A magnetic field exerts a force on the moving electrons in a current carrying wire. What exerts the force on a wire?

12. There are regions where the magnetic field of earth is almost perpendicular to the surface of Earth. What difficulty does this cause in the use of a compass?

11.6 The Hall Effect

13. Hall potentials are much larger for poor conductors than for good conductors. Why?

11.7 Applications of Magnetic Forces and Fields

14. Describe the primary function of the electric field and the magnetic field in a cyclotron.

Problems

11.2 Magnetic Fields and Lines

15. What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases?