CHAPTER 10
Direct-Current Circuits

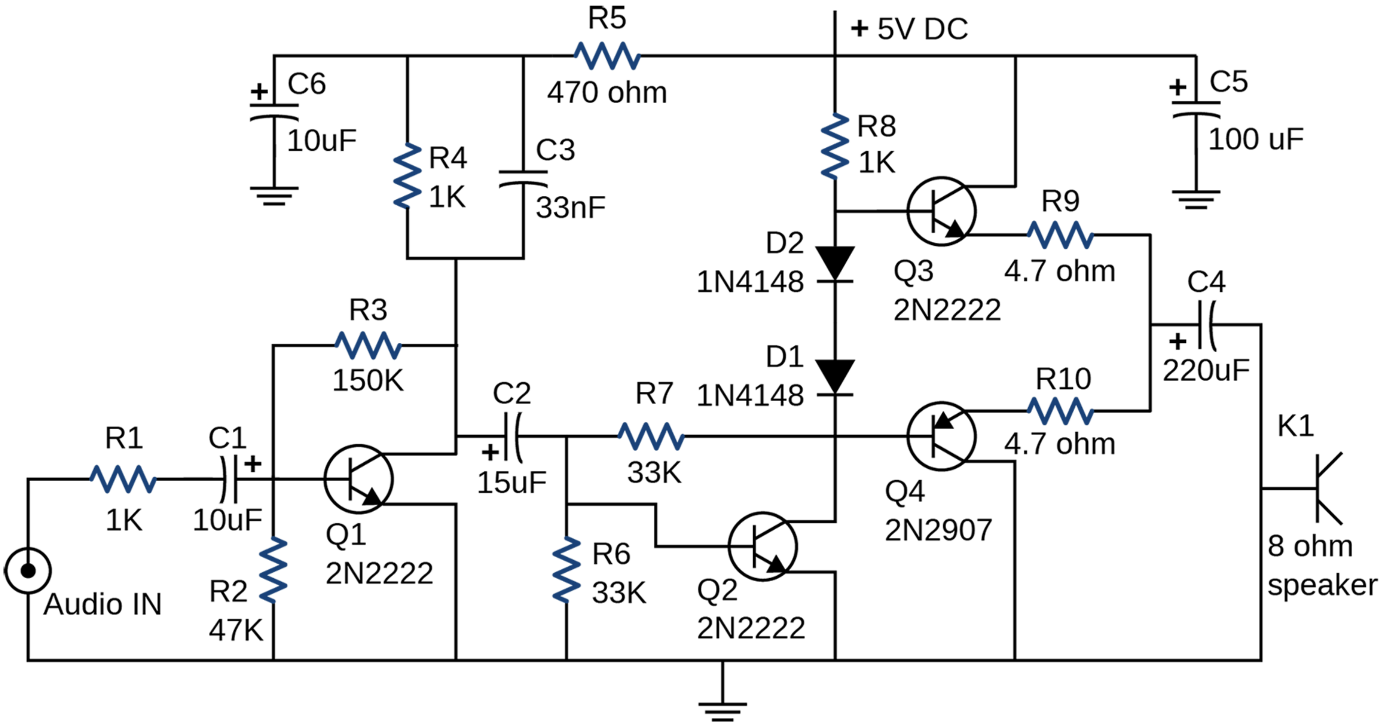


Figure 10.1 This circuit shown is used to amplify small signals and power the earbud speakers attached to a cellular phone. This circuit's components include resistors, capacitors, and diodes, all of which have been covered in previous chapters, as well as transistors, which are semi-conducting devices covered in Condensed Matter Physics. Circuits using similar components are found in all types of equipment and appliances you encounter in everyday life, such as alarm clocks, televisions, computers, and refrigerators.

Chapter Outline

INTRODUCTION In the preceding few chapters, we discussed electric components, including capacitors, resistors, and diodes. In this chapter, we use these electric components in circuits. A circuit is a collection of electrical components connected to accomplish a specific task. Figure 10.1 shows an amplifier circuit, which takes a small-amplitude signal and amplifies it to power the speakers in earbuds. Although the circuit looks complex, it actually consists of a set of series, parallel, and series-parallel circuits. The second section of this chapter covers the analysis of series and parallel circuits that consist of resistors. Later in this chapter, we introduce the basic equations and techniques to analyze any circuit, including those that are not reducible through simplifying parallel and series elements. But first, we need to understand how to power a circuit.

10.1 Electromotive Force

Learning Objectives

By the end of the section, you will be able to:

- Describe the electromotive force (emf) and the internal resistance of a battery
- Explain the basic operation of a battery

If you forget to turn off your car lights, they slowly dim as the battery runs down. Why don't they suddenly blink off when the battery's energy is gone? Their gradual dimming implies that the battery output voltage decreases as the battery is depleted. The reason for the decrease in output voltage for depleted batteries is that all voltage sources have two fundamental parts—a source of electrical energy and an internal resistance. In this section, we examine the energy source and the internal resistance.

Introduction to Electromotive Force

Voltage has many sources, a few of which are shown in Figure 10.2. All such devices create a **potential difference** and can supply current if connected to a circuit. A special type of potential difference is known as **electromotive force (emf)**. The emf is not a force at all, but the term 'electromotive force' is used for historical reasons. It was coined by Alessandro Volta in the 1800s, when he invented the first battery, also known as the voltaic pile. Because the electromotive force is not a force, it is common to refer to these sources simply as sources of emf (pronounced as the letters "ee-em-eff"), instead of sources of electromotive force.

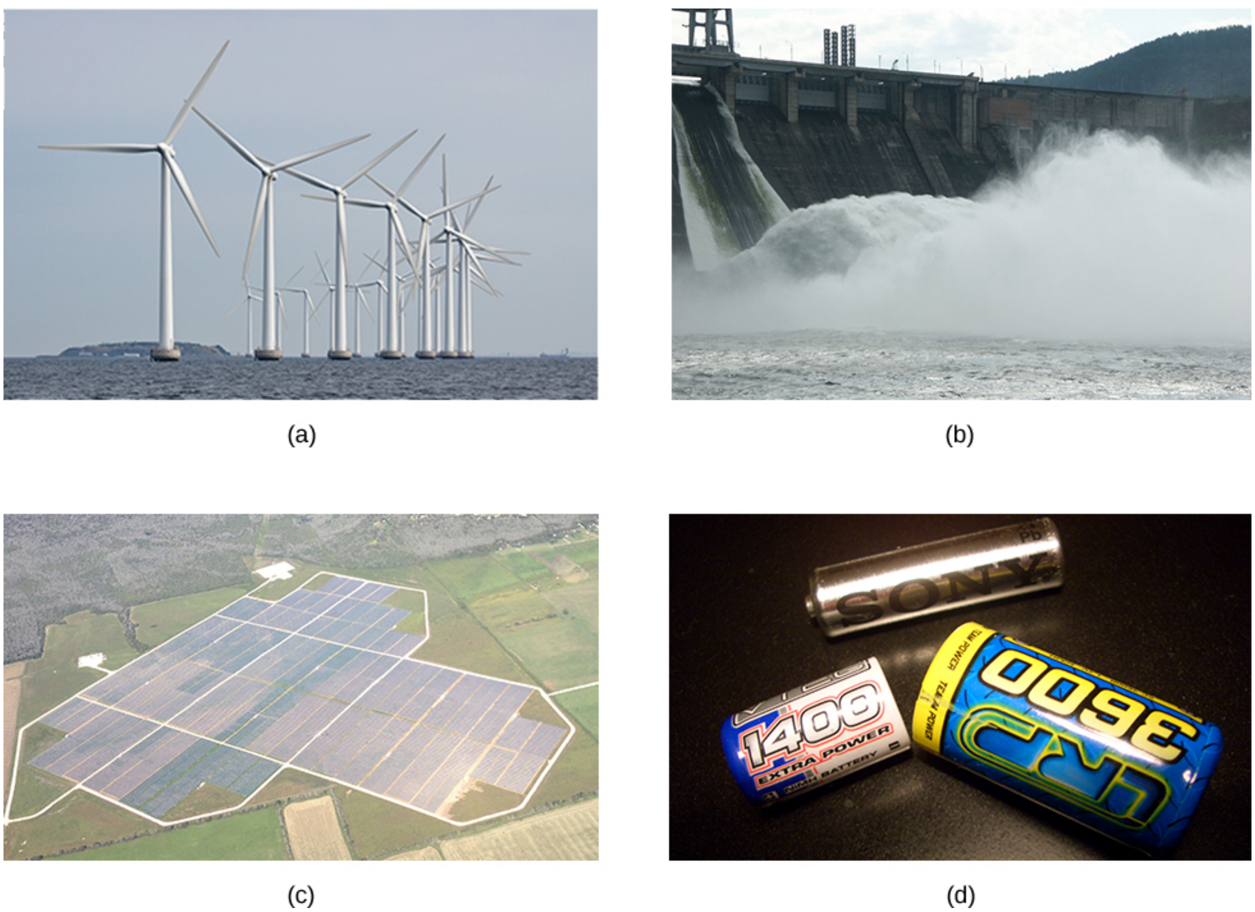


Figure 10.2 A variety of voltage sources. (a) The Brazos Wind Farm in Fluvanna, Texas; (b) the Krasnoyarsk Dam in Russia; (c) a solar

farm; (d) a group of nickel metal hydride batteries. The voltage output of each device depends on its construction and load. The voltage output equals emf only if there is no load. (credit a: modification of work by Stig Nygaard; credit b: modification of work by "vadimpl"/Wikimedia Commons; credit c: modification of work by "The tdog"/Wikimedia Commons; credit d: modification of work by "Itrados"/Wikimedia Commons)

If the electromotive force is not a force at all, then what is the emf and what is a source of emf? To answer these questions, consider a simple circuit of a 12-V lamp attached to a 12-V battery, as shown in Figure 10.3. The battery can be modeled as a two-terminal device that keeps one terminal at a higher electric potential than the second terminal. The higher electric potential is sometimes called the positive terminal and is labeled with a plus sign. The lower-potential terminal is sometimes called the negative terminal and labeled with a minus sign. This is the source of the emf.

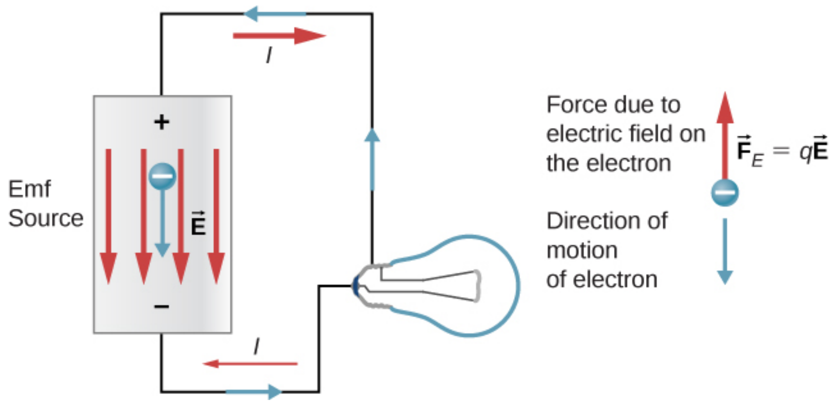


Figure 10.3 A source of emf maintains one terminal at a higher electric potential than the other terminal, acting as a source of current in a circuit.

When the emf source is not connected to the lamp, there is no net flow of charge within the emf source. Once the battery is connected to the lamp, charges flow from one terminal of the battery, through the lamp (causing the lamp to light), and back to the other terminal of the battery. If we consider positive (conventional) current flow, positive charges leave the positive terminal, travel through the lamp, and enter the negative terminal.

Positive current flow is useful for most of the circuit analysis in this chapter, but in metallic wires and resistors, electrons contribute the most to current, flowing in the opposite direction of positive current flow. Therefore, it is more realistic to consider the movement of electrons for the analysis of the circuit in Figure 10.3. The electrons leave the negative terminal, travel through the lamp, and return to the positive terminal. In order for the emf source to maintain the potential difference between the two terminals, negative charges (electrons) must be moved from the positive terminal to the negative terminal. The emf source acts as a charge pump, moving negative charges from the positive terminal to the negative terminal to maintain the potential difference. This increases the potential energy of the charges and, therefore, the electric potential of the charges.

The force on the negative charge from the electric field is in the opposite direction of the electric field, as shown in Figure 10.3. In order for the negative charges to be moved to the negative terminal, work must be done on the negative charges. This requires energy, which comes from chemical reactions in the battery. The potential is kept high on the positive terminal and low on the negative terminal to maintain the potential difference between the two terminals. The emf is equal to the work done on the charge per unit charge $\left(\varepsilon = \frac{dW}{dq}\right)$ when there is no current flowing. Since the unit for work is the joule and the unit for charge is the coulomb, the unit for emf is the volt $(1\text{ V} = 1\text{ J/C})$.

The **terminal voltage** V_{terminal} of a battery is voltage measured across the terminals of the battery. An ideal battery is an emf source that maintains a constant terminal voltage, independent of the current between the two terminals. An ideal battery has no internal resistance, and the terminal voltage is equal to the emf of the battery. In the next section, we will show that a real battery does have internal resistance and the terminal voltage is always less than the emf of the battery.

The Origin of Battery Potential

The combination of chemicals and the makeup of the terminals in a battery determine its emf. The lead acid battery used in cars and other vehicles is one of the most common combinations of chemicals. Figure 10.4 shows a single cell (one of six) of this battery. The cathode (positive) terminal of the cell is connected to a lead oxide plate, whereas the anode (negative) terminal is connected to a lead plate. Both plates are immersed in sulfuric acid, the electrolyte for the system.

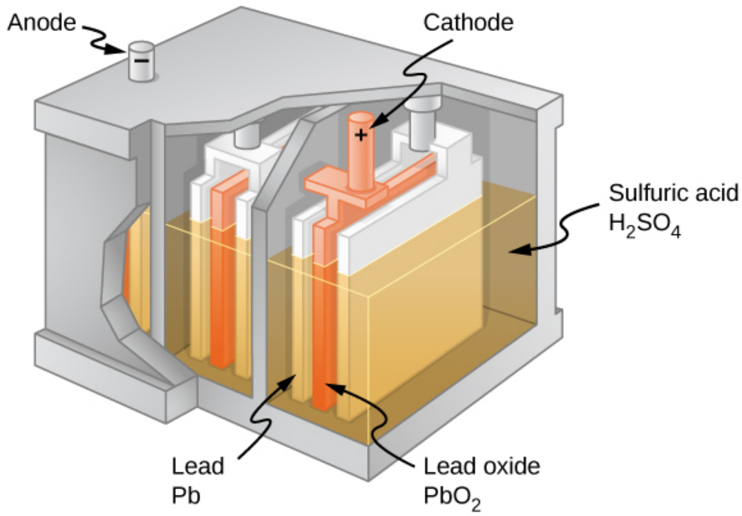


Figure 10.4 Chemical reactions in a lead-acid cell separate charge, sending negative charge to the anode, which is connected to the lead plates. The lead oxide plates are connected to the positive or cathode terminal of the cell. Sulfuric acid conducts the charge, as well as participates in the chemical reaction.

Knowing a little about how the chemicals in a lead-acid battery interact helps in understanding the potential created by the battery. Figure 10.5 shows the result of a single chemical reaction. Two electrons are placed on the anode, making it negative, provided that the cathode supplies two electrons. This leaves the cathode positively charged, because it has lost two electrons. In short, a separation of charge has been driven by a chemical reaction.

Note that the reaction does not take place unless there is a complete circuit to allow two electrons to be supplied to the cathode. Under many circumstances, these electrons come from the anode, flow through a resistance, and return to the cathode. Note also that since the chemical reactions involve substances with resistance, it is not possible to create the emf without an internal resistance.

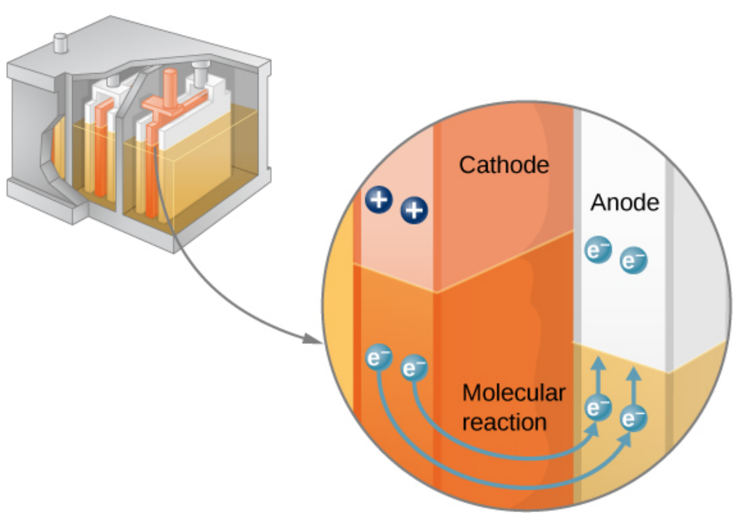


Figure 10.5 In a lead-acid battery, two electrons are forced onto the anode of a cell, and two electrons are removed from the cathode of the cell. The chemical reaction in a lead-acid battery places two electrons on the anode and removes two from the cathode. It requires a

closed circuit to proceed, since the two electrons must be supplied to the cathode.

Internal Resistance and Terminal Voltage

The amount of resistance to the flow of current within the voltage source is called the **internal resistance**. The internal resistance r of a battery can behave in complex ways. It generally increases as a battery is depleted, due to the oxidation of the plates or the reduction of the acidity of the electrolyte. However, internal resistance may also depend on the magnitude and direction of the current through a voltage source, its temperature, and even its history. The internal resistance of rechargeable nickel-cadmium cells, for example, depends on how many times and how deeply they have been depleted. A simple model for a battery consists of an idealized emf source ε and an internal resistance r (Figure 10.6).

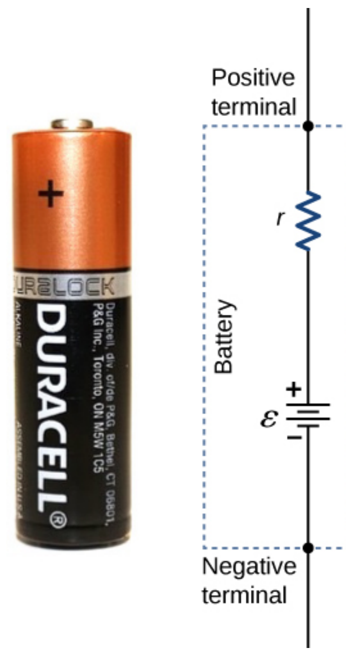


Figure 10.6 A battery can be modeled as an idealized emf (ε) with an internal resistance (r). The terminal voltage of the battery is $V_{\text{terminal}} = \varepsilon - Ir$.

Suppose an external resistor, known as the load resistance R, is connected to a voltage source such as a battery, as in Figure 10.7. The figure shows a model of a battery with an emf ε, an internal resistance r, and a load resistor R connected across its terminals. Using conventional current flow, positive charges leave the positive terminal of the battery, travel through the resistor, and return to the negative terminal of the battery. The terminal voltage of the battery depends on the emf, the internal resistance, and the current, and is equal to

$$V_{\text{terminal}} = \varepsilon - Ir. \tag{10.1}$$

For a given emf and internal resistance, the terminal voltage decreases as the current increases due to the potential drop Ir of the internal resistance.

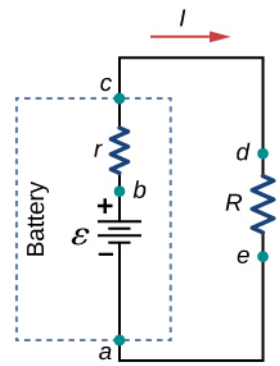


Figure 10.7 Schematic of a voltage source and its load resistor *R*. Since the internal resistance *r* is in series with the load, it can significantly affect the terminal voltage and the current delivered to the load.

A graph of the potential difference across each element the circuit is shown in Figure 10.8. A current I runs through the circuit, and the potential drop across the internal resistor is equal to *Ir*. The terminal voltage is equal to $\varepsilon - Ir$, which is equal to the **potential drop** across the load resistor $IR = \varepsilon - Ir$. As with potential energy, it is the change in voltage that is important. When the term “voltage” is used, we assume that it is actually the change in the potential, or ΔV. However, Δ is often omitted for convenience.

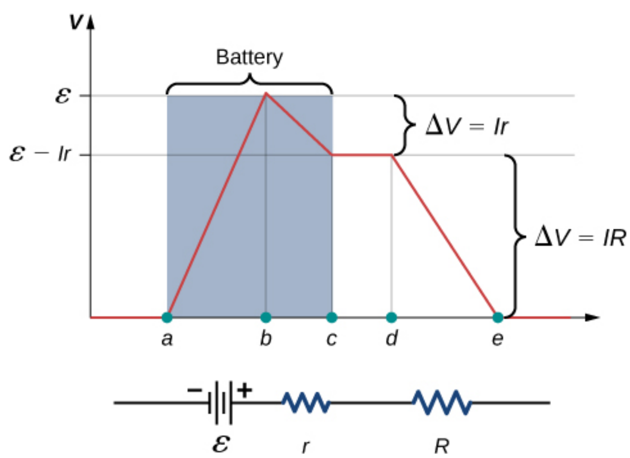


Figure 10.8 A graph of the voltage through the circuit of a battery and a load resistance. The electric potential increases the emf of the battery due to the chemical reactions doing work on the charges. There is a decrease in the electric potential in the battery due to the internal resistance. The potential decreases due to the internal resistance $(-Ir)$, making the terminal voltage of the battery equal to $(\varepsilon - Ir)$. The voltage then decreases by (*IR*). The current is equal to $I = \frac{\varepsilon}{r+R}$.

The current through the load resistor is $I = \frac{\varepsilon}{r+R}$. We see from this expression that the smaller the internal resistance r, the greater the current the voltage source supplies to its load R. As batteries are depleted, r increases. If r becomes a significant fraction of the load resistance, then the current is significantly reduced, as the following example illustrates.

EXAMPLE 10.1

Analyzing a Circuit with a Battery and a Load

A given battery has a 12.00-V emf and an internal resistance of $0.100\ \Omega$. (a) Calculate its terminal voltage when connected to a $10.00\text{-}\Omega$ load. (b) What is the terminal voltage when connected to a $0.500\text{-}\Omega$ load? (c) What power does the $0.500\text{-}\Omega$ load dissipate? (d) If the internal resistance grows to $0.500\ \Omega$, find the current, terminal voltage, and power dissipated by a $0.500\text{-}\Omega$ load.

Strategy

The analysis above gave an expression for current when internal resistance is taken into account. Once the current is found, the terminal voltage can be calculated by using the equation $V_{\text{terminal}} = \varepsilon - Ir$. Once current is found, we can also find the power dissipated by the resistor.

Solution

a. Entering the given values for the emf, load resistance, and internal resistance into the expression above yields

$$I = \frac{\varepsilon}{R + r} = \frac{12.00\ \text{V}}{10.10\ \Omega} = 1.188\ \text{A}.$$

Enter the known values into the equation $V_{\text{terminal}} = \varepsilon - Ir$ to get the terminal voltage:

$$V_{\text{terminal}} = \varepsilon - Ir = 12.00\ \text{V} - (1.188\ \text{A})(0.100\ \Omega) = 11.90\ \text{V}.$$

The terminal voltage here is only slightly lower than the emf, implying that the current drawn by this light load is not significant.

b. Similarly, with $R_{\text{load}} = 0.500\ \Omega$, the current is

$$I = \frac{\varepsilon}{R + r} = \frac{12.00\ \text{V}}{0.600\ \Omega} = 20.00\ \text{A}.$$

The terminal voltage is no

$$V_{\text{terminal}} = \varepsilon - Ir = 12.00\ \text{V} - (20.00\ \text{A})\,(0.100\ \Omega) = 10.00\ \text{V}.$$

The terminal voltage exhibits a more significant reduction compared with emf, implying $0.500\ \Omega$ is a heavy load for this battery. A “heavy load” signifies a larger draw of current from the source but not a larger resistance.

c. The power dissipated by the $0.500\text{-}\Omega$ load can be found using the formula $P = I^2R$. Entering the known values gives

$$P = I^2R = (20.0\ \text{A})^2(0.500\ \Omega) = 2.00 \times 10^2\ \text{W}.$$

Note that this power can also be obtained using the expression $\frac{V^2}{R}$ or IV, where V is the terminal voltage (10.0 V in this case).

d. Here, the internal resistance has increased, perhaps due to the depletion of the battery, to the point where it is as great as the load resistance. As before, we first find the current by entering the known values into the expression, yielding

$$I = \frac{\varepsilon}{R + r} = \frac{12.00\ \text{V}}{1.00\ \Omega} = 12.00\ \text{A}.$$

Now the terminal voltage is

$$V_{\text{terminal}} = \varepsilon - Ir = 12.00\ \text{V} - (12.00\ \text{A})(0.500\ \Omega) = 6.00\ \text{V},$$

and the power dissipated by the load is

$$P = I^2R = (12.00\ \text{A})^2(0.500\ \Omega) = 72.00\ \text{W}.$$

We see that the increased internal resistance has significantly decreased the terminal voltage, current, and power delivered to a load.

Significance

The internal resistance of a battery can increase for many reasons. For example, the internal resistance of a rechargeable battery increases as the number of times the battery is recharged increases. The increased internal resistance may have two effects on the battery. First, the terminal voltage will decrease. Second, the battery may overheat due to the increased power dissipated by the internal resistance.

CHECK YOUR UNDERSTANDING 10.1

If you place a wire directly across the two terminal of a battery, effectively shorting out the terminals, the battery will begin to get hot. Why do you suppose this happens?

10.2 Resistors in Series and Parallel

Learning Objectives

By the end of this section, you will be able to:

- Define the term equivalent resistance
- Calculate the equivalent resistance of resistors connected in series
- Calculate the equivalent resistance of resistors connected in parallel

In Current and Resistance, we described the term 'resistance' and explained the basic design of a resistor. Basically, a resistor limits the flow of charge in a circuit and is an ohmic device where $V = IR$. Most circuits have more than one resistor. If several resistors are connected together and connected to a battery, the current supplied by the battery depends on the **equivalent resistance** of the circuit.

The equivalent resistance of a combination of resistors depends on both their individual values and how they are connected. The simplest combinations of resistors are series and parallel connections (Figure 10.11). In a series circuit, the output current of the first resistor flows into the input of the second resistor; therefore, the current is the same in each resistor. In a parallel circuit, all of the resistor leads on one side of the resistors are connected together and all the leads on the other side are connected together. In the case of a parallel configuration, each resistor has the same potential drop across it, and the currents through each resistor may be different, depending on the resistor. The sum of the individual currents equals the current that flows into the parallel connections.

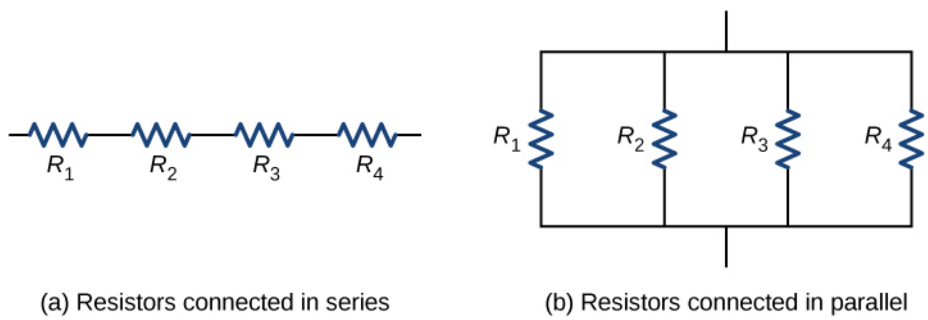


Figure 10.11 (a) For a series connection of resistors, the current is the same in each resistor. (b) For a parallel connection of resistors, the voltage is the same across each resistor.

Resistors in Series

Resistors are said to be in series whenever the current flows through the resistors sequentially. Consider Figure 10.12, which shows three resistors in series with an applied voltage equal to V_{ab}. Since there is only one path for the charges to flow through, the current is the same through each resistor. The equivalent resistance of a set of resistors in a series connection is equal to the algebraic sum of the individual resistances.

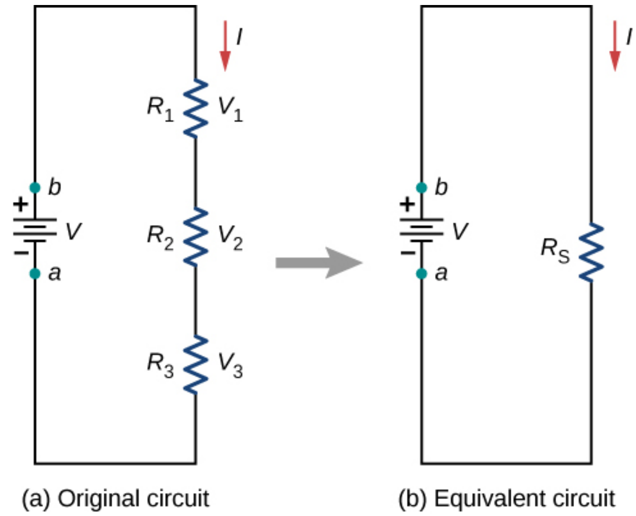


Figure 10.12 (a) Three resistors connected in series to a voltage source. (b) The original circuit is reduced to an equivalent resistance and a voltage source.

In Figure 10.12, the current coming from the voltage source flows through each resistor, so the current through each resistor is the same. The current through the circuit depends on the voltage supplied by the voltage source and the resistance of the resistors. For each resistor, a potential drop occurs that is equal to the loss of electric potential energy as a current travels through each resistor. According to Ohm's law, the potential drop V across a resistor when a current flows through it is calculated using the equation $V = IR$, where I is the current in amps (A) and R is the resistance in ohms (Ω). Since energy is conserved, and the voltage is equal to the potential energy per charge, the sum of the voltage applied to the circuit by the source and the potential drops across the individual resistors around a loop should be equal to zero:

$$\sum_{i=1}^{N} V_i = 0.$$

This equation is often referred to as Kirchhoff's loop law, which we will look at in more detail later in this chapter. For Figure 10.12, the sum of the potential drop of each resistor and the voltage supplied by the voltage source should equal zero:

$$\begin{aligned} V - V_1 - V_2 - V_3 &= 0, \\ V &= V_1 + V_2 + V_3, \\ &= IR_1 + IR_2 + IR_3, \\ I &= \frac{V}{R_1 + R_2 + R_3} = \frac{V}{R_S}. \end{aligned}$$

Since the current through each component is the same, the equality can be simplified to an equivalent resistance, which is just the sum of the resistances of the individual resistors.

Any number of resistors can be connected in series. If N resistors are connected in series, the equivalent resistance is

$$R_S = R_1 + R_2 + R_3 + \cdots + R_{N-1} + R_N = \sum_{i=1}^{N} R_i. \qquad 10.2$$

One result of components connected in a series circuit is that if something happens to one component, it affects all the other components. For example, if several lamps are connected in series and one bulb burns out, all the other lamps go dark.

EXAMPLE 10.2

Equivalent Resistance, Current, and Power in a Series Circuit

A battery with a terminal voltage of 9 V is connected to a circuit consisting of four $20\text{-}\Omega$ and one $10\text{-}\Omega$ resistors all in series (Figure 10.13). Assume the battery has negligible internal resistance. (a) Calculate the equivalent resistance of the circuit. (b) Calculate the current through each resistor. (c) Calculate the potential drop across each resistor. (d) Determine the total power dissipated by the resistors and the power supplied by the battery.

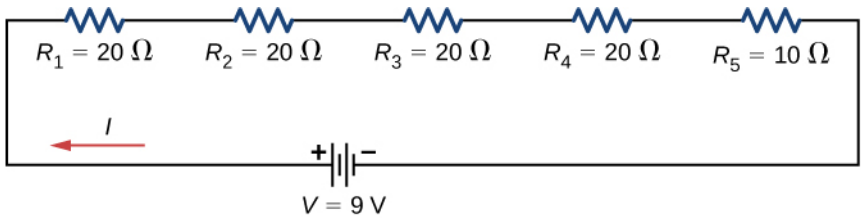


Figure 10.13 A simple series circuit with five resistors.

Strategy

In a series circuit, the equivalent resistance is the algebraic sum of the resistances. The current through the circuit can be found from Ohm's law and is equal to the voltage divided by the equivalent resistance. The potential drop across each resistor can be found using Ohm's law. The power dissipated by each resistor can be found using $P = I^2 R$, and the total power dissipated by the resistors is equal to the sum of the power dissipated by each resistor. The power supplied by the battery can be found using $P = I\varepsilon$.

Solution

a. The equivalent resistance is the algebraic sum of the resistances:

$$R_S = R_1 + R_2 + R_3 + R_4 + R_5 = 20\ \Omega + 20\ \Omega + 20\ \Omega + 20\ \Omega + 10\ \Omega = 90\ \Omega.$$

b. The current through the circuit is the same for each resistor in a series circuit and is equal to the applied voltage divided by the equivalent resistance:

$$I = \frac{V}{R_S} = \frac{9\ \text{V}}{90\ \Omega} = 0.1\ \text{A}.$$

c. The potential drop across each resistor can be found using Ohm's law:

$$\begin{aligned} V_1 = V_2 = V_3 = V_4 &= (0.1\ \text{A})\,20\ \Omega = 2\ \text{V}, \\ V_5 &= (0.1\ \text{A})\,10\ \Omega = 1\ \text{V}, \\ V_1 + V_2 + V_3 + V_4 + V_5 &= 9\ \text{V}. \end{aligned}$$

Note that the sum of the potential drops across each resistor is equal to the voltage supplied by the battery.

d. The power dissipated by a resistor is equal to $P = I^2 R$, and the power supplied by the battery is equal to $P = I\varepsilon$:

$$\begin{aligned} P_1 = P_2 = P_3 = P_4 &= (0.1\ \text{A})^2\,(20\ \Omega) = 0.2\ \text{W}, \\ P_5 &= (0.1\ \text{A})^2\,(10\ \Omega) = 0.1\ \text{W}, \\ P_{\text{dissipated}} &= 0.2\ \text{W} + 0.2\ \text{W} + 0.2\ \text{W} + 0.2\ \text{W} + 0.1\ \text{W} = 0.9\ \text{W}, \\ P_{\text{source}} &= I\varepsilon = (0.1\ \text{A})\,(9\ \text{V}) = 0.9\ \text{W}. \end{aligned}$$

Significance

There are several reasons why we would use multiple resistors instead of just one resistor with a resistance equal to the equivalent resistance of the circuit. Perhaps a resistor of the required size is not available, or we need to dissipate the heat generated, or we want to minimize the cost of resistors. Each resistor may cost a few cents to a few dollars, but when multiplied by thousands of units, the cost saving may be appreciable.

CHECK YOUR UNDERSTANDING 10.2

Some strings of miniature holiday lights are made to short out when a bulb burns out. The device that causes the short is called a shunt, which allows current to flow around the open circuit. A “short” is like putting a piece of wire across the component. The bulbs are usually grouped in series of nine bulbs. If too many bulbs burn out, the shunts eventually open. What causes this?

Let’s briefly summarize the major features of resistors in series:

1. Series resistances add together to get the equivalent resistance:

$$R_S = R_1 + R_2 + R_3 + \cdots + R_{N-1} + R_N = \sum_{i=1}^{N} R_i.$$

2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it. The total potential drop across a series configuration of resistors is equal to the sum of the potential drops across each resistor.

Resistors in Parallel

Figure 10.14 shows resistors in parallel, wired to a voltage source. Resistors are in parallel when one end of all the resistors are connected by a continuous wire of negligible resistance and the other end of all the resistors are also connected to one another through a continuous wire of negligible resistance. The potential drop across each resistor is the same. Current through each resistor can be found using Ohm’s law $I = V/R$, where the voltage is constant across each resistor. For example, an automobile’s headlights, radio, and other systems are wired in parallel, so that each subsystem utilizes the full voltage of the source and can operate completely independently. The same is true of the wiring in your house or any building.

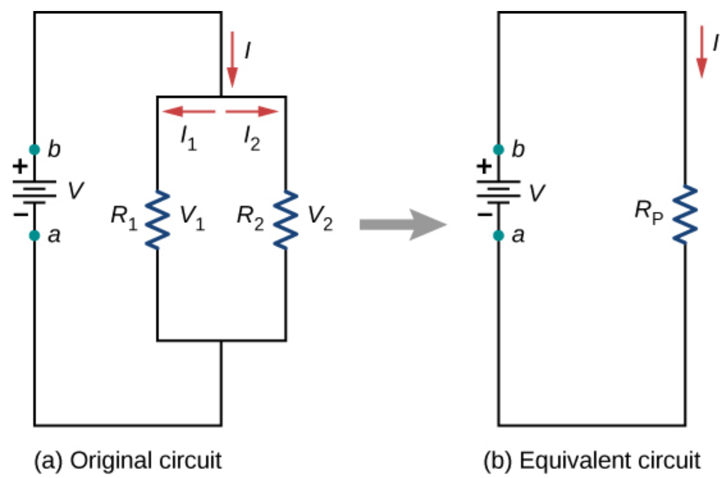


Figure 10.14 (a) Two resistors connected in parallel to a voltage source. (b) The original circuit is reduced to an equivalent resistance and a voltage source.

The current flowing from the voltage source in Figure 10.14 depends on the voltage supplied by the voltage source and the equivalent resistance of the circuit. In this case, the current flows from the voltage source and enters a junction, or node, where the circuit splits flowing through resistors R_1 and R_2. As the charges flow from the battery, some go through resistor R_1 and some flow through resistor R_2. The sum of the currents flowing into a junction must be equal to the sum of the currents flowing out of the junction:

$$\sum I_{in} = \sum I_{out}.$$

This equation is referred to as Kirchhoff’s junction rule and will be discussed in detail in the next section. In Figure 10.14, the junction rule gives $I = I_1 + I_2$. There are two loops in this circuit, which leads to the equations $V = I_1 R_1$ and $I_1 R_1 = I_2 R_2$. Note the voltage across the resistors in parallel are the same $(V = V_1 = V_2)$ and the current is additive:

$$\begin{aligned} I &= I_1 + I_2 \\ &= \frac{V_1}{R_1} + \frac{V_2}{R_2} \\ &= \frac{V}{R_1} + \frac{V}{R_2} \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_P} \\ R_P &= \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}. \end{aligned}$$

Generalizing to any number of N resistors, the equivalent resistance R_P of a parallel connection is related to the individual resistances by

$$R_P = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_{N-1}} + \frac{1}{R_N}\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1}. \quad 10.3$$

This relationship results in an equivalent resistance R_P that is less than the smallest of the individual resistances. When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, so the total resistance is lower.

EXAMPLE 10.3

Analysis of a Parallel Circuit

Three resistors $R_1 = 1.00\,\Omega$, $R_2 = 2.00\,\Omega$, and $R_3 = 2.00\,\Omega$, are connected in parallel. The parallel connection is attached to a $V = 3.00\text{ V}$ voltage source. (a) What is the equivalent resistance? (b) Find the current supplied by the source to the parallel circuit. (c) Calculate the currents in each resistor and show that these add together to equal the current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source and show that it equals the total power dissipated by the resistors.

Strategy

(a) The total resistance for a parallel combination of resistors is found using $R_P = \left(\sum_i \frac{1}{R_i}\right)^{-1}$.

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

(b) The current supplied by the source can be found from Ohm's law, substituting R_P for the total resistance $I = \frac{V}{R_P}$.

(c) The individual currents are easily calculated from Ohm's law $\left(I_i = \frac{V_i}{R_i}\right)$, since each resistor gets the full voltage. The total current is the sum of the individual currents: $I = \sum_i I_i$.

(d) The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P_i = V^2/R_i$, since each resistor gets full voltage.

(e) The total power can also be calculated in several ways, use $P = IV$.

Solution

a. The total resistance for a parallel combination of resistors is found using Equation 10.3. Entering known values gives

$$R_P = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{1.00\,\Omega} + \frac{1}{2.00\,\Omega} + \frac{1}{2.00\,\Omega}\right)^{-1} = 0.50\,\Omega.$$

The total resistance with the correct number of significant digits is $R_P = 0.50\,\Omega$. As predicted, R_P is less than the smallest individual resistance.

b. The total current can be found from Ohm's law, substituting R_P for the total resistance. This gives

$$I = \frac{V}{R_P} = \frac{3.00\,\text{V}}{0.50\,\Omega} = 6.00\,\text{A}.$$

Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.

c. The individual currents are easily calculated from Ohm's law, since each resistor gets the full voltage. Thus,

$$I_1 = \frac{V}{R_1} = \frac{3.00\,\text{V}}{1.00\,\Omega} = 3.00\,\text{A}.$$

Similarly,

$$I_2 = \frac{V}{R_2} = \frac{3.00\,\text{V}}{2.00\,\Omega} = 1.50\,\text{A}$$

and

$$I_3 = \frac{V}{R_3} = \frac{3.00\,\text{V}}{2.00\,\Omega} = 1.50\,\text{A}.$$

The total current is the sum of the individual currents:

$$I_1 + I_2 + I_3 = 6.00\,\text{A}.$$

d. The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P = V^2/R$, since each resistor gets full voltage. Thus,

$$P_1 = \frac{V^2}{R_1} = \frac{(3.00\,\text{V})^2}{1.00\,\Omega} = 9.00\,\text{W}.$$

Similarly,

$$P_2 = \frac{V^2}{R_2} = \frac{(3.00\,\text{V})^2}{2.00\,\Omega} = 4.50\,\text{W}$$

and

$$P_3 = \frac{V^2}{R_3} = \frac{(3.00\,\text{V})^2}{2.00\,\Omega} = 4.50\,\text{W}.$$

e. The total power can also be calculated in several ways. Choosing $P = IV$ and entering the total current yields

$$P = IV = (6.00\,\text{A})(3.00\,\text{V}) = 18.00\,\text{W}.$$

Significance

Total power dissipated by the resistors is also 18.00 W:

$$P_1 + P_2 + P_3 = 9.00\,\text{W} + 4.50\,\text{W} + 4.50\,\text{W} = 18.00\,\text{W}.$$

Notice that the total power dissipated by the resistors equals the power supplied by the source.

CHECK YOUR UNDERSTANDING 10.3

Consider the same potential difference ($V = 3.00\,\text{V}$) applied to the same three resistors connected in series. Would the equivalent resistance of the series circuit be higher, lower, or equal to the three resistor in parallel? Would the current through the series circuit be higher, lower, or equal to the current provided by the same voltage applied to the parallel circuit? How would the power dissipated by the resistor in series compare to the power dissipated by the resistors in parallel?

CHECK YOUR UNDERSTANDING 10.4

How would you use a river and two waterfalls to model a parallel configuration of two resistors? How does this analogy break down?

Let us summarize the major features of resistors in parallel:

1. Equivalent resistance is found from

$$R_P = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_{N-1}} + \frac{1}{R_N}\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1},$$

and is smaller than any individual resistance in the combination.
2. The potential drop across each resistor in parallel is the same.
3. Parallel resistors do not each get the total current; they divide it. The current entering a parallel combination of resistors is equal to the sum of the current through each resistor in parallel.

In this chapter, we introduced the equivalent resistance of resistors connect in series and resistors connected in parallel. You may recall that in Capacitance, we introduced the equivalent capacitance of capacitors connected in series and parallel. Circuits often contain both capacitors and resistors. Table 10.1 summarizes the equations used for the equivalent resistance and equivalent capacitance for series and parallel connections.

	Series combination	Parallel combination
Equivalent capacitance	$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$	$C_P = C_1 + C_2 + C_3 + \cdots$
Equivalent resistance	$R_S = R_1 + R_2 + R_3 + \cdots = \sum_{i=1}^{N} R_i$	$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

Table 10.1 Summary for Equivalent Resistance and Capacitance in Series and Parallel Combinations

Combinations of Series and Parallel

More complex connections of resistors are often just combinations of series and parallel connections. Such combinations are common, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in Figure 10.15. Various parts can be identified as either series or parallel connections, reduced to their equivalent resistances, and then further reduced until a single equivalent resistance is left. The process is more time consuming than difficult. Here, we note the equivalent resistance as R_{eq}.

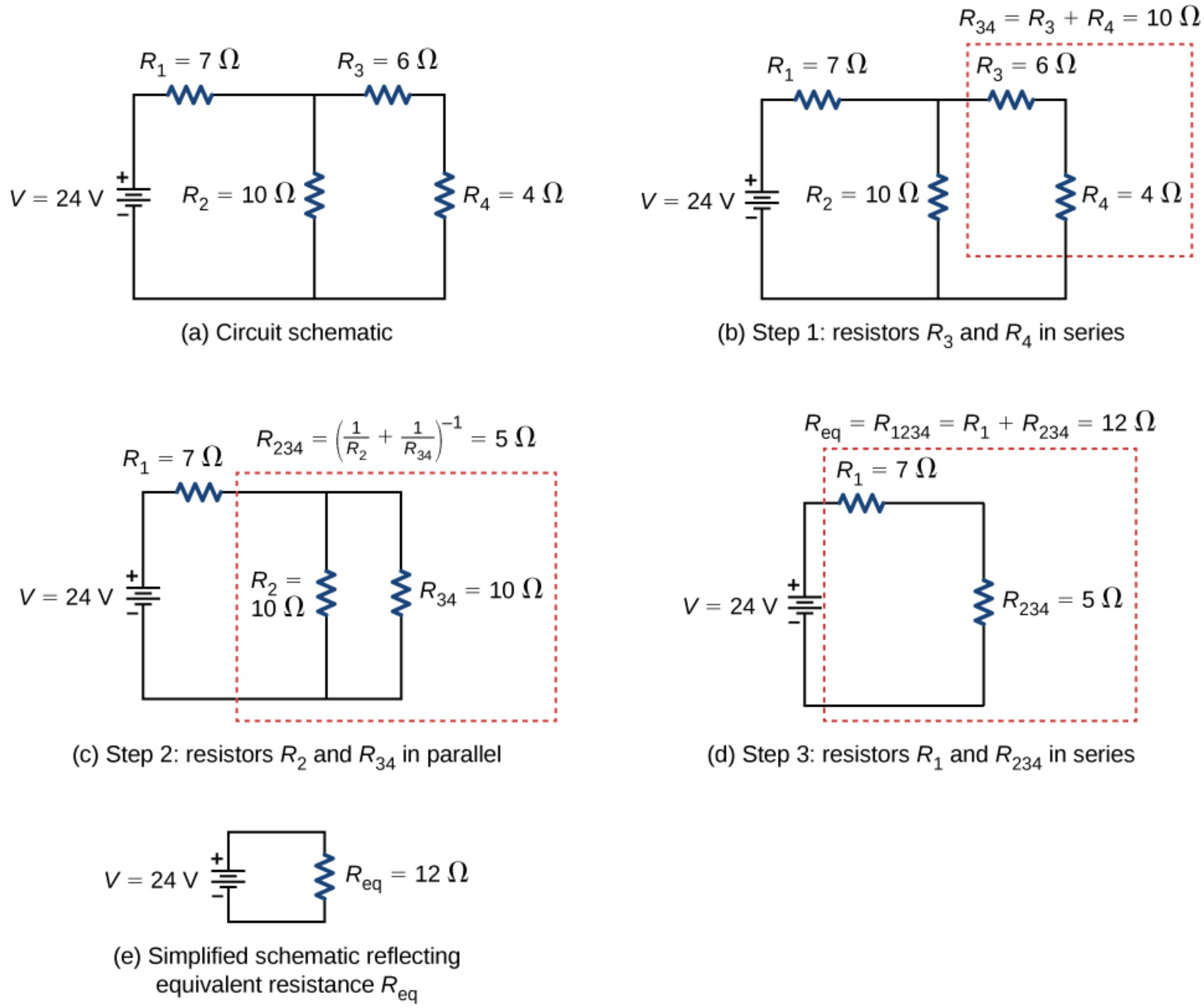


Figure 10.15 (a) The original circuit of four resistors. (b) Step 1: The resistors R_3 and R_4 are in series and the equivalent resistance is $R_{34} = 10\,\Omega$. (c) Step 2: The reduced circuit shows resistors R_2 and R_{34} are in parallel, with an equivalent resistance of $R_{234} = 5\,\Omega$. (d) Step 3: The reduced circuit shows that R_1 and R_{234} are in series with an equivalent resistance of $R_{1234} = 12\,\Omega$, which is the equivalent resistance R_{eq}. (e) The reduced circuit with a voltage source of $V = 24\text{ V}$ with an equivalent resistance of $R_{\text{eq}} = 12\,\Omega$. This results in a current of $I = 2\text{ A}$ from the voltage source.

Notice that resistors R_3 and R_4 are in series. They can be combined into a single equivalent resistance. One method of keeping track of the process is to include the resistors as subscripts. Here the equivalent resistance of R_3 and R_4 is

$$R_{34} = R_3 + R_4 = 6\,\Omega + 4\,\Omega = 10\,\Omega.$$

The circuit now reduces to three resistors, shown in Figure 10.15(c). Redrawing, we now see that resistors R_2 and R_{34} constitute a parallel circuit. Those two resistors can be reduced to an equivalent resistance:

$$R_{234} = \left(\frac{1}{R_2} + \frac{1}{R_{34}}\right)^{-1} = \left(\frac{1}{10\,\Omega} + \frac{1}{10\,\Omega}\right)^{-1} = 5\,\Omega.$$

This step of the process reduces the circuit to two resistors, shown in in Figure 10.15(d). Here, the circuit reduces to two resistors, which in this case are in series. These two resistors can be reduced to an equivalent resistance, which is the equivalent resistance of the circuit:

$$R_{\text{eq}} = R_{1234} = R_1 + R_{234} = 7\,\Omega + 5\,\Omega = 12\,\Omega.$$

The main goal of this circuit analysis is reached, and the circuit is now reduced to a single resistor and single voltage source.

Now we can analyze the circuit. The current provided by the voltage source is $I = \frac{V}{R_{eq}} = \frac{24\text{ V}}{12\,\Omega} = 2\text{ A}$. This current runs through resistor R_1 and is designated as I_1. The potential drop across R_1 can be found using Ohm's law:

$$V_1 = I_1 R_1 = (2\text{ A})(7\,\Omega) = 14\text{ V}.$$

Looking at Figure 10.15(c), this leaves $24\text{ V} - 14\text{ V} = 10\text{ V}$ to be dropped across the parallel combination of R_2 and R_{34}. The current through R_2 can be found using Ohm's law:

$$I_2 = \frac{V_2}{R_2} = \frac{10\text{ V}}{10\,\Omega} = 1\text{ A}.$$

The resistors R_3 and R_4 are in series so the currents I_3 and I_4 are equal to

$$I_3 = I_4 = I - I_2 = 2\text{ A} - 1\text{ A} = 1\text{ A}.$$

Using Ohm's law, we can find the potential drop across the last two resistors. The potential drops are $V_3 = I_3 R_3 = 6\text{ V}$ and $V_4 = I_4 R_4 = 4\text{ V}$. The final analysis is to look at the power supplied by the voltage source and the power dissipated by the resistors. The power dissipated by the resistors is

$$\begin{aligned} P_1 &= I_1^2 R_1 = (2\text{ A})^2\,(7\,\Omega) = 28\text{ W}, \\ P_2 &= I_2^2 R_2 = (1\text{ A})^2\,(10\,\Omega) = 10\text{ W}, \\ P_3 &= I_3^2 R_3 = (1\text{ A})^2\,(6\,\Omega) = 6\text{ W}, \\ P_4 &= I_4^2 R_4 = (1\text{ A})^2\,(4\,\Omega) = 4\text{ W}, \\ P_{\text{dissipated}} &= P_1 + P_2 + P_3 + P_4 = 48\text{ W}. \end{aligned}$$

The total energy is constant in any process. Therefore, the power supplied by the voltage source is $P_s = IV = (2\text{ A})(24\text{ V}) = 48\text{ W}$. Analyzing the power supplied to the circuit and the power dissipated by the resistors is a good check for the validity of the analysis; they should be equal.

EXAMPLE 10.4

Combining Series and Parallel Circuits

Figure 10.16 shows resistors wired in a combination of series and parallel. We can consider R_1 to be the resistance of wires leading to R_2 and R_3. (a) Find the equivalent resistance of the circuit. (b) What is the potential drop V_1 across resistor R_1? (c) Find the current I_2 through resistor R_2. (d) What power is dissipated by R_2?

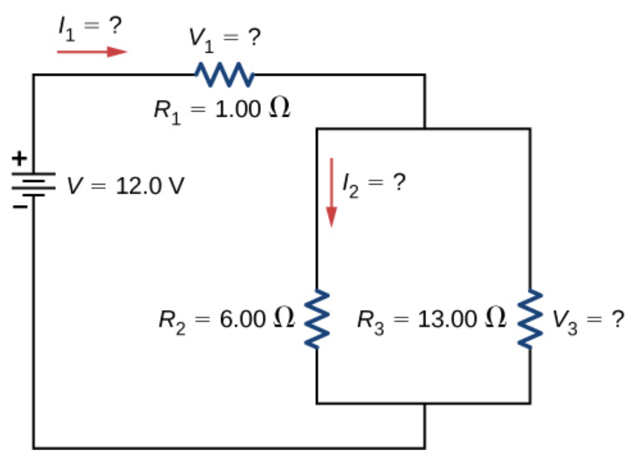


Figure 10.16 These three resistors are connected to a voltage source so that R_2 and R_3 are in parallel with one another and that combination is in series with R_1.

Strategy

(a) To find the equivalent resistance, first find the equivalent resistance of the parallel connection of R_2 and R_3. Then use this result to find the equivalent resistance of the series connection with R_1.

(b) The current through R_1 can be found using Ohm's law and the voltage applied. The current through R_1 is equal to the current from the battery. The potential drop V_1 across the resistor R_1 (which represents the resistance in the connecting wires) can be found using Ohm's law.

(c) The current through R_2 can be found using Ohm's law $I_2 = \frac{V_2}{R_2}$. The voltage across R_2 can be found using $V_2 = V - V_1$.

(d) Using Ohm's law ($V_2 = I_2 R_2$), the power dissipated by the resistor can also be found using $P_2 = I_2^2 R_2 = \frac{V_2^2}{R_2}$.

Solution

a. To find the equivalent resistance of the circuit, notice that the parallel connection of R_2 and R_3 is in series with R_1, so the equivalent resistance is

$$R_{\text{eq}} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = 1.00\,\Omega + \left(\frac{1}{6.00\,\Omega} + \frac{1}{13.00\,\Omega}\right)^{-1} = 5.10\,\Omega.$$

The total resistance of this combination is intermediate between the pure series and pure parallel values ($20.0\,\Omega$ and $0.804\,\Omega$, respectively).

b. The current through R_1 is equal to the current supplied by the battery:

$$I_1 = I = \frac{V}{R_{\text{eq}}} = \frac{12.0\text{ V}}{5.10\,\Omega} = 2.35\text{ A}.$$

The voltage across R_1 is

$$V_1 = I_1 R_1 = (2.35\text{ A})(1\,\Omega) = 2.35\text{ V}.$$

The voltage applied to R_2 and R_3 is less than the voltage supplied by the battery by an amount V_1. When wire resistance is large, it can significantly affect the operation of the devices represented by R_2 and R_3.

c. To find the current through R_2, we must first find the voltage applied to it. The voltage across the two resistors in parallel is the same:

$$V_2 = V_3 = V - V_1 = 12.0\text{ V} - 2.35\text{ V} = 9.65\text{ V}.$$

Now we can find the current I_2 through resistance R_2 using Ohm's law:

$$I_2 = \frac{V_2}{R_2} = \frac{9.65\text{ V}}{6.00\,\Omega} = 1.61\text{ A}.$$

The current is less than the 2.00 A that flowed through R_2 when it was connected in parallel to the battery in the previous parallel circuit example.

d. The power dissipated by R_2 is given by

$$P_2 = I_2^2 R_2 = (1.61\text{ A})^2(6.00\,\Omega) = 15.5\text{ W}.$$

Significance

The analysis of complex circuits can often be simplified by reducing the circuit to a voltage source and an equivalent resistance. Even if the entire circuit cannot be reduced to a single voltage source and a single equivalent resistance, portions of the circuit may be reduced, greatly simplifying the analysis.

CHECK YOUR UNDERSTANDING 10.5

Consider the electrical circuits in your home. Give at least two examples of circuits that must use a combination of series and parallel circuits to operate efficiently.

Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the *IR* drop in the wires can also be significant and may become apparent from the heat generated in the cord.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in Figure 10.17. The device represented by R_3 has a very low resistance, so when it is switched on, a large current flows. This increased current causes a larger *IR* drop in the wires represented by R_1, reducing the voltage across the light bulb (which is R_2), which then dims noticeably.

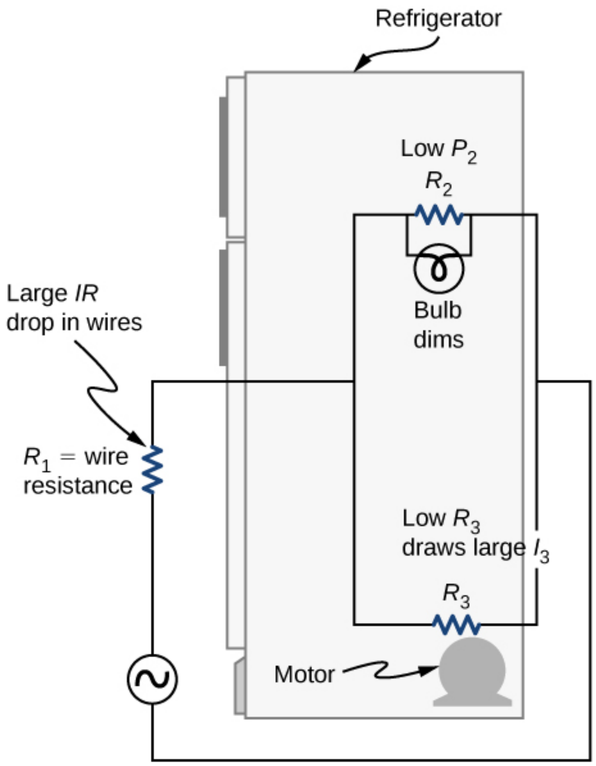


Figure 10.17 Why do lights dim when a large appliance is switched on? The answer is that the large current the appliance motor draws causes a significant *IR* drop in the wires and reduces the voltage across the light.

PROBLEM-SOLVING STRATEGY

Series and Parallel Resistors

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the known values for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel.

5. Check to see whether the answers are reasonable and consistent.

EXAMPLE 10.5

Combining Series and Parallel Circuits

Two resistors connected in series (R_1, R_2) are connected to two resistors that are connected in parallel (R_3, R_4). The series-parallel combination is connected to a battery. Each resistor has a resistance of 10.00 Ohms. The wires connecting the resistors and battery have negligible resistance. A current of 2.00 Amps runs through resistor R_1. What is the voltage supplied by the voltage source?

Strategy

Use the steps in the preceding problem-solving strategy to find the solution for this example.

Solution

1. Draw a clear circuit diagram (Figure 10.18).

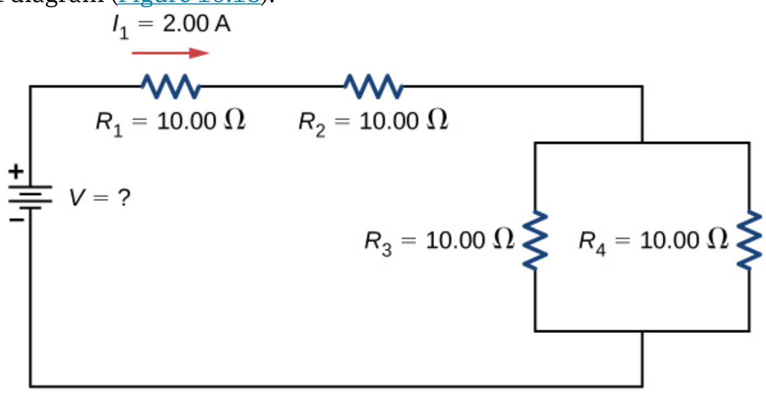


Figure 10.18 To find the unknown voltage, we must first find the equivalent resistance of the circuit.

2. The unknown is the voltage of the battery. In order to find the voltage supplied by the battery, the equivalent resistance must be found.
3. In this circuit, we already know that the resistors R_1 and R_2 are in series and the resistors R_3 and R_4 are in parallel. The equivalent resistance of the parallel configuration of the resistors R_3 and R_4 is in series with the series configuration of resistors R_1 and R_2.
4. The voltage supplied by the battery can be found by multiplying the current from the battery and the equivalent resistance of the circuit. The current from the battery is equal to the current through R_1 and is equal to 2.00 A. We need to find the equivalent resistance by reducing the circuit. To reduce the circuit, first consider the two resistors in parallel. The equivalent resistance is $R_{34} = \left(\frac{1}{10.00\ \Omega} + \frac{1}{10.00\ \Omega}\right)^{-1} = 5.00\ \Omega$. This parallel combination is in series with the other two resistors, so the equivalent resistance of the circuit is $R_{eq} = R_1 + R_2 + R_{34} = 25.00\ \Omega$. The voltage supplied by the battery is therefore $V = IR_{eq} = 2.00\ \text{A}\ (25.00\ \Omega) = 50.00\ \text{V}$.
5. One way to check the consistency of your results is to calculate the power supplied by the battery and the power dissipated by the resistors. The power supplied by the battery is $P_{batt} = IV = 100.00\ \text{W}$. Since they are in series, the current through R_2 equals the current through R_1. Since $R_3 = R_4$, the current through each will be 1.00 Amps. The power dissipated by the resistors is equal to the sum of the power dissipated by each resistor:

$$P = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4 = 40.00\ \text{W} + 40.00\ \text{W} + 10.00\ \text{W} + 10.00\ \text{W} = 100.00\ \text{W}.$$

Since the power dissipated by the resistors equals the power supplied by the battery, our solution seems consistent.

Significance

If a problem has a combination of series and parallel, as in this example, it can be reduced in steps by using the preceding problem-solving strategy and by considering individual groups of series or parallel connections. When finding R_{eq} for a parallel connection, the reciprocal must be taken with care. In addition, units and numerical results must be reasonable. Equivalent series resistance should be greater, whereas equivalent parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

10.3 Kirchhoff's Rules

Learning Objectives

By the end of this section, you will be able to:

- State Kirchhoff's junction rule
- State Kirchhoff's loop rule
- Analyze complex circuits using Kirchhoff's rules

We have just seen that some circuits may be analyzed by reducing a circuit to a single voltage source and an equivalent resistance. Many complex circuits cannot be analyzed with the series-parallel techniques developed in the preceding sections. In this section, we elaborate on the use of Kirchhoff's rules to analyze more complex circuits. For example, the circuit in Figure 10.19 is known as a multi-loop circuit, which consists of junctions. A junction, also known as a node, is a connection of three or more wires. In this circuit, the previous methods cannot be used, because not all the resistors are in clear series or parallel configurations that can be reduced. Give it a try. The resistors R_1 and R_2 are in series and can be reduced to an equivalent resistance. The same is true of resistors R_4 and R_5. But what do you do then?

Even though this circuit cannot be analyzed using the methods already learned, two circuit analysis rules can be used to analyze any circuit, simple or complex. The rules are known as **Kirchhoff's rules**, after their inventor Gustav Kirchhoff (1824–1887).

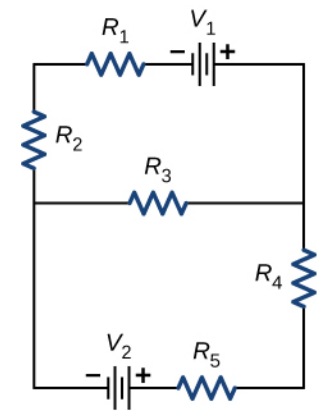


Figure 10.19 This circuit cannot be reduced to a combination of series and parallel connections. However, we can use Kirchhoff's rules to analyze it.

Kirchhoff's Rules

- Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction:

$$\sum I_{in} = \sum I_{out}. \tag{10.4}$$

- Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero:

$$\sum V = 0. \tag{10.5}$$

INTERACTIVE

In this virtual lab (https://openstax.org/l/21cirreslabsim) simulation, you may construct circuits with resistors, voltage sources, ammeters and voltmeters to test your knowledge of circuit design.

Ohmmeters

An ohmmeter is an instrument used to measure the resistance of a component or device. The operation of the ohmmeter is based on Ohm's law. Traditional ohmmeters contained an internal voltage source (such as a battery) that would be connected across the component to be tested, producing a current through the component. A galvanometer was then used to measure the current and the resistance was deduced using Ohm's law. Modern digital meters use a constant current source to pass current through the component, and the voltage difference across the component is measured. In either case, the resistance is measured using Ohm's law $(R = V/I)$, where the voltage is known and the current is measured, or the current is known and the voltage is measured.

The component of interest should be isolated from the circuit; otherwise, you will be measuring the equivalent resistance of the circuit. An ohmmeter should never be connected to a "live" circuit, one with a voltage source connected to it and current running through it. Doing so can damage the meter.

10.5 RC Circuits

Learning Objectives

By the end of this section, you will be able to:

- Describe the charging process of a capacitor
- Describe the discharging process of a capacitor
- List some applications of RC circuits

When you use a flash camera, it takes a few seconds to charge the capacitor that powers the flash. The light flash discharges the capacitor in a tiny fraction of a second. Why does charging take longer than discharging? This question and several other phenomena that involve charging and discharging capacitors are discussed in this module.

Circuits with Resistance and Capacitance

An ***RC* circuit** is a circuit containing resistance and capacitance. As presented in Capacitance, the capacitor is an electrical component that stores electric charge, storing energy in an electric field.

Figure 10.38(a) shows a simple *RC* circuit that employs a dc (direct current) voltage source ε, a resistor *R*, a capacitor *C*, and a two-position switch. The circuit allows the capacitor to be charged or discharged, depending on the position of the switch. When the switch is moved to position *A*, the capacitor charges, resulting in the circuit in part (b). When the switch is moved to position *B*, the capacitor discharges through the resistor.

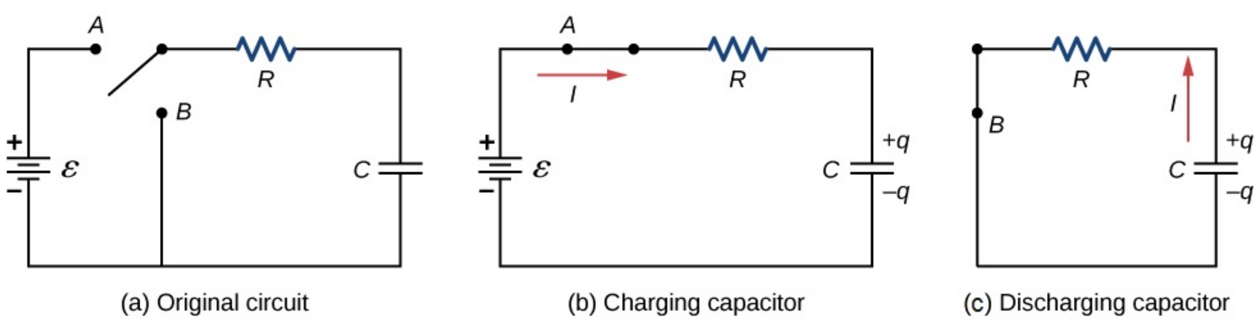


Figure 10.38 (a) An *RC* circuit with a two-pole switch that can be used to charge and discharge a capacitor. (b) When the switch is moved to position *A*, the circuit reduces to a simple series connection of the voltage source, the resistor, the capacitor, and the switch. (c) When the switch is moved to position *B*, the circuit reduces to a simple series connection of the resistor, the capacitor, and the switch. The voltage source is removed from the circuit.

Charging a Capacitor

We can use Kirchhoff's loop rule to understand the charging of the capacitor. This results in the equation $\varepsilon - V_R - V_c = 0$. This equation can be used to model the charge as a function of time as the capacitor charges. Capacitance is defined as $C = q/V$, so the voltage across the capacitor is $V_C = \frac{q}{C}$. Using Ohm's law, the potential drop across the resistor is $V_R = IR$, and the current is defined as $I = dq/dt$.

$$\begin{aligned} &\varepsilon - V_R - V_c = 0, \\ &\varepsilon - IR - \frac{q}{C} = 0, \\ &\varepsilon - R\frac{dq}{dt} - \frac{q}{C} = 0. \end{aligned}$$

This differential equation can be integrated to find an equation for the charge on the capacitor as a function of time.

$$\begin{aligned} &\varepsilon - R\frac{dq}{dt} - \frac{q}{C} = 0, \\ &\frac{dq}{dt} = \frac{\varepsilon C - q}{RC}, \\ &\int_0^q \frac{dq}{\varepsilon C - q} = \frac{1}{RC}\int_0^t dt. \end{aligned}$$

Let $u = \varepsilon C - q$, then $du = -dq$. The result is

$$\begin{aligned} &-\int_0^q \frac{du}{u} = \frac{1}{RC}\int_0^t dt, \\ &\ln\left(\frac{\varepsilon C - q}{\varepsilon C}\right) = -\frac{1}{RC}t, \\ &\frac{\varepsilon C - q}{\varepsilon C} = e^{-t/RC}. \end{aligned}$$

Simplifying results in an equation for the charge on the charging capacitor as a function of time:

$$q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{t}{\tau}}\right). \quad \text{10.8}$$

A graph of the charge on the capacitor versus time is shown in Figure 10.39(a). First note that as time approaches infinity, the exponential goes to zero, so the charge approaches the maximum charge $Q = C\varepsilon$ and has units of coulombs. The units of RC are seconds, units of time. This quantity is known as the time constant:

$$\tau = RC. \quad \text{10.9}$$

At time $t = \tau = RC$, the charge is equal to $1 - e^{-1} = 1 - 0.368 = 0.632$ of the maximum charge $Q = C\varepsilon$. Notice that the time rate change of the charge is the slope at a point of the charge versus time plot. The slope of the graph is large at time $t = 0.0$ s and approaches zero as time increases.

As the charge on the capacitor increases, the current through the resistor decreases, as shown in Figure

10.39(b). The current through the resistor can be found by taking the time derivative of the charge.

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}\left[C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)\right],$$

$$I(t) = C\varepsilon\left(\frac{1}{RC}\right)e^{-\frac{t}{RC}} = \frac{\varepsilon}{R}e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}},$$

$$I(t) = I_0 e^{-t/\tau}. \quad (10.10)$$

At time $t = 0.00$ s, the current through the resistor is $I_0 = \frac{\varepsilon}{R}$. As time approaches infinity, the current approaches zero. At time $t = \tau$, the current through the resistor is $I(t = \tau) = I_0 e^{-1} = 0.368 I_0$.

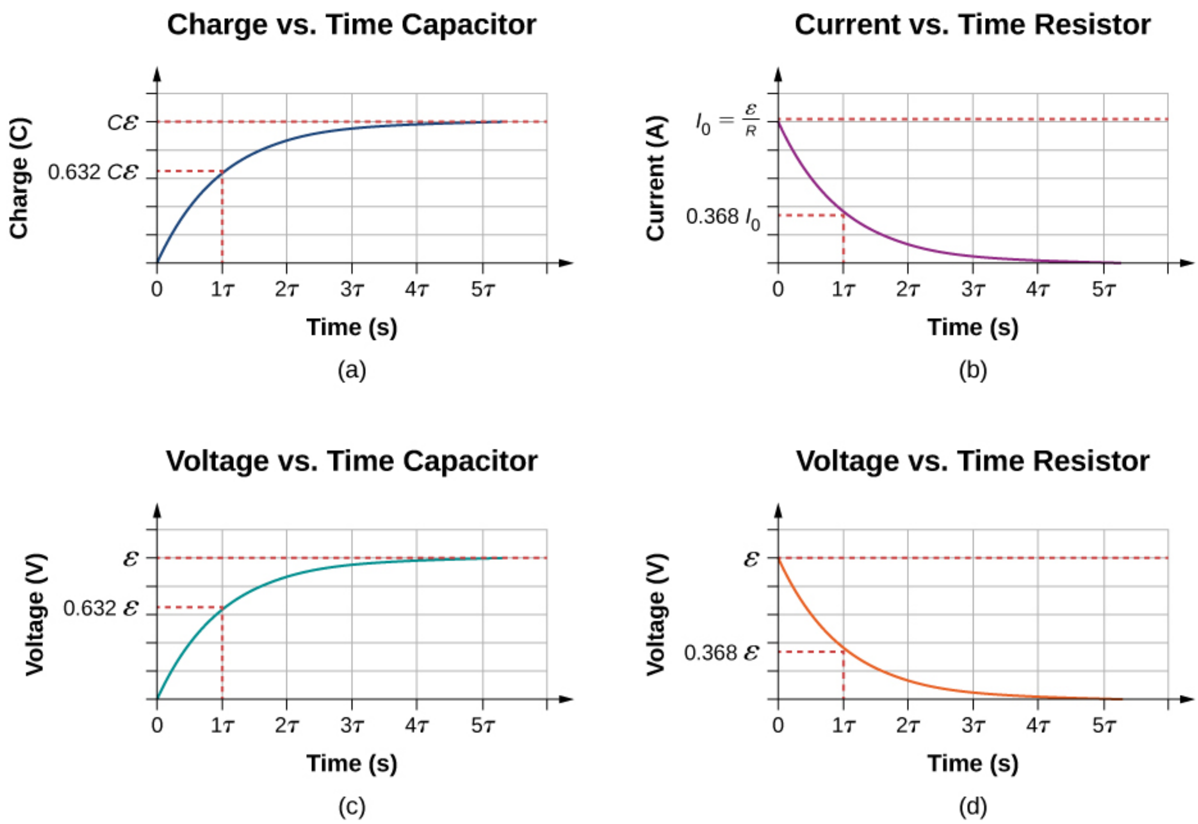


Figure 10.39 (a) Charge on the capacitor versus time as the capacitor charges. (b) Current through the resistor versus time. (c) Voltage difference across the capacitor. (d) Voltage difference across the resistor.

Figure 10.39(c) and Figure 10.39(d) show the voltage differences across the capacitor and the resistor, respectively. As the charge on the capacitor increases, the current decreases, as does the voltage difference across the resistor $V_R(t) = (I_0 R)e^{-t/\tau} = \varepsilon e^{-t/\tau}$. The voltage difference across the capacitor increases as $V_C(t) = \varepsilon\left(1 - e^{-t/\tau}\right)$.

Discharging a Capacitor

When the switch in Figure 10.38(a) is moved to position *B*, the circuit reduces to the circuit in part (c), and the charged capacitor is allowed to discharge through the resistor. A graph of the charge on the capacitor as a function of time is shown in Figure 10.40(a). Using Kirchhoff's loop rule to analyze the circuit as the capacitor discharges results in the equation $-V_R - V_c = 0$, which simplifies to $IR + \frac{q}{C} = 0$. Using the definition of current $\frac{dq}{dt}R = -\frac{q}{C}$ and integrating the loop equation yields an equation for the charge on the capacitor as a function of time:

$$q(t) = Qe^{-t/\tau}. \quad (10.11)$$

Here, Q is the initial charge on the capacitor and $\tau = RC$ is the time constant of the circuit. As shown in the graph, the charge decreases exponentially from the initial charge, approaching zero as time approaches infinity.

The current as a function of time can be found by taking the time derivative of the charge:

$$I(t) = -\frac{Q}{RC}e^{-t/\tau}. \quad (10.12)$$

The negative sign shows that the current flows in the opposite direction of the current found when the capacitor is charging. Figure 10.40(b) shows an example of a plot of charge versus time and current versus time. A plot of the voltage difference across the capacitor and the voltage difference across the resistor as a function of time are shown in parts (c) and (d) of the figure. Note that the magnitudes of the charge, current, and voltage all decrease exponentially, approaching zero as time increases.

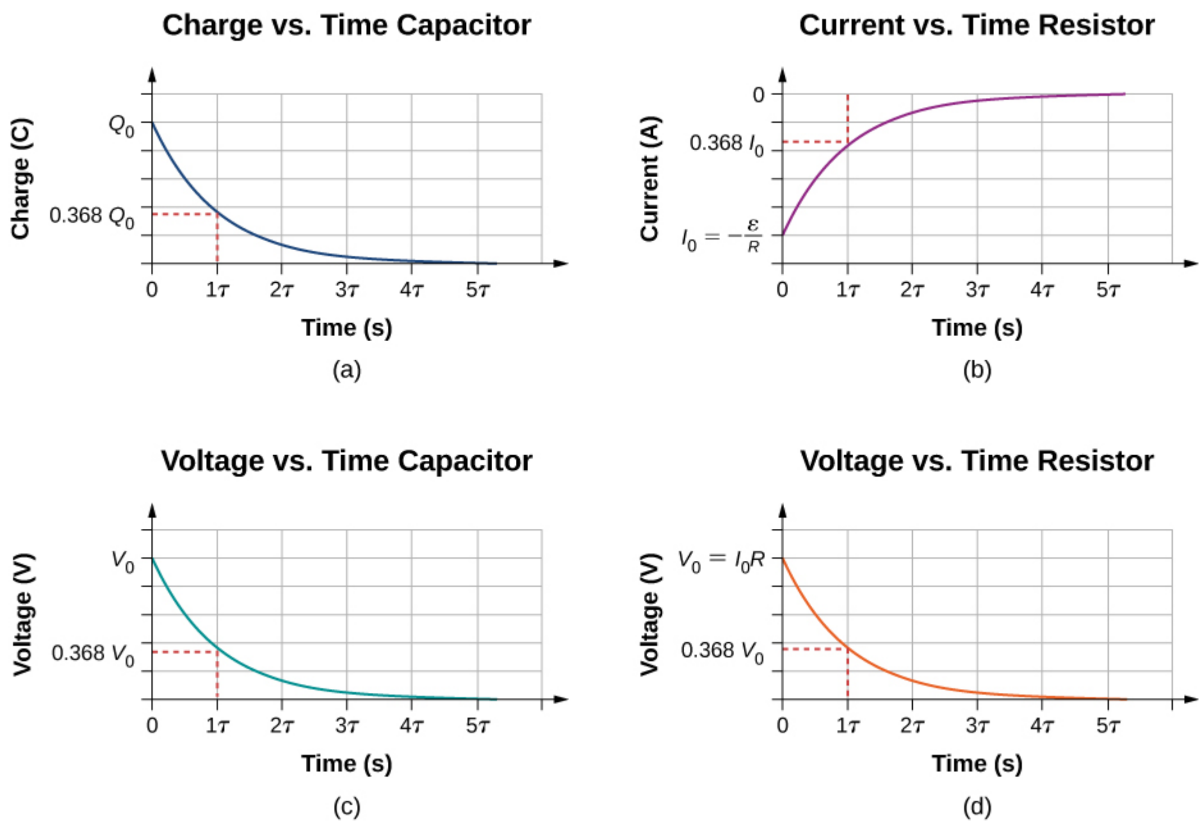


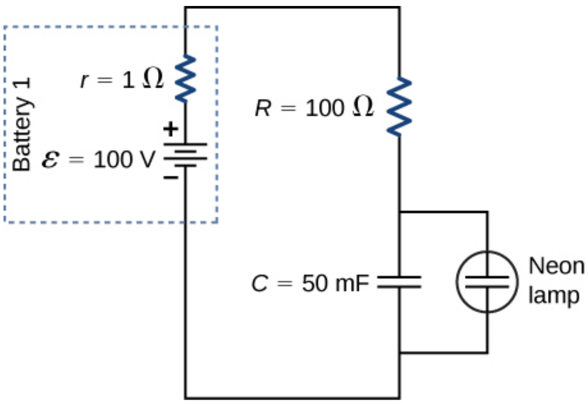
Figure 10.40 (a) Charge on the capacitor versus time as the capacitor discharges. (b) Current through the resistor versus time. (c) Voltage difference across the capacitor. (d) Voltage difference across the resistor.

Now we can explain why the flash camera mentioned at the beginning of this section takes so much longer to charge than discharge: The resistance while charging is significantly greater than while discharging. The internal resistance of the battery accounts for most of the resistance while charging. As the battery ages, the increasing internal resistance makes the charging process even slower.

EXAMPLE 10.8

The Relaxation Oscillator

One application of an RC circuit is the relaxation oscillator, as shown below. The relaxation oscillator consists of a voltage source, a resistor, a capacitor, and a neon lamp. The neon lamp acts like an open circuit (infinite resistance) until the potential difference across the neon lamp reaches a specific voltage. At that voltage, the lamp acts like a short circuit (zero resistance), and the capacitor discharges through the neon lamp and produces light. In the relaxation oscillator shown, the voltage source charges the capacitor until the voltage across the capacitor is 80 V. When this happens, the neon in the lamp breaks down and allows the capacitor to discharge through the lamp, producing a bright flash. After the capacitor fully discharges through the neon lamp, it begins to charge again, and the process repeats. Assuming that the time it takes the capacitor to discharge is negligible, what is the time interval between flashes?



Strategy

The time period can be found from considering the equation $V_C\,(t) = \varepsilon\left(1 - e^{-t/\tau}\right)$, where $\tau = (R + r)\,C$.

Solution

The neon lamp flashes when the voltage across the capacitor reaches 80 V. The *RC* time constant is equal to $\tau = (R + r)\,C = (101\ \Omega)\left(50\ \times\ 10^{-3}\mathrm{F}\right) = 5.05$ s. We can solve the voltage equation for the time it takes the capacitor to reach 80 V:

$$\begin{aligned} V_C\,(t) &= \varepsilon\left(1 - e^{-t/\tau}\right), \\ e^{-t/\tau} &= 1 - \frac{V_C(t)}{\varepsilon}, \\ \ln\left(e^{-t/\tau}\right) &= \ln\left(1 - \frac{V_C(t)}{\varepsilon}\right), \\ t &= -\tau\ln\left(1 - \frac{V_C(t)}{\varepsilon}\right) = -5.05\ \mathrm{s} \cdot \ln\left(1 - \frac{80\ \mathrm{V}}{100\ \mathrm{V}}\right) = 8.13\ \mathrm{s}. \end{aligned}$$

Significance

One application of the relaxation oscillator is for controlling indicator lights that flash at a frequency determined by the values for *R* and *C*. In this example, the neon lamp will flash every 8.13 seconds, a frequency of $f = \frac{1}{T} = \frac{1}{8.13\ \mathrm{s}} = 0.123$ Hz. The relaxation oscillator has many other practical uses. It is often used in electronic circuits, where the neon lamp is replaced by a transistor or a device known as a tunnel diode. The description of the transistor and tunnel diode is beyond the scope of this chapter, but you can think of them as voltage controlled switches. They are normally open switches, but when the right voltage is applied, the switch closes and conducts. The “switch” can be used to turn on another circuit, turn on a light, or run a small motor. A relaxation oscillator can be used to make the turn signals of your car blink or your cell phone to vibrate.

RC circuits have many applications. They can be used effectively as timers for applications such as intermittent windshield wipers, pace makers, and strobe lights. Some models of intermittent windshield wipers use a variable resistor to adjust the interval between sweeps of the wiper. Increasing the resistance increases the *RC* time constant, which increases the time between the operation of the wipers.

Another application is the pacemaker. The heart rate is normally controlled by electrical signals, which cause the muscles of the heart to contract and pump blood. When the heart rhythm is abnormal (the heartbeat is too high or too low), pace makers can be used to correct this abnormality. Pacemakers have sensors that detect body motion and breathing to increase the heart rate during physical activities, thus meeting the increased need for blood and oxygen, and an *RC* timing circuit can be used to control the time between voltage signals to the heart.

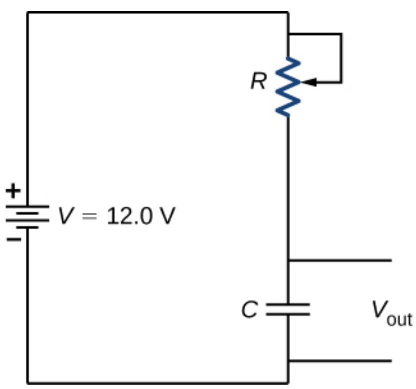
Looking ahead to the study of ac circuits (Alternating-Current Circuits), ac voltages vary as sine functions with specific frequencies. Periodic variations in voltage, or electric signals, are often recorded by scientists. These voltage signals could come from music recorded by a microphone or atmospheric data collected by radar. Occasionally, these signals can contain unwanted frequencies known as "noise." *RC* filters can be used to filter out the unwanted frequencies.

In the study of electronics, a popular device known as a 555 timer provides timed voltage pulses. The time between pulses is controlled by an *RC* circuit. These are just a few of the countless applications of *RC* circuits.

EXAMPLE 10.9

Intermittent Windshield Wipers

A relaxation oscillator is used to control a pair of windshield wipers. The relaxation oscillator consists of a 10.00-mF capacitor and a $10.00\text{-k}\Omega$ variable resistor known as a rheostat. A knob connected to the variable resistor allows the resistance to be adjusted from $0.00\ \Omega$ to $10.00\ \text{k}\Omega$. The output of the capacitor is used to control a voltage-controlled switch. The switch is normally open, but when the output voltage reaches 10.00 V, the switch closes, energizing an electric motor and discharging the capacitor. The motor causes the windshield wipers to sweep once across the windshield and the capacitor begins to charge again. To what resistance should the rheostat be adjusted for the period of the wiper blades be 10.00 seconds?



Strategy

The resistance considers the equation $V_{\text{out}}(t) = V\left(1 - e^{-t/\tau}\right)$, where $\tau = RC$. The capacitance, output voltage, and voltage of the battery are given. We need to solve this equation for the resistance.

Solution

The output voltage will be 10.00 V and the voltage of the battery is 12.00 V. The capacitance is given as 10.00 mF. Solving for the resistance yields

CHAPTER REVIEW

Key Terms

ammeter instrument that measures current

electromotive force (emf) energy produced per unit charge, drawn from a source that produces an electrical current

equivalent resistance resistance of a combination of resistors; it can be thought of as the resistance of a single resistor that can replace a combination of resistors in a series and/or parallel circuit

internal resistance amount of resistance to the flow of current within the voltage source

junction rule sum of all currents entering a junction must equal the sum of all currents leaving the junction

Kirchhoff's rules set of two rules governing current and changes in potential in an electric circuit

loop rule algebraic sum of changes in potential around any closed circuit path (loop) must be zero

potential difference difference in electric potential between two points in an electric circuit, measured in volts

potential drop loss of electric potential energy as a current travels across a resistor, wire, or other component

***RC* circuit** circuit that contains both a resistor and a capacitor

shock hazard hazard in which an electric current passes through a person

terminal voltage potential difference measured across the terminals of a source when there is no load attached

thermal hazard hazard in which an excessive electric current causes undesired thermal effects

three-wire system wiring system used at present for safety reasons, with live, neutral, and ground wires

voltmeter instrument that measures voltage

Key Equations

Terminal voltage of a single voltage source	$V_{\text{terminal}} = \varepsilon - Ir_{\text{eq}}$
Equivalent resistance of a series circuit	$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots + R_{N-1} + R_N = \sum_{i=1}^{N} R_i$
Equivalent resistance of a parallel circuit	$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R2} + \cdots + \frac{1}{R_N}\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1}$
Junction rule	$\sum I_{\text{in}} = \sum I_{\text{out}}$
Loop rule	$\sum V = 0$
Terminal voltage of N voltage sources in series	$V_{\text{terminal}} = \sum_{i=1}^{N} \varepsilon_i - I \sum_{i=1}^{N} r_i = \sum_{i=1}^{N} \varepsilon_i - Ir_{\text{eq}}$
Terminal voltage of N voltage sources in parallel	$V_{\text{terminal}} = \varepsilon - I \sum_{i=1}^{N} \left(\frac{1}{r_i}\right)^{-1} = \varepsilon - Ir_{\text{eq}}$
Charge on a charging capacitor	$q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{t}{\tau}}\right)$
Time constant	$\tau = RC$

Current during charging of a capacitor	$I = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}}$
Charge on a discharging capacitor	$q(t) = Qe^{-\frac{t}{\tau}}$
Current during discharging of a capacitor	$I(t) = -\frac{Q}{RC} e^{-\frac{t}{\tau}}$

Summary

10.1 Electromotive Force

- All voltage sources have two fundamental parts: a source of electrical energy that has a characteristic electromotive force (emf), and an internal resistance r. The emf is the work done per charge to keep the potential difference of a source constant. The emf is equal to the potential difference across the terminals when no current is flowing. The internal resistance r of a voltage source affects the output voltage when a current flows.
- The voltage output of a device is called its terminal voltage V_{terminal} and is given by $V_{\text{terminal}} = \varepsilon - Ir$, where I is the electric current and is positive when flowing away from the positive terminal of the voltage source and r is the internal resistance.

10.2 Resistors in Series and Parallel

- The equivalent resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances:
$$R_s = R_1 + R_2 + R_3 + \cdots = \sum_{i=1}^{N} R_i .$$
- Each resistor in a series circuit has the same amount of current flowing through it.
- The potential drop, or power dissipation, across each individual resistor in a series is different, and their combined total is the power source input.
- The equivalent resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula
$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1} .$$
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

10.3 Kirchhoff's Rules

- Kirchhoff's rules can be used to analyze any circuit, simple or complex. The simpler series and parallel connection rules are special cases of Kirchhoff's rules.
- Kirchhoff's first rule, also known as the junction rule, applies to the charge to a junction. Current is the flow of charge; thus, whatever charge flows into the junction must flow out.
- Kirchhoff's second rule, also known as the loop rule, states that the voltage drop around a loop is zero.
- When calculating potential and current using Kirchhoff's rules, a set of conventions must be followed for determining the correct signs of various terms.
- When multiple voltage sources are in series, their internal resistances add together and their emfs add together to get the total values.
- When multiple voltage sources are in parallel, their internal resistances combine to an equivalent resistance that is less than the individual resistance and provides a higher current than a single cell.
- Solar cells can be wired in series or parallel to provide increased voltage or current, respectively.

10.4 Electrical Measuring Instruments

- Voltmeters measure voltage, and ammeters measure current. Analog meters are based on the combination of a resistor and a galvanometer, a device that gives an analog reading of current or voltage. Digital meters are based on analog-to-digital converters and

provide a discrete or digital measurement of the current or voltage.
- A voltmeter is placed in parallel with the voltage source to receive full voltage and must have a large resistance to limit its effect on the circuit.
- An ammeter is placed in series to get the full current flowing through a branch and must have a small resistance to limit its effect on the circuit.
- Standard voltmeters and ammeters alter the circuit they are connected to and are thus limited in accuracy.
- Ohmmeters are used to measure resistance. The component in which the resistance is to be measured should be isolated (removed) from the circuit.

10.5 RC Circuits

- An *RC* circuit is one that has both a resistor and a capacitor.
- The time constant τ for an *RC* circuit is $\tau = RC$.
- When an initially uncharged ($q = 0$ at $t = 0$) capacitor in series with a resistor is charged by a dc voltage source, the capacitor asymptotically approaches the maximum charge.
- As the charge on the capacitor increases, the current exponentially decreases from the initial current: $I_0 = \varepsilon / R$.
- If a capacitor with an initial charge Q is discharged through a resistor starting at $t = 0$, then its charge decreases exponentially. The current flows in the opposite direction, compared to when it charges, and the magnitude of the charge decreases with time.

10.6 Household Wiring and Electrical Safety

- The two types of electric hazards are thermal (excessive power) and shock (current through a person). Electrical safety systems and devices are employed to prevent thermal and shock hazards.
- Shock severity is determined by current, path, duration, and ac frequency.
- Circuit breakers and fuses interrupt excessive currents to prevent thermal hazards.
- The three-wire system guards against thermal and shock hazards, utilizing live/hot, neutral, and ground wires, and grounding the neutral wire and case of the appliance.
- A ground fault circuit interrupter (GFCI) prevents shock by detecting the loss of current to unintentional paths.

Conceptual Questions

10.1 Electromotive Force

1. What effect will the internal resistance of a rechargeable battery have on the energy being used to recharge the battery?

2. A battery with an internal resistance of r and an emf of 10.00 V is connected to a load resistor $R = r$. As the battery ages, the internal resistance triples. How much is the current through the load resistor reduced?

3. Show that the power dissipated by the load resistor is maximum when the resistance of the load resistor is equal to the internal resistance of the battery.

10.2 Resistors in Series and Parallel

4. A voltage occurs across an open switch. What is the power dissipated by the open switch?

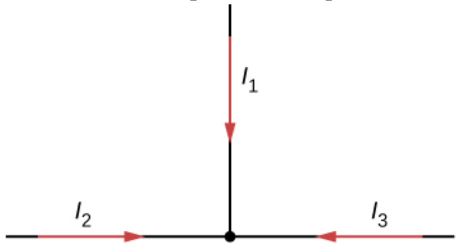
5. The severity of a shock depends on the magnitude of the current through your body. Would you prefer to be in series or in parallel with a resistance, such as the heating element of a toaster, if you were shocked by it? Explain.

6. Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

7. Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

10.3 Kirchhoff's Rules

8. Can all of the currents going into the junction shown below be positive? Explain.



9. Consider the circuit shown below. Does the