CHAPTER 9
Current and Resistance

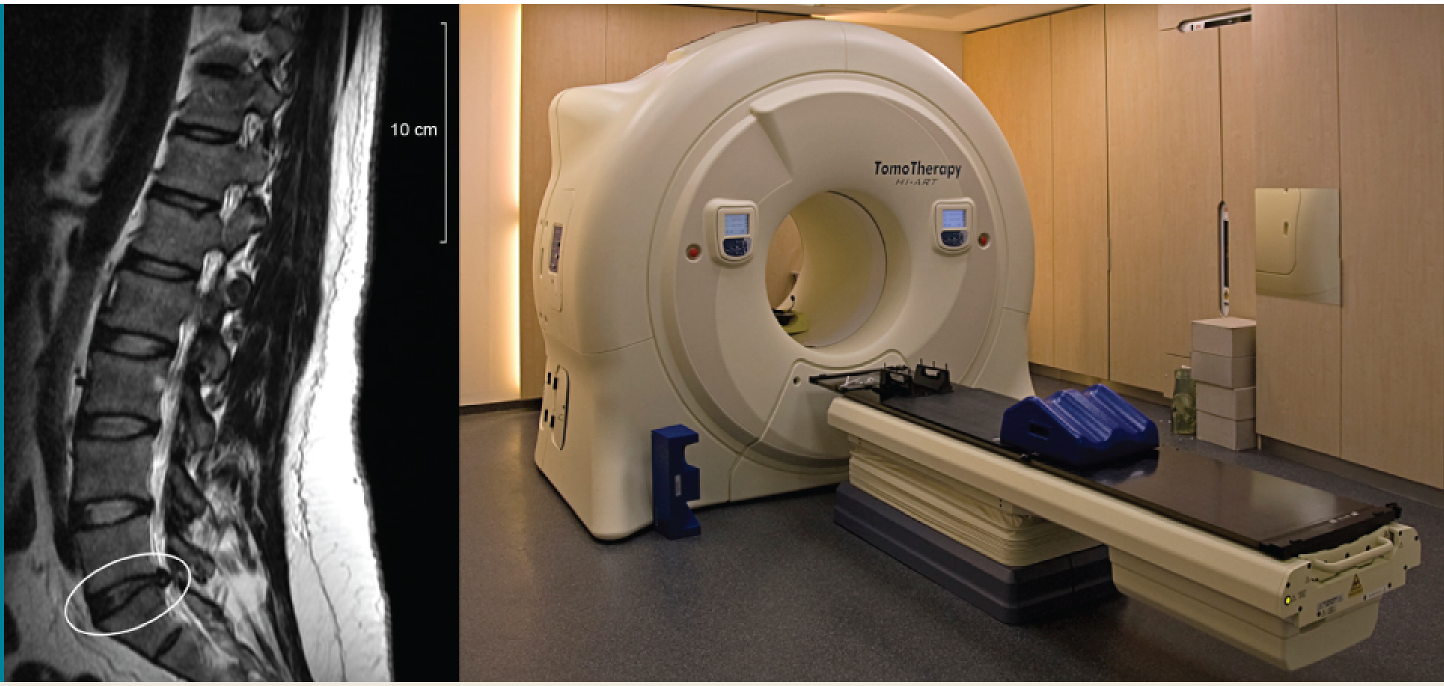


Figure 9.1 Magnetic resonance imaging (MRI) uses superconducting magnets and produces high-resolution images without the danger of radiation. The image on the left shows the spacing of vertebrae along a human spinal column, with the circle indicating where the vertebrae are too close due to a ruptured disc. On the right is a picture of the MRI instrument, which surrounds the patient on all sides. A large amount of electrical current is required to operate the electromagnets (credit right: modification of work by “digital cat”/Flickr).

Chapter Outline

INTRODUCTION In this chapter, we study the electrical current through a material, where the electrical current is the rate of flow of charge. We also examine a characteristic of materials known as the resistance. Resistance is a measure of how much a material impedes the flow of charge, and it will be shown that the resistance depends on temperature. In general, a good conductor, such as copper, gold, or silver, has very low resistance. Some materials, called superconductors, have zero resistance at very low temperatures.

High currents are required for the operation of electromagnets. Superconductors can be used to make electromagnets that are 10 times stronger than the strongest conventional electromagnets. These superconducting magnets are used in the construction of magnetic resonance imaging (MRI) devices that can be used to make high-resolution images of the human body. The chapter-opening picture shows an MRI image of the vertebrae of a human subject and the MRI device itself. Superconducting magnets have many other uses. For example, superconducting magnets are used in the Large Hadron Collider (LHC) to curve the path of protons in the ring.

9.1 Electrical Current

Learning Objectives

By the end of this section, you will be able to:

- Describe an electrical current
- Define the unit of electrical current
- Explain the direction of current flow

Up to now, we have considered primarily static charges. When charges did move, they were accelerated in response to an electrical field created by a voltage difference. The charges lost potential energy and gained kinetic energy as they traveled through a potential difference where the electrical field did work on the charge.

Although charges do not require a material to flow through, the majority of this chapter deals with understanding the movement of charges through a material. The rate at which the charges flow past a location—that is, the amount of charge per unit time—is known as the *electrical current*. When charges flow through a medium, the current depends on the voltage applied, the material through which the charges flow, and the state of the material. Of particular interest is the motion of charges in a conducting wire. In previous chapters, charges were accelerated due to the force provided by an electrical field, losing potential energy and gaining kinetic energy. In this chapter, we discuss the situation of the force provided by an electrical field in a conductor, where charges lose kinetic energy to the material reaching a constant velocity, known as the "*drift velocity*." This is analogous to an object falling through the atmosphere and losing kinetic energy to the air, reaching a constant terminal velocity.

If you have ever taken a course in first aid or safety, you may have heard that in the event of electric shock, it is the current, not the voltage, which is the important factor on the severity of the shock and the amount of damage to the human body. Current is measured in units called amperes; you may have noticed that circuit breakers in your home and fuses in your car are rated in amps (or amperes). But what is the ampere and what does it measure?

Defining Current and the Ampere

Electrical current is defined to be the rate at which charge flows. When there is a large current present, such as that used to run a refrigerator, a large amount of charge moves through the wire in a small amount of time. If the current is small, such as that used to operate a handheld calculator, a small amount of charge moves through the circuit over a long period of time.

Electrical Current

The average electrical current I is the rate at which charge flows,

$$I_{\text{ave}} = \frac{\Delta Q}{\Delta t}, \quad (9.1)$$

where ΔQ is the amount of net charge passing through a given cross-sectional area in time Δt (Figure 9.2). The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \frac{\Delta Q}{\Delta t}$, we see that an ampere is defined as one coulomb of charge passing through a given area per second:

$$1\,\text{A} \equiv 1\frac{\text{C}}{\text{s}}. \tag{9.2}$$

The instantaneous electrical current, or simply the **electrical current**, is the time derivative of the charge that flows and is found by taking the limit of the average electrical current as $\Delta t \to 0$:

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}. \tag{9.3}$$

Most electrical appliances are rated in amperes (or amps) required for proper operation, as are fuses and circuit breakers.

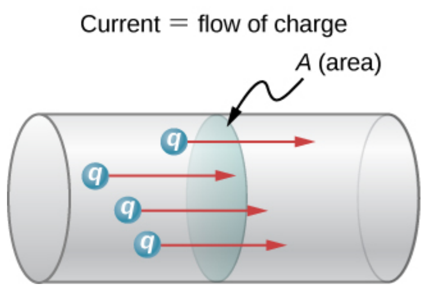


Figure 9.2 The rate of flow of charge is current. An ampere is the flow of one coulomb of charge through an area in one second. A current of one amp would result from 6.25×10^{18} electrons flowing through the area *A* each second.

EXAMPLE 9.1

Calculating the Average Current

The main purpose of a battery in a car or truck is to run the electric starter motor, which starts the engine. The operation of starting the vehicle requires a large current to be supplied by the battery. Once the engine starts, a device called an alternator takes over supplying the electric power required for running the vehicle and for charging the battery.

(a) What is the average current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow from the battery?

Strategy

We can use the definition of the average current in the equation $I = \frac{\Delta Q}{\Delta t}$ to find the average current in part (a), since charge and time are given. For part (b), once we know the average current, we can its definition $I = \frac{\Delta Q}{\Delta t}$ to find the time required for 1.00 C of charge to flow from the battery.

Solution

a. Entering the given values for charge and time into the definition of current gives

$$I = \frac{\Delta Q}{\Delta t} = \frac{720\,\text{C}}{4.00\,\text{s}} = 180\,\text{C/s} = 180\,\text{A}.$$

b. Solving the relationship $I = \frac{\Delta Q}{\Delta t}$ for time Δt and entering the known values for charge and current gives

$$\Delta t = \frac{\Delta Q}{I} = \frac{1.00\,\text{C}}{180\,\text{C/s}} = 5.56 \times 10^{-3}\,\text{s} = 5.56\,\text{ms}.$$

Significance

a. This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these “starter motors” are fairly large to overcome the inertia of the engine. b. A high current requires a short time to supply a large amount of charge. This large current is needed to supply the large amount of energy needed to start the engine.

EXAMPLE 9.2

Calculating Instantaneous Currents

Consider a charge moving through a cross-section of a wire where the charge is modeled as $Q(t) = Q_M\left(1 - e^{-t/\tau}\right)$. Here, Q_M is the charge after a long period of time, as time approaches infinity, with units of coulombs, and τ is a time constant with units of seconds (see Figure 9.3). What is the current through the wire?

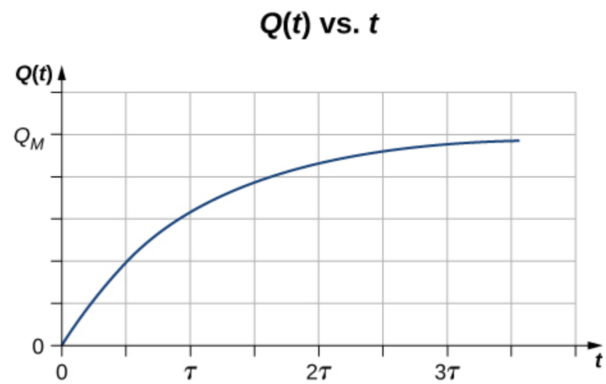


Figure 9.3 A graph of the charge moving through a cross-section of a wire over time.

Strategy

The current through the cross-section can be found from $I = \frac{dQ}{dt}$. Notice from the figure that the charge increases to Q_M and the derivative decreases, approaching zero, as time increases (Figure 9.4).

Solution

The derivative can be found using $\frac{d}{dx}e^u = e^u\frac{du}{dx}$.

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left[Q_M\left(1 - e^{-t/\tau}\right)\right] = \frac{Q_M}{\tau}e^{-t/\tau}.$$

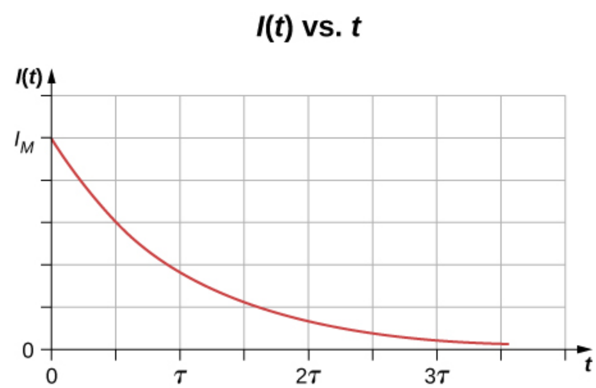


Figure 9.4 A graph of the current flowing through the wire over time.

Significance

The current through the wire in question decreases exponentially, as shown in Figure 9.4. In later chapters, it will be shown that a time-dependent current appears when a capacitor charges or discharges through a resistor. Recall that a capacitor is a device that stores charge. You will learn about the resistor in Model of Conduction in Metals.

CHECK YOUR UNDERSTANDING 9.1

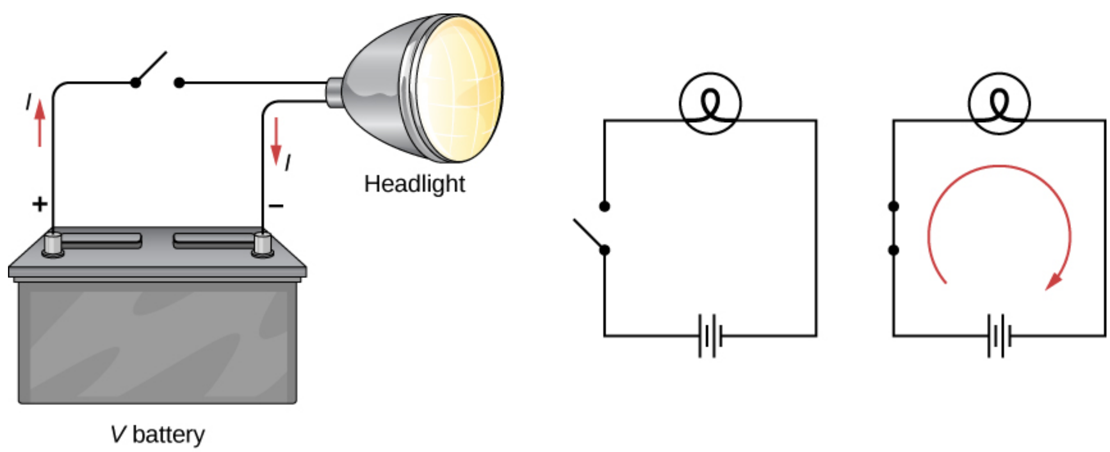
Handheld calculators often use small solar cells to supply the energy required to complete the calculations needed to complete your next physics exam. The current needed to run your calculator can be as small as 0.30 mA. How long would it take for 1.00 C of charge to flow from the solar cells? Can solar cells be used, instead of batteries, to start traditional internal combustion engines presently used in most cars and trucks?

CHECK YOUR UNDERSTANDING 9.2

Circuit breakers in a home are rated in amperes, normally in a range from 10 amps to 30 amps, and are used to protect the residents from harm and their appliances from damage due to large currents. A single 15-amp circuit breaker may be used to protect several outlets in the living room, whereas a single 20-amp circuit breaker may be used to protect the refrigerator in the kitchen. What can you deduce from this about current used by the various appliances?

Current in a Circuit

In the previous paragraphs, we defined the current as the charge that flows through a cross-sectional area per unit time. In order for charge to flow through an appliance, such as the headlight shown in Figure 9.5, there must be a complete path (or **circuit**) from the positive terminal to the negative terminal. Consider a simple circuit of a car battery, a switch, a headlight lamp, and wires that provide a current path between the components. In order for the lamp to light, there must be a complete path for current flow. In other words, a charge must be able to leave the positive terminal of the battery, travel through the component, and back to the negative terminal of the battery. The switch is there to control the circuit. Part (a) of the figure shows the simple circuit of a car battery, a switch, a conducting path, and a headlight lamp. Also shown is the **schematic** of the circuit [part (b)]. A schematic is a graphical representation of a circuit and is very useful in visualizing the main features of a circuit. Schematics use standardized symbols to represent the components in a circuits and solid lines to represent the wires connecting the components. The battery is shown as a series of long and short lines, representing the historic voltaic pile. The lamp is shown as a circle with a loop inside, representing the filament of an incandescent bulb. The switch is shown as two points with a conducting bar to connect the two points and the wires connecting the components are shown as solid lines. The schematic in part (c) shows the direction of current flow when the switch is closed.



(a) (b) (c)

Figure 9.5 (a) A simple electric circuit of a headlight (lamp), a battery, and a switch. When the switch is closed, an uninterrupted path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by parallel lines, which resemble plates in the original design of a battery. The longer lines indicate the positive terminal. The conducting wires are shown as solid lines. The switch is shown, in the open position, as two terminals with a line representing a conducting bar that can make contact between the two terminals. The lamp is represented by a circle encompassing a filament, as would be seen in an incandescent light bulb. (c) When the switch is closed, the circuit is complete and current flows from the positive terminal to the negative terminal of the battery.

When the switch is closed in Figure 9.5(c), there is a complete path for charges to flow, from the positive terminal of the battery, through the switch, then through the headlight and back to the negative terminal of the battery. Note that the direction of current flow is from positive to negative. The direction of **conventional current** is always represented in the direction that positive charge would flow, from the positive terminal to the negative terminal.

The conventional current flows from the positive terminal to the negative terminal, but depending on the actual situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator, used for nuclear research, can produce a current of pure positive charges, such as protons. In the Tevatron Accelerator at Fermilab, before it was shut down in 2011, beams of protons and antiprotons traveling in opposite directions were collided. The protons are positive and therefore their current is in the same direction as they travel. The antiprotons are negativity charged and thus their current is in the opposite direction that the actual particles travel.

A closer look at the current flowing through a wire is shown in Figure 9.6. The figure illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American scientist and statesman Benjamin Franklin in the 1700s. Having no knowledge of the particles that make up the atom (namely the proton, electron, and neutron), Franklin believed that electrical current flowed from a material that had more of an "electrical fluid" and to a material that had less of this "electrical fluid." He coined the term *positive* for the material that had more of this electrical fluid and *negative* for the material that lacked the electrical fluid. He surmised that current would flow from the material with more electrical fluid—the positive material—to the negative material, which has less electrical fluid. Franklin called this direction of current a positive current flow. This was pretty advanced thinking for a man who knew nothing about the atom.

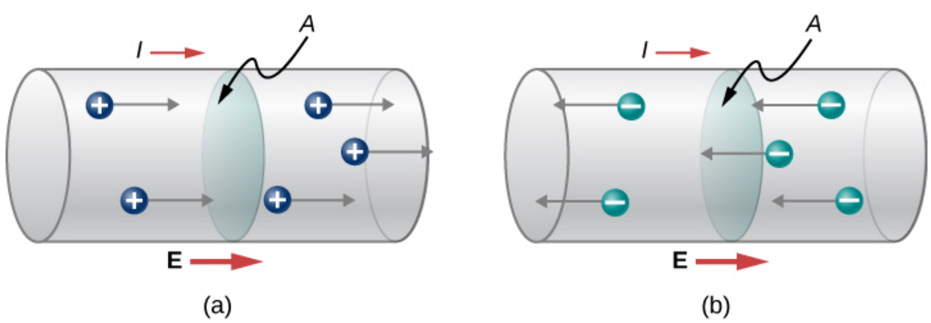


Figure 9.6 Current *I* is the rate at which charge moves through an area *A*, such as the cross-section of a wire. Conventional current is defined to move in the direction of the electrical field. (a) Positive charges move in the direction of the electrical field, which is the same direction as conventional current. (b) Negative charges move in the direction opposite to the electrical field. Conventional current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

We now know that a material is positive if it has a greater number of protons than electrons, and it is negative if it has a greater number of electrons than protons. In a conducting metal, the current flow is due primarily to electrons flowing from the negative material to the positive material, but for historical reasons, we consider the positive current flow and the current is shown to flow from the positive terminal of the battery to the negative terminal.

It is important to realize that an electrical field is present in conductors and is responsible for producing the current (Figure 9.6). In previous chapters, we considered the static electrical case, where charges in a conductor quickly redistribute themselves on the surface of the conductor in order to cancel out the external electrical field and restore equilibrium. In the case of an electrical circuit, the charges are prevented from ever reaching equilibrium by an external source of electric potential, such as a battery. The energy needed to move the charge is supplied by the electric potential from the battery.

Although the electrical field is responsible for the motion of the charges in the conductor, the work done on the charges by the electrical field does not increase the kinetic energy of the charges. We will show that the electrical field is responsible for keeping the electric charges moving at a "drift velocity."

9.2 Model of Conduction in Metals

Learning Objectives

By the end of this section, you will be able to:

- Define the drift velocity of charges moving through a metal
- Define the vector current density
- Describe the operation of an incandescent lamp

When electrons move through a conducting wire, they do not move at a constant velocity, that is, the electrons do not move in a straight line at a constant speed. Rather, they interact with and collide with atoms and other free electrons in the conductor. Thus, the electrons move in a zig-zag fashion and drift through the wire. We should also note that even though it is convenient to discuss the direction of current, current is a scalar quantity. When discussing the velocity of charges in a current, it is more appropriate to discuss the current density. We will come back to this idea at the end of this section.

Drift Velocity

Electrical signals move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a light switch is moved to the 'on' position. Most electrical signals carried by currents travel at speeds on the order of 10^8 m/s, a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move much slower on average, typically drifting at speeds on the order of 10^{-4} m/s. How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in Figure 9.7, the incoming charge pushes other charges ahead of it due to the repulsive force between like charges. These moving charges push on charges farther down the line. The density of charge in a system cannot easily be increased, so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this fast-moving signal, or shock wave, is a rapidly propagating change in the electrical field.

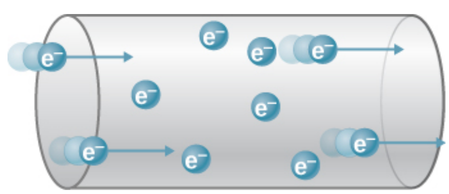


Figure 9.7 When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Good conductors have large numbers of free charges. In metals, the free charges are free electrons. (In fact, good electrical conductors are often good heat conductors too, because large numbers of free electrons can transport thermal energy as well as carry electrical current.) Figure 9.8 shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electrical field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity** $\vec{\mathbf{v}}_d$ is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.

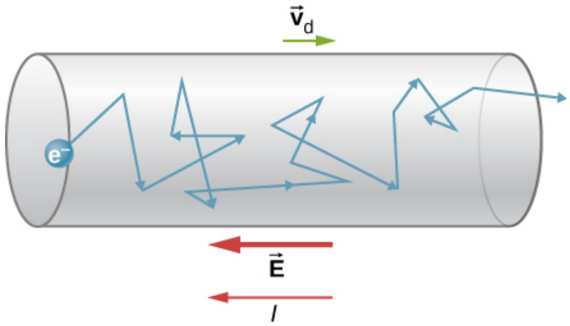


Figure 9.8 Free electrons moving in a conductor make many collisions with other electrons and other particles. A typical path of one electron is shown. The average velocity of the free charges is called the drift velocity $\vec{v}_d$ and for electrons, it is in the direction opposite to the electrical field. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

Free-electron collisions transfer energy to the atoms of the conductor. The electrical field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed) of the electrons. The work is transferred to the conductor's atoms, often increasing temperature. Thus, a continuous power input is required to keep a current flowing. (An exception is superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings.) For a conductor that is not a superconductor, the supply of energy can be useful, as in an incandescent light bulb filament (Figure 9.9). The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

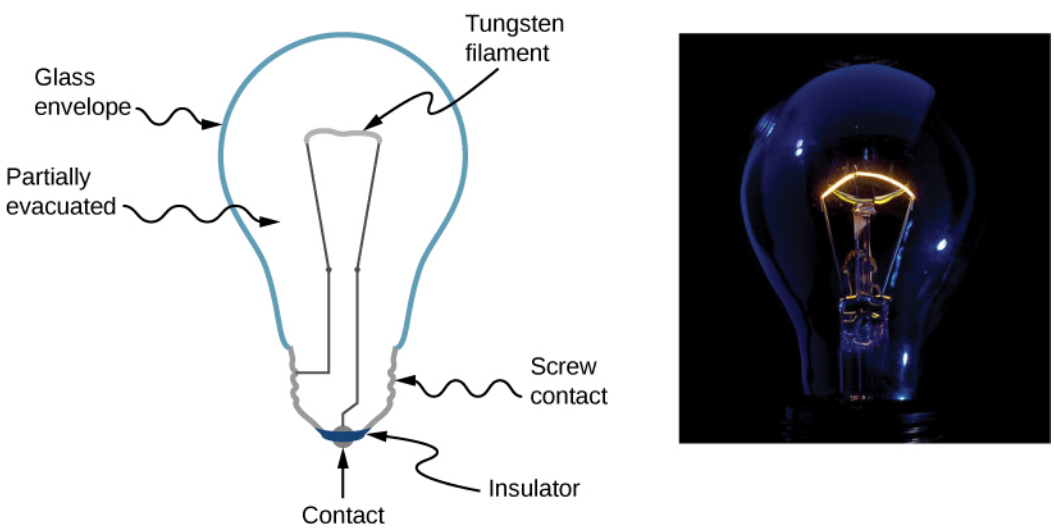


Figure 9.9 The incandescent lamp is a simple design. A tungsten filament is placed in a partially evacuated glass envelope. One end of the filament is attached to the screw base, which is made out of a conducting material. The second end of the filament is attached to a second contact in the base of the bulb. The two contacts are separated by an insulating material. Current flows through the filament, and the temperature of the filament becomes large enough to cause the filament to glow and produce light. However, these bulbs are not very energy efficient, as evident from the heat coming from the bulb. In the year 2012, the United States, along with many other countries, began to phase out incandescent lamps in favor of more energy-efficient lamps, such as light-emitting diode (LED) lamps and compact fluorescent lamps (CFL) (credit right: modification of work by Serge Saint).

We can obtain an expression for the relationship between current and drift velocity by considering the number of free charges in a segment of wire, as illustrated in Figure 9.10. The number of free charges per unit volume, or the number density of free charges, is given the symbol n where $n = \frac{\text{number of charges}}{\text{volume}}$. The value of n depends on the material. The shaded segment has a volume $Av_d\,dt$, so that the number of free charges in the volume is $nAv_d\,dt$. The charge dQ in this segment is thus $qnAv_d\,dt$, where q is the amount of charge on each carrier. (The magnitude of the charge of electrons is $q = 1.60 \times 10^{-19}$ C.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time *dt*, the current is

$$I = \frac{dQ}{dt} = qnAv_{\text{d}}.$$

Rearranging terms gives

$$v_{\text{d}} = \frac{I}{nqA} \qquad (9.4)$$

where v_{d} is the drift velocity, n is the free charge density, A is the cross-sectional area of the wire, and I is the current through the wire. The carriers of the current each have charge q and move with a drift velocity of magnitude v_{d}.

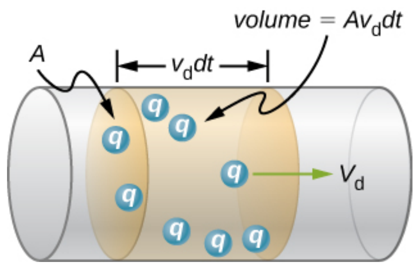


Figure 9.10 All the charges in the shaded volume of this wire move out in a time *dt*, having a drift velocity of magnitude v_{d}.

Note that simple drift velocity is not the entire story. The speed of an electron is sometimes much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do move might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons?

Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as strongly as the inner electrons do. These are the free electrons. They are not bound to a single atom but can instead move freely among the atoms in a "sea" of electrons. When an electrical field is applied, these free electrons respond by accelerating. As they move, they collide with the atoms in the lattice and with other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

As you know, electric power is usually supplied to equipment and appliances through round wires made of a conducting material (copper, aluminum, silver, or gold) that are stranded or solid. The diameter of the wire determines the current-carrying capacity—the larger the diameter, the greater the current-carrying capacity. Even though the current-carrying capacity is determined by the diameter, wire is not normally characterized by the diameter directly. Instead, wire is commonly sold in a unit known as "gauge." Wires are manufactured by passing the material through circular forms called "drawing dies." In order to make thinner wires, manufacturers draw the wires through multiple dies of successively thinner diameter. Historically, the gauge of the wire was related to the number of drawing processes required to manufacture the wire. For this reason, the larger the gauge, the smaller the diameter. In the United States, the American Wire Gauge (AWG) was developed to standardize the system. Household wiring commonly consists of 10-gauge (2.588-mm diameter) to 14-gauge (1.628-mm diameter) wire. A device used to measure the gauge of wire is shown in Figure 9.11.

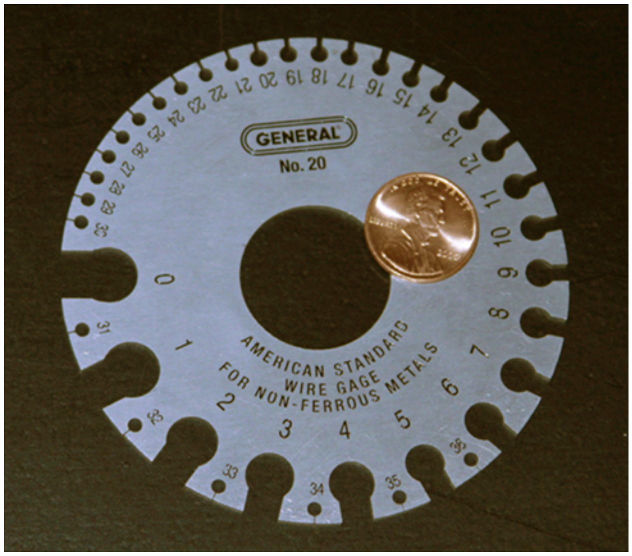


Figure 9.11 A device for measuring the gauge of electrical wire. As you can see, higher gauge numbers indicate thinner wires. (credit: Joseph J. Trout)

EXAMPLE 9.3

Calculating Drift Velocity in a Common Wire

Calculate the drift velocity of electrons in a copper wire with a diameter of 2.053 mm (12-gauge) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20.0 A.) The density of copper is $8.80 \times 10^3 \text{ kg/m}^3$ and the atomic mass of copper is 63.54 g/mol.

Strategy

We can calculate the drift velocity using the equation $I = nqAv_\text{d}$. The current is $I = 20.00 \text{ A}$ and $q = 1.60 \times 10^{-19} \text{C}$ is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula $A = \pi r^2$, where r is one-half the diameter. The given diameter is 2.053 mm, so r is 1.0265 mm. We are given the density of copper, $8.80 \times 10^3 \text{ kg/m}^3$, and the atomic mass of copper is 63.54 g/mol. We can use these two quantities along with Avogadro's number, 6.02×10^{23} atoms/mol, to determine n, the number of free electrons per cubic meter.

Solution

First, we calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, the number of free electrons is the same as the number of copper atoms per m^3. We can now find n as follows:

$$\begin{aligned} n &= \frac{1\ e^-}{\text{atom}} \times \frac{6.02 \times 10^{23} \text{atoms}}{\text{mol}} \times \frac{1 \text{ mol}}{63.54 \text{ g}} \times \frac{1000 \text{ g}}{\text{kg}} \times \frac{8.80 \times 10^3 \text{kg}}{1 \text{ m}^3} \\ &= 8.34 \times 10^{28} e^-/\text{m}^3. \end{aligned}$$

The cross-sectional area of the wire is

$$A = \pi r^2 = \pi \left(\frac{2.05 \times 10^{-3} \text{m}}{2} \right)^2 = 3.30 \times 10^{-6} \text{m}^2.$$

Rearranging $I = nqAv_\text{d}$ to isolate drift velocity gives

$$v_\text{d} = \frac{I}{nqA} = \frac{20.00 \text{ A}}{(8.34 \times 10^{28}/\text{m}^3)(-1.60 \times 10^{-19} \text{C})(3.30 \times 10^{-6} \text{m}^2)} = -4.54 \times 10^{-4} \text{ m/s}.$$

Significance

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of 10^{-4} m/s) confirms that the signal moves on the order of 10^{12} times faster (about 10^{8} m/s) than the charges that carry it.

CHECK YOUR UNDERSTANDING 9.3

In Example 9.4, the drift velocity was calculated for a 2.053-mm diameter (12-gauge) copper wire carrying a 20-amp current. Would the drift velocity change for a 1.628-mm diameter (14-gauge) wire carrying the same 20-amp current?

Current Density

Although it is often convenient to attach a negative or positive sign to indicate the overall direction of motion of the charges, current is a scalar quantity, $I = \frac{dQ}{dt}$. It is often necessary to discuss the details of the motion of the charge, instead of discussing the overall motion of the charges. In such cases, it is necessary to discuss the current density, $\vec{\mathbf{J}}$, a vector quantity. The **current density** is the flow of charge through an infinitesimal area, divided by the area. The current density must take into account the local magnitude and direction of the charge flow, which varies from point to point. The unit of current density is ampere per meter squared, and the direction is defined as the direction of net flow of positive charges through the area.

The relationship between the current and the current density can be seen in Figure 9.12. The differential current flow through the area $d\vec{\mathbf{A}}$ is found as

$$dI = \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} = J\,dA \cos\theta,$$

where θ is the angle between the area and the current density. The total current passing through area $d\vec{\mathbf{A}}$ can be found by integrating over the area,

$$I = \iint_{\text{area}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}. \qquad (9.5)$$

Consider the magnitude of the current density, which is the current divided by the area:

$$J = \frac{I}{A} = \frac{n\,|q|\,Av_{\text{d}}}{A} = n\,|q|\,v_{\text{d}}.$$

Thus, the current density is $\vec{\mathbf{J}} = nq\vec{\mathbf{v}}_{\text{d}}$. If q is positive, $\vec{\mathbf{v}}_{\text{d}}$ is in the same direction as the electrical field $\vec{\mathbf{E}}$. If q is negative, $\vec{\mathbf{v}}_{\text{d}}$ is in the opposite direction of $\vec{\mathbf{E}}$. Either way, the direction of the current density $\vec{\mathbf{J}}$ is in the direction of the electrical field $\vec{\mathbf{E}}$.

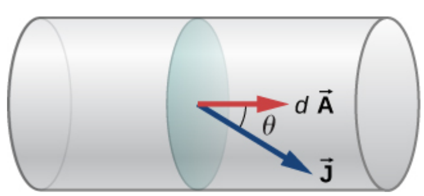


Figure 9.12 The current density $\vec{\mathbf{J}}$ is defined as the current passing through an infinitesimal cross-sectional area divided by the area. The direction of the current density is the direction of the net flow of positive charges and the magnitude is equal to the current divided by the infinitesimal area.

EXAMPLE 9.4

Calculating the Current Density in a Wire

The current supplied to a lamp with a 100-W light bulb is 0.87 amps. The lamp is wired using a copper wire with diameter 2.588 mm (10-gauge). Find the magnitude of the current density.

Strategy

The current density is the current moving through an infinitesimal cross-sectional area divided by the area. We can calculate the magnitude of the current density using $J = \frac{I}{A}$. The current is given as 0.87 A. The cross-sectional area can be calculated to be $A = 5.26 \text{ mm}^2$.

Solution

Calculate the current density using the given current $I = 0.87 \text{ A}$ and the area, found to be $A = 5.26 \text{ mm}^2$.

$$J = \frac{I}{A} = \frac{0.87 \text{ A}}{5.26 \times 10^{-6} \text{m}^2} = 1.65 \times 10^5 \frac{\text{A}}{\text{m}^2}.$$

Significance

The current density in a conducting wire depends on the current through the conducting wire and the cross-sectional area of the wire. For a given current, as the diameter of the wire increases, the charge density decreases.

CHECK YOUR UNDERSTANDING 9.4

The current density is proportional to the current and inversely proportional to the area. If the current density in a conducting wire increases, what would happen to the drift velocity of the charges in the wire?

What is the significance of the current density? The current density is proportional to the current, and the current is the number of charges that pass through a cross-sectional area per second. The charges move through the conductor, accelerated by the electric force provided by the electrical field. The electrical field is created when a voltage is applied across the conductor. In Ohm's Law, we will use this relationship between the current density and the electrical field to examine the relationship between the current through a conductor and the voltage applied.

9.3 Resistivity and Resistance

Learning Objectives

By the end of this section, you will be able to:

- Differentiate between resistance and resistivity
- Define the term conductivity
- Describe the electrical component known as a resistor
- State the relationship between resistance of a resistor and its length, cross-sectional area, and resistivity
- State the relationship between resistivity and temperature

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—that are necessary to maintain a current. All such devices create a potential difference and are referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electrical field. The electrical field, in turn, exerts force on free charges, causing current. The amount of current depends not only on the magnitude of the voltage, but also on the characteristics of the material that the current is flowing through. The material can resist the flow of the charges, and the measure of how much a material resists the flow of charges is known as the *resistivity*. This resistivity is crudely analogous to the friction between two materials that resists motion.

Resistivity

When a voltage is applied to a conductor, an electrical field $\vec{\mathbf{E}}$ is created, and charges in the conductor feel a force due to the electrical field. The current density $\vec{\mathbf{J}}$ that results depends on the electrical field and the properties of the material. This dependence can be very complex. In some materials, including metals at a given temperature, the current density is approximately proportional to the electrical field. In these cases, the current density can be modeled as

$$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}},$$

where σ is the **electrical conductivity**. The electrical conductivity is analogous to thermal conductivity and is a measure of a material's ability to conduct or transmit electricity. Conductors have a higher electrical conductivity than insulators. Since the electrical conductivity is $\sigma = J/E$, the units are

$$\sigma = \frac{[J]}{[E]} = \frac{\text{A/m}^2}{\text{V/m}} = \frac{\text{A}}{\text{V}\cdot\text{m}}.$$

Here, we define a unit named the **ohm** with the Greek symbol uppercase omega, Ω. The unit is named after Georg Simon Ohm, whom we will discuss later in this chapter. The Ω is used to avoid confusion with the number 0. One ohm equals one volt per amp: $1\ \Omega = 1\ \text{V/A}$. The units of electrical conductivity are therefore $(\Omega\cdot\text{m})^{-1}$.

Conductivity is an intrinsic property of a material. Another intrinsic property of a material is the **resistivity**, or electrical resistivity. The resistivity of a material is a measure of how strongly a material opposes the flow of electrical current. The symbol for resistivity is the lowercase Greek letter rho, ρ, and resistivity is the reciprocal of electrical conductivity:

$$\rho = \frac{1}{\sigma}.$$

The unit of resistivity in SI units is the ohm-meter $(\Omega\cdot\text{m})$. We can define the resistivity in terms of the electrical field and the current density,

$$\rho = \frac{E}{J}. \qquad (9.6)$$

The greater the resistivity, the larger the field needed to produce a given current density. The lower the resistivity, the larger the current density produced by a given electrical field. Good conductors have a high conductivity and low resistivity. Good insulators have a low conductivity and a high resistivity. Table 9.1 lists resistivity and conductivity values for various materials.

Material	Conductivity, σ $(\Omega\cdot\text{m})^{-1}$	Resistivity, ρ $(\Omega\cdot\text{m})$	Temperature Coefficient, α $(^\circ\text{C})^{-1}$
Conductors			
Silver	6.29×10^{7}	1.59×10^{-8}	0.0038
Copper	5.95×10^{7}	1.68×10^{-8}	0.0039
Gold	4.10×10^{7}	2.44×10^{-8}	0.0034
Aluminum	3.77×10^{7}	2.65×10^{-8}	0.0039
Tungsten	1.79×10^{7}	5.60×10^{-8}	0.0045

Material	Conductivity, σ $(\Omega \cdot m)^{-1}$	Resistivity, ρ $(\Omega \cdot m)$	Temperature Coefficient, α $(°C)^{-1}$
Iron	1.03×10^7	9.71×10^{-8}	0.0065
Platinum	0.94×10^7	10.60×10^{-8}	0.0039
Steel	0.50×10^7	20.00×10^{-8}	
Lead	0.45×10^7	22.00×10^{-8}	
Manganin (Cu, Mn, Ni alloy)	0.21×10^7	48.20×10^{-8}	0.000002
Constantan (Cu, Ni alloy)	0.20×10^7	49.00×10^{-8}	0.00003
Mercury	0.10×10^7	98.00×10^{-8}	0.0009
Nichrome (Ni, Fe, Cr alloy)	0.10×10^7	100.00×10^{-8}	0.0004
Semiconductors[1]			
Carbon (pure)	2.86×10^4	3.50×10^{-5}	−0.0005
Carbon	$(2.86 - 1.67) \times 10^{-6}$	$(3.5 - 60) \times 10^{-5}$	−0.0005
Germanium (pure)		600×10^{-3}	−0.048
Germanium		$(1 - 600) \times 10^{-3}$	−0.050
Silicon (pure)		2300	−0.075
Silicon		$0.1 - 2300$	−0.07
Insulators			
Amber	2.00×10^{-15}	5×10^{14}	
Glass	$10^{-9} - 10^{-14}$	$10^9 - 10^{14}$	
Lucite	$<10^{-13}$	$>10^{13}$	
Mica	$10^{-11} - 10^{-15}$	$10^{11} - 10^{15}$	
Quartz (fused)	1.33×10^{-18}	75×10^{16}	
Rubber (hard)	$10^{-13} - 10^{-16}$	$10^{13} - 10^{16}$	
Sulfur	10^{-15}	10^{15}	

Material	Conductivity, σ $(\Omega \cdot \mathrm{m})^{-1}$	Resistivity, ρ $(\Omega \cdot \mathrm{m})$	Temperature Coefficient, α $(°\mathrm{C})^{-1}$
Teflon™	$<10^{-13}$	$>10^{13}$	
Wood	$10^{-8} - 10^{-11}$	$10^{8} - 10^{11}$	

Table 9.1 Resistivities and Conductivities of Various Materials at 20 °C [1] Values depend strongly on amounts and types of impurities.

The materials listed in the table are separated into categories of conductors, semiconductors, and insulators, based on broad groupings of resistivity. Conductors have the smallest resistivity, and insulators have the largest; semiconductors have intermediate resistivity. Conductors have varying but large, free charge densities, whereas most charges in insulators are bound to atoms and are not free to move. Semiconductors are intermediate, having far fewer free charges than conductors, but having properties that make the number of free charges depend strongly on the type and amount of impurities in the semiconductor. These unique properties of semiconductors are put to use in modern electronics, as we will explore in later chapters.

CHECK YOUR UNDERSTANDING 9.5

Copper wires use routinely used for extension cords and house wiring for several reasons. Copper has the highest electrical conductivity rating, and therefore the lowest resistivity rating, of all nonprecious metals. Also important is the tensile strength, where the tensile strength is a measure of the force required to pull an object to the point where it breaks. The tensile strength of a material is the maximum amount of tensile stress it can take before breaking. Copper has a high tensile strength, $2 \times 10^8 \frac{\mathrm{N}}{\mathrm{m}^2}$. A third important characteristic is ductility. Ductility is a measure of a material's ability to be drawn into wires and a measure of the flexibility of the material, and copper has a high ductility. Summarizing, for a conductor to be a suitable candidate for making wire, there are at least three important characteristics: low resistivity, high tensile strength, and high ductility. What other materials are used for wiring and what are the advantages and disadvantages?

INTERACTIVE

View this interactive simulation (https://openstax.org/l/21resistwire) to see what the effects of the cross-sectional area, the length, and the resistivity of a wire are on the resistance of a conductor. Adjust the variables using slide bars and see if the resistance becomes smaller or larger.

Temperature Dependence of Resistivity

Looking back at Table 9.1, you will see a column labeled "Temperature Coefficient." The resistivity of some materials has a strong temperature dependence. In some materials, such as copper, the resistivity increases with increasing temperature. In fact, in most conducting metals, the resistivity increases with increasing temperature. The increasing temperature causes increased vibrations of the atoms in the lattice structure of the metals, which impede the motion of the electrons. In other materials, such as carbon, the resistivity decreases with increasing temperature. In many materials, the dependence is approximately linear and can be modeled using a linear equation:

$$\rho \approx \rho_0 \left[1 + \alpha (T - T_0)\right], \tag{9.7}$$

where ρ is the resistivity of the material at temperature T, α is the temperature coefficient of the material, and ρ_0 is the resistivity at T_0, usually taken as $T_0 = 20.00\ °\mathrm{C}$.

Note also that the temperature coefficient α is negative for the semiconductors listed in Table 9.1, meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing ρ with temperature is also related to the type and amount of impurities present in the semiconductors.

Resistance

We now consider the resistance of a wire or component. The resistance is a measure of how difficult it is to pass current through a wire or component. Resistance depends on the resistivity. The resistivity is a characteristic of the material used to fabricate a wire or other electrical component, whereas the resistance is a characteristic of the wire or component.

To calculate the resistance, consider a section of conducting wire with cross-sectional area A, length L, and resistivity ρ. A battery is connected across the conductor, providing a potential difference ΔV across it (Figure 9.13). The potential difference produces an electrical field that is proportional to the current density, according to $\vec{\mathbf{E}} = \rho\vec{\mathbf{J}}$.

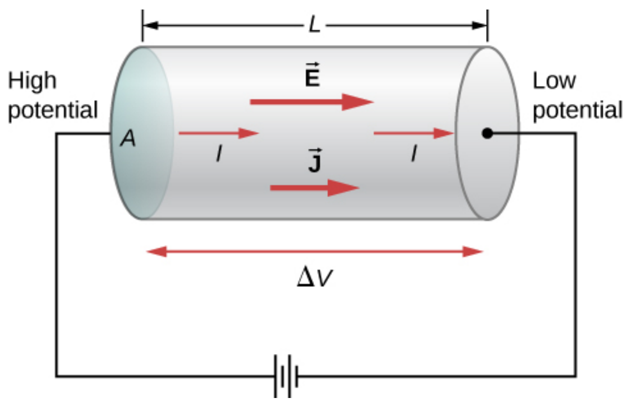


Figure 9.13 A potential provided by a battery is applied to a segment of a conductor with a cross-sectional area A and a length L.

The magnitude of the electrical field across the segment of the conductor is equal to the voltage divided by the length, $E = V/L$, and the magnitude of the current density is equal to the current divided by the cross-sectional area, $J = I/A$. Using this information and recalling that the electrical field is proportional to the resistivity and the current density, we can see that the voltage is proportional to the current:

$$E = \rho J$$
$$\frac{V}{L} = \rho\frac{I}{A}$$
$$V = \left(\rho\frac{L}{A}\right) I.$$

Resistance

The ratio of the voltage to the current is defined as the **resistance** R:

$$R \equiv \frac{V}{I}. \quad (9.8)$$

The resistance of a cylindrical segment of a conductor is equal to the resistivity of the material times the length divided by the area:

$$R \equiv \frac{V}{I} = \rho\frac{L}{A}. \quad (9.9)$$

The unit of resistance is the ohm, Ω. For a given voltage, the higher the resistance, the lower the current.

Resistors

A common component in electronic circuits is the resistor. The resistor can be used to reduce current flow or provide a voltage drop. Figure 9.14 shows the symbols used for a resistor in schematic diagrams of a circuit. Two commonly used standards for circuit diagrams are provided by the American National Standard Institute (ANSI, pronounced "AN-see") and the International Electrotechnical Commission (IEC). Both systems are commonly used. We use the ANSI standard in this text for its visual recognition, but we note that for larger, more complex circuits, the IEC standard may have a cleaner presentation, making it easier to read.

Figure 9.14 Symbols for a resistor used in circuit diagrams. (a) The ANSI symbol; (b) the IEC symbol.

Material and shape dependence of resistance

A resistor can be modeled as a cylinder with a cross-sectional area A and a length L, made of a material with a resistivity ρ (Figure 9.15). The resistance of the resistor is $R = \rho \frac{L}{A}$.

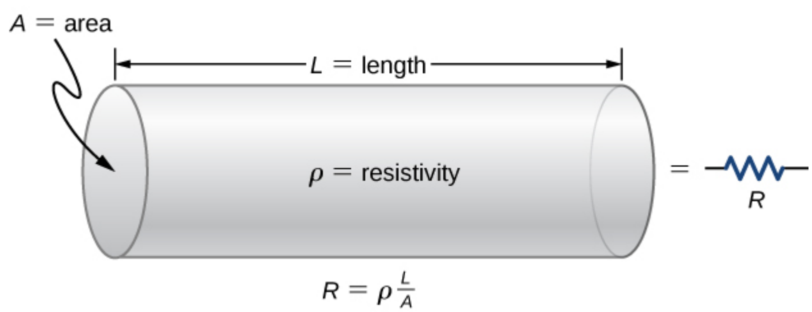


Figure 9.15 A model of a resistor as a uniform cylinder of length L and cross-sectional area A. Its resistance to the flow of current is analogous to the resistance posed by a pipe to fluid flow. The longer the cylinder, the greater its resistance. The larger its cross-sectional area A, the smaller its resistance.

The most common material used to make a resistor is carbon. A carbon track is wrapped around a ceramic core, and two copper leads are attached. A second type of resistor is the metal film resistor, which also has a ceramic core. The track is made from a metal oxide material, which has semiconductive properties similar to carbon. Again, copper leads are inserted into the ends of the resistor. The resistor is then painted and marked for identification. A resistor has four colored bands, as shown in Figure 9.16.

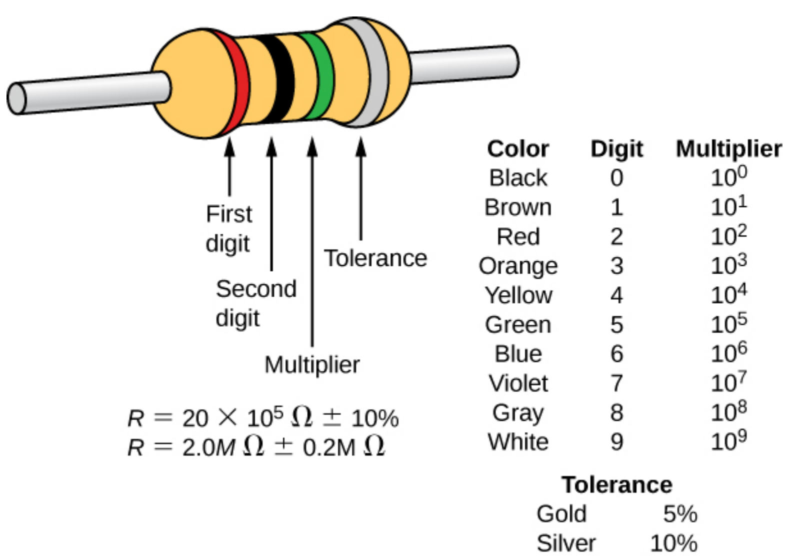


Figure 9.16 Many resistors resemble the figure shown above. The four bands are used to identify the resistor. The first two colored bands represent the first two digits of the resistance of the resistor. The third color is the multiplier. The fourth color represents the tolerance of

the resistor. The resistor shown has a resistance of $20 \times 10^5\ \Omega \pm 10\%$.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12}\ \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^5\ \Omega$, whereas the resistance of the human heart is about $10^3\ \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\ \Omega$, and superconductors have no resistance at all at low temperatures. As we have seen, resistance is related to the shape of an object and the material of which it is composed.

EXAMPLE 9.5

Current Density, Resistance, and Electrical field for a Current-Carrying Wire

Calculate the current density, resistance, and electrical field of a 5-m length of copper wire with a diameter of 2.053 mm (12-gauge) carrying a current of $I = 10\text{ mA}$.

Strategy

We can calculate the current density by first finding the cross-sectional area of the wire, which is $A = 3.31\text{ mm}^2$, and the definition of current density $J = \frac{I}{A}$. The resistance can be found using the length of the wire $L = 5.00\text{ m}$, the area, and the resistivity of copper $\rho = 1.68 \times 10^{-8}\ \Omega \cdot \text{m}$, where $R = \rho \frac{L}{A}$. The resistivity and current density can be used to find the electrical field.

Solution

First, we calculate the current density:

$$J = \frac{I}{A} = \frac{10 \times 10^{-3}\text{A}}{3.31 \times 10^{-6}\text{m}^2} = 3.02 \times 10^3 \frac{\text{A}}{\text{m}^2}.$$

The resistance of the wire is

$$R = \rho \frac{L}{A} = \left(1.68 \times 10^{-8}\ \Omega \cdot \text{m}\right) \frac{5.00\text{ m}}{3.31 \times 10^{-6}\text{m}^2} = 0.025\,\Omega\,.$$

Finally, we can find the electrical field:

$$E = \rho J = 1.68 \times 10^{-8}\ \Omega \cdot \text{m} \left(3.02 \times 10^3 \frac{\text{A}}{\text{m}^2}\right) = 5.07 \times 10^{-5} \frac{\text{V}}{\text{m}}.$$

Significance

From these results, it is not surprising that copper is used for wires for carrying current because the resistance is quite small. Note that the current density and electrical field are independent of the length of the wire, but the voltage depends on the length.

The resistance of an object also depends on temperature, since R_0 is directly proportional to ρ. For a cylinder, we know $R = \rho \frac{L}{A}$, so if L and A do not change greatly with temperature, R has the same temperature dependence as ρ. (Examination of the coefficients of linear expansion shows them to be about two orders of magnitude less than typical temperature coefficients of resistivity, so the effect of temperature on L and A is about two orders of magnitude less than on ρ.) Thus,

$$R = R_0(1 + \alpha \Delta T) \tag{9.10}$$

is the temperature dependence of the resistance of an object, where R_0 is the original resistance (usually taken to be $20.00\ °\text{C}$) and R is the resistance after a temperature change ΔT. The color code gives the resistance of the resistor at a temperature of $T = 20.00\ °\text{C}$.

Numerous thermometers are based on the effect of temperature on resistance (Figure 9.17). One of the most common thermometers is based on the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small, so that it quickly comes into thermal equilibrium with the part of a person it touches.

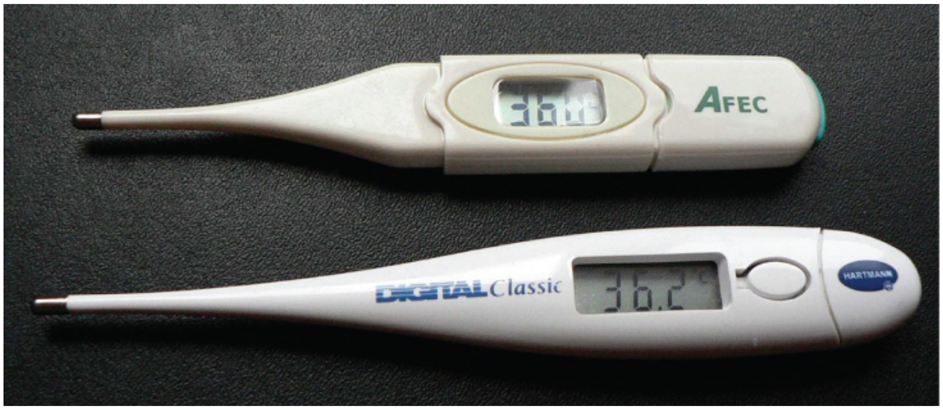


Figure 9.17 These familiar thermometers are based on the automated measurement of a thermistor's temperature-dependent resistance.

EXAMPLE 9.6

Calculating Resistance

Although caution must be used in applying $\rho = \rho_0(1 + \alpha\Delta T)$ and $R = R_0(1 + \alpha\Delta T)$ for temperature changes greater than 100 °C, for tungsten, the equations work reasonably well for very large temperature changes. A tungsten filament at 20 °C has a resistance of 0.350 Ω. What would the resistance be if the temperature is increased to 2850 °C?

Strategy

This is a straightforward application of $R = R_0(1 + \alpha\Delta T)$, since the original resistance of the filament is given as $R_0 = 0.350\ \Omega$ and the temperature change is $\Delta T = 2830\ °\text{C}$.

Solution

The resistance of the hotter filament R is obtained by entering known values into the above equation:

$$R = R_0\,(1 + \alpha\Delta T) = (0.350\ \Omega)\left[1 + \left(\frac{4.5\ \times\ 10^{-3}}{°\text{C}}\right)(2830\ °\text{C})\right] = 4.8\ \Omega\,.$$

Significance

Notice that the resistance changes by more than a factor of 10 as the filament warms to the high temperature and the current through the filament depends on the resistance of the filament and the voltage applied. If the filament is used in an incandescent light bulb, the initial current through the filament when the bulb is first energized will be higher than the current after the filament reaches the operating temperature.

CHECK YOUR UNDERSTANDING 9.6

A strain gauge is an electrical device to measure strain, as shown below. It consists of a flexible, insulating backing that supports a conduction foil pattern. The resistance of the foil changes as the backing is stretched. How does the strain gauge resistance change? Is the strain gauge affected by temperature changes?

$$\begin{aligned} dR &= \frac{\rho}{A}dr = \frac{\rho}{2\pi rL}dr, \\ R &= \int_{r_i}^{r_o} dR = \int_{r_i}^{r_o} \frac{\rho}{2\pi rL}dr = \frac{\rho}{2\pi L}\int_{r_i}^{r_o} \frac{1}{r}dr = \frac{\rho}{2\pi L}\ln\frac{r_o}{r_i}. \end{aligned}$$

Significance

The resistance of a coaxial cable depends on its length, the inner and outer radii, and the resistivity of the material separating the two conductors. Since this resistance is not infinite, a small leakage current occurs between the two conductors. This leakage current leads to the attenuation (or weakening) of the signal being sent through the cable.

CHECK YOUR UNDERSTANDING 9.7

The resistance between the two conductors of a coaxial cable depends on the resistivity of the material separating the two conductors, the length of the cable and the inner and outer radius of the two conductor. If you are designing a coaxial cable, how does the resistance between the two conductors depend on these variables?

INTERACTIVE

View this simulation (https://openstax.org/l/21batteryresist) to see how the voltage applied and the resistance of the material the current flows through affects the current through the material. You can visualize the collisions of the electrons and the atoms of the material effect the temperature of the material.

9.4 Ohm's Law

Learning Objectives

By the end of this section, you will be able to:

- Describe Ohm’s law
- Recognize when Ohm’s law applies and when it does not

We have been discussing three electrical properties so far in this chapter: current, voltage, and resistance. It turns out that many materials exhibit a simple relationship among the values for these properties, known as Ohm’s law. Many other materials do not show this relationship, so despite being called Ohm’s law, it is not considered a law of nature, like Newton’s laws or the laws of thermodynamics. But it is very useful for calculations involving materials that do obey Ohm’s law.

Description of Ohm’s Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

$$I \propto V.$$

This important relationship is the basis for **Ohm’s law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law, which is to say that it is an experimentally observed phenomenon, like friction. Such a linear relationship doesn’t always occur. Any material, component, or device that obeys Ohm’s law, where the current through the device is proportional to the voltage applied, is known as an **ohmic** material or ohmic component. Any material or component that does not obey Ohm’s law is known as a **nonohmic** material or nonohmic component.

Ohm's Experiment

In a paper published in 1827, Georg Ohm described an experiment in which he measured voltage across and current through various simple electrical circuits containing various lengths of wire. A similar experiment is shown in Figure 9.19. This experiment is used to observe the current through a resistor that results from an applied voltage. In this simple circuit, a resistor is connected in series with a battery. The voltage is measured with a voltmeter, which must be placed across the resistor (in parallel with the resistor). The current is measured with an ammeter, which must be in line with the resistor (in series with the resistor).

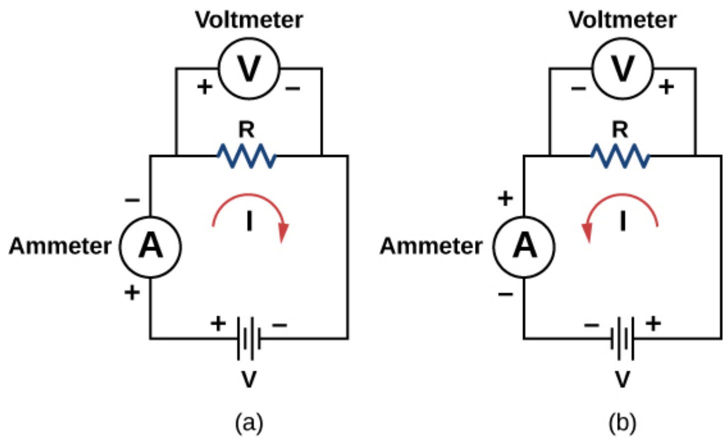


Figure 9.19 The experimental set-up used to determine if a resistor is an ohmic or nonohmic device. (a) When the battery is attached, the current flows in the clockwise direction and the voltmeter and ammeter have positive readings. (b) When the leads of the battery are switched, the current flows in the counterclockwise direction and the voltmeter and ammeter have negative readings.

In this updated version of Ohm's original experiment, several measurements of the current were made for several different voltages. When the battery was hooked up as in Figure 9.19(a), the current flowed in the clockwise direction and the readings of the voltmeter and ammeter were positive. Does the behavior of the current change if the current flowed in the opposite direction? To get the current to flow in the opposite direction, the leads of the battery can be switched. When the leads of the battery were switched, the readings of the voltmeter and ammeter readings were negative because the current flowed in the opposite direction, in this case, counterclockwise. Results of a similar experiment are shown in Figure 9.20.

I(A)	V(V)
−2.74	−10.00
−2.25	−9.00
−2.00	−8.00
−1.75	−7.00
−1.50	−6.00
−1.49	−5.00
−1.00	−4.00
−0.51	−3.00
−0.74	−2.00
−0.49	−1.00
+0.00	+0.00
+0.49	+1.00
+0.50	+2.00
+0.99	+3.00
+0.76	+4.00
+1.01	+5.00
+1.74	+6.00
+1.75	+7.00
+2.00	+8.00
+2.49	+9.00
+2.50	+10.00

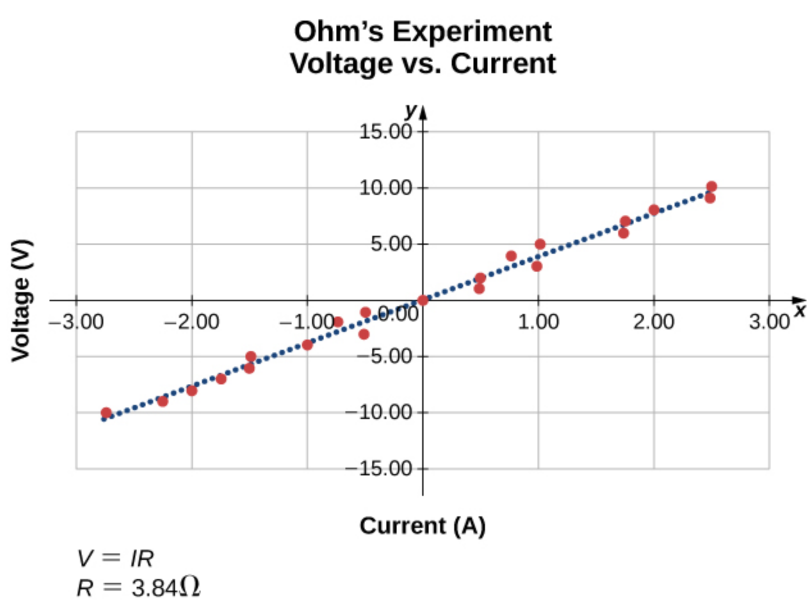


Figure 9.20 A resistor is placed in a circuit with a battery. The voltage applied varies from −10.00 V to +10.00 V, increased by 1.00-V

increments. A plot shows values of the voltage versus the current typical of what a casual experimenter might find.

In this experiment, the voltage applied across the resistor varies from −10.00 to +10.00 V, by increments of 1.00 V. The current through the resistor and the voltage across the resistor are measured. A plot is made of the voltage versus the current, and the result is approximately linear. The slope of the line is the resistance, or the voltage divided by the current. This result is known as Ohm's law:

$$V = IR, \tag{9.11}$$

where V is the voltage measured in volts across the object in question, I is the current measured through the object in amps, and R is the resistance in units of ohms. As stated previously, any device that shows a linear relationship between the voltage and the current is known as an ohmic device. A resistor is therefore an ohmic device.

EXAMPLE 9.8

Measuring Resistance

A carbon resistor at room temperature (20 °C) is attached to a 9.00-V battery and the current measured through the resistor is 3.00 mA. (a) What is the resistance of the resistor measured in ohms? (b) If the temperature of the resistor is increased to 60 °C by heating the resistor, what is the current through the resistor?

Strategy

(a) The resistance can be found using Ohm's law. Ohm's law states that $V = IR$, so the resistance can be found using $R = V/I$.

(b) First, the resistance is temperature dependent so the new resistance after the resistor has been heated can be found using $R = R_0(1 + \alpha\Delta T)$. The current can be found using Ohm's law in the form $I = V/R$.

Solution

a. Using Ohm's law and solving for the resistance yields the resistance at room temperature:

$$R = \frac{V}{I} = \frac{9.00\text{ V}}{3.00 \times 10^{-3}\text{ A}} = 3.00 \times 10^{3}\ \Omega = 3.00\text{ k}\Omega.$$

b. The resistance at 60 °C can be found using $R = R_0(1 + \alpha\Delta T)$ where the temperature coefficient for carbon is $\alpha = -0.0005$. $R = R_0(1 + \alpha\Delta T) = 3.00 \times 10^3(1 - 0.0005(60\,°\text{C} - 20\,°\text{C})) = 2.94\text{ k}\Omega$.
The current through the heated resistor is

$$I = \frac{V}{R} = \frac{9.00\text{ V}}{2.94 \times 10^{3}\ \Omega} = 3.06 \times 10^{-3}\text{ A} = 3.06\text{ mA}.$$

Significance

A change in temperature of 40 °C resulted in a 2.00% change in current. This may not seem like a very great change, but changing electrical characteristics can have a strong effect on the circuits. For this reason, many electronic appliances, such as computers, contain fans to remove the heat dissipated by components in the electric circuits.

CHECK YOUR UNDERSTANDING 9.8

The voltage supplied to your house varies as $V(t) = V_{\text{max}} \sin(2\pi f t)$. If a resistor is connected across this voltage, will Ohm's law $V = IR$ still be valid?

INTERACTIVE

See how the equation form of Ohm's law (https://openstax.org/l/21ohmslaw) relates to a simple circuit. Adjust current density, the conductivity, and the electrical field. This microscopic view suggests the proportionality $V \propto I$ comes from the drift velocity of the free electrons in the metal that results from an applied electrical field. As stated earlier, the current density is proportional to the applied electrical field. The reformulation of Ohm's law is credited to Gustav Kirchhoff, whose name we will see again in the next chapter.

9.5 Electrical Energy and Power

Learning Objectives

By the end of this section, you will be able to:

- Express electrical power in terms of the voltage and the current
- Describe the power dissipated by a resistor in an electric circuit
- Calculate the energy efficiency and cost effectiveness of appliances and equipment

In an electric circuit, electrical energy is continuously converted into other forms of energy. For example, when a current flows in a conductor, electrical energy is converted into thermal energy within the conductor. The electrical field, supplied by the voltage source, accelerates the free electrons, increasing their kinetic energy for a short time. This increased kinetic energy is converted into thermal energy through collisions with the ions of the lattice structure of the conductor. In Work and Kinetic Energy, we defined power as the rate at which work is done by a force measured in watts. Power can also be defined as the rate at which energy is transferred. In this section, we discuss the time rate of energy transfer, or power, in an electric circuit.

Power in Electric Circuits

Power is associated by many people with electricity. Power transmission lines might come to mind. We also think of light bulbs in terms of their power ratings in watts. What is the expression for **electric power**?

Let us compare a 25-W bulb with a 60-W bulb (Figure 9.23(a)). The 60-W bulb glows brighter than the 25-W bulb. Although it is not shown, a 60-W light bulb is also warmer than the 25-W bulb. The heat and light is produced by from the conversion of electrical energy. The kinetic energy lost by the electrons in collisions is converted into the internal energy of the conductor and radiation. How are voltage, current, and resistance related to electric power?

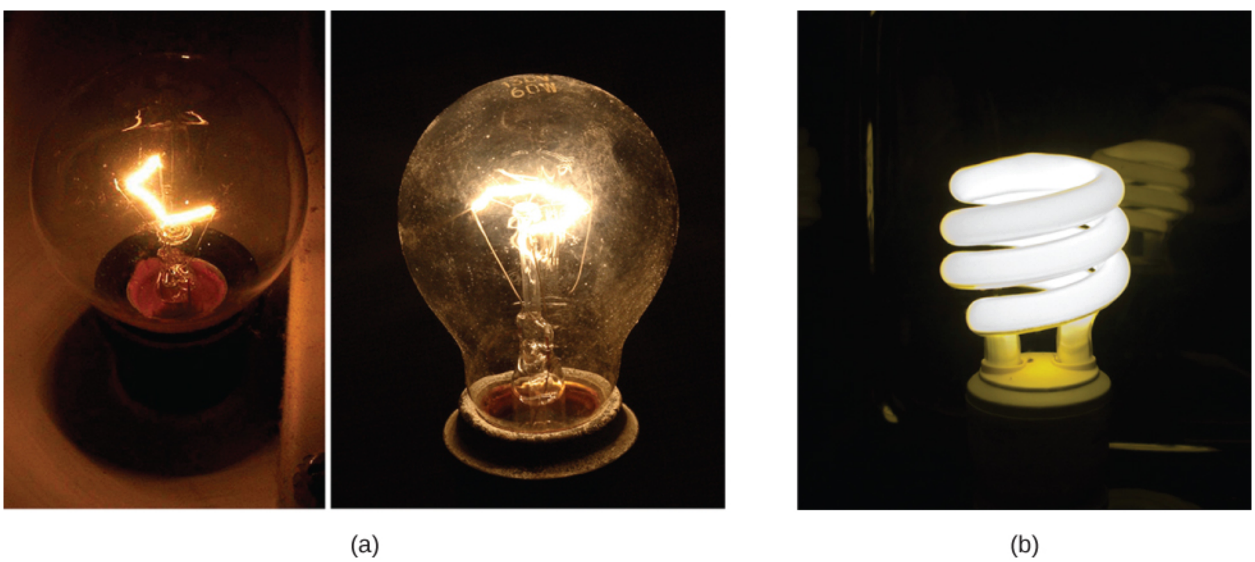


Figure 9.23 (a) Pictured above are two incandescent bulbs: a 25-W bulb (left) and a 60-W bulb (right). The 60-W bulb provides a higher intensity light than the 25-W bulb. The electrical energy supplied to the light bulbs is converted into heat and light. (b) This compact fluorescent light (CFL) bulb puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit a: modification of works by "Dickbauch"/Wikimedia Commons and Greg Westfall; credit b: modification of work by "dbgg1979"/Flickr)

To calculate electric power, consider a voltage difference existing across a material (Figure 9.24). The electric potential V_1 is higher than the electric potential at V_2, and the voltage difference is negative $V = V_2 - V_1$. As discussed in Electric Potential, an electrical field exists between the two potentials, which points from the higher potential to the lower potential. Recall that the electrical potential is defined as the potential energy per charge, $V = \Delta U/q$, and the charge ΔQ loses potential energy moving through the potential difference.

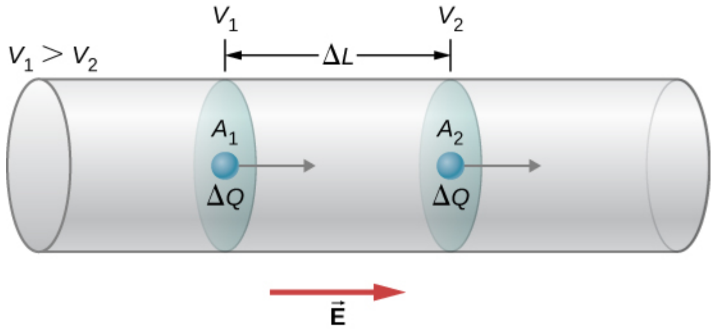


Figure 9.24 When there is a potential difference across a conductor, an electrical field is present that points in the direction from the higher potential to the lower potential.

If the charge is positive, the charge experiences a force due to the electrical field $\vec{\mathbf{F}} = m\vec{\mathbf{a}} = \Delta Q\vec{\mathbf{E}}$. This force is necessary to keep the charge moving. This force does not act to accelerate the charge through the entire distance ΔL because of the interactions of the charge with atoms and free electrons in the material. The speed, and therefore the kinetic energy, of the charge do not increase during the entire trip across ΔL, and charge passing through area A_2 has the same drift velocity v_{d} as the charge that passes through area A_1. However, work is done on the charge, by the electrical field, which changes the potential energy. Since the change in the electrical potential difference is negative, the electrical field is found to be

$$E = -\frac{(V_2 - V_1)}{\Delta L} = \frac{V}{\Delta L}.$$

The work done on the charge is equal to the electric force times the length at which the force is applied,

$$W = F\Delta L = (\Delta QE)\,\Delta L = \left(\Delta Q\frac{V}{\Delta L}\right)\Delta L = \Delta QV = \Delta U.$$

The charge moves at a drift velocity v_{d} so the work done on the charge results in a loss of potential energy, but the average kinetic energy remains constant. The lost electrical potential energy appears as thermal energy in the material. On a microscopic scale, the energy transfer is due to collisions between the charge and the molecules of the material, which leads to an increase in temperature in the material. The loss of potential energy results in an increase in the temperature of the material, which is dissipated as radiation. In a resistor, it is dissipated as heat, and in a light bulb, it is dissipated as heat and light.

The power dissipated by the material as heat and light is equal to the time rate of change of the work:

$$P = \frac{\Delta U}{\Delta t} = -\frac{\Delta QV}{\Delta t} = IV.$$

With a resistor, the voltage drop across the resistor is dissipated as heat. Ohm's law states that the voltage across the resistor is equal to the current times the resistance, $V = IR$. The power dissipated by the resistor is therefore

$$P = IV = I\,(IR) = I^2R \text{ or } P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}.$$

If a resistor is connected to a battery, the power dissipated as radiant energy by the wires and the resistor is equal to $P = IV = I^2R = \frac{V^2}{R}$. The power supplied from the battery is equal to current times the voltage, $P = IV$.

Electric Power

The electric power gained or lost by any device has the form

$$P = IV. \tag{9.12}$$

The power dissipated by a resistor has the form

$$P = I^2 R = \frac{V^2}{R}. \tag{9.13}$$

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature, its resistance is higher, too.

EXAMPLE 9.9

Calculating Power in Electric Devices

A DC winch motor is rated at 20.00 A with a voltage of 115 V. When the motor is running at its maximum power, it can lift an object with a weight of 4900.00 N a distance of 10.00 m, in 30.00 s, at a constant speed. (a) What is the power consumed by the motor? (b) What is the power used in lifting the object? Ignore air resistance. (c) Assuming that the difference in the power consumed by the motor and the power used lifting the object are dissipated as heat by the resistance of the motor, estimate the resistance of the motor?

Strategy

(a) The power consumed by the motor can be found using $P = IV$. (b) The power used in lifting the object at a constant speed can be found using $P = Fv$, where the speed is the distance divided by the time. The upward force supplied by the motor is equal to the weight of the object because the acceleration is zero. (c) The resistance of the motor can be found using $P = I^2 R$.

Solution

a. The power consumed by the motor is equal to $P = IV$ and the current is given as 20.00 A and the voltage is 115.00 V:

$$P = IV = (20.00\ \text{A})\ 115.00\ \text{V} = 2300.00\ \text{W}.$$

b. The power used lifting the object is equal to $P = Fv$ where the force is equal to the weight of the object (1960 N) and the magnitude of the velocity is $v = \frac{10.00\ \text{m}}{30.00\ \text{s}} = 0.33\frac{\text{m}}{\text{s}}$,

$$P = Fv = (4900\ \text{N})\ 0.33\ \text{m/s} = 1633.33\ \text{W}.$$

c. The difference in the power equals 2300.00 W − 1633.33 W = 666.67 W and the resistance can be found using $P = I^2 R$:

$$R = \frac{P}{I^2} = \frac{666.67\ \text{W}}{(20.00\ \text{A})^2} = 1.67\ \Omega.$$

Significance

The resistance of the motor is quite small. The resistance of the motor is due to many windings of copper wire. The power dissipated by the motor can be significant since the thermal power dissipated by the motor is proportional to the square of the current $\left(P = I^2 R\right)$.

CHECK YOUR UNDERSTANDING 9.9

Electric motors have a reasonably high efficiency. A 100-hp motor can have an efficiency of 90% and a 1-hp motor can have an efficiency of 80%. Why is it important to use high-performance motors?

A fuse (Figure 9.25) is a device that protects a circuit from currents that are too high. A fuse is basically a short piece of wire between two contacts. As we have seen, when a current is running through a conductor, the kinetic energy of the charge carriers is converted into thermal energy in the conductor. The piece of wire in the fuse is under tension and has a low melting point. The wire is designed to heat up and break at the rated current. The fuse is destroyed and must be replaced, but it protects the rest of the circuit. Fuses act quickly, but there is a small time delay while the wire heats up and breaks.

Figure 9.25 A fuse consists of a piece of wire between two contacts. When a current passes through the wire that is greater than the rated current, the wire melts, breaking the connection. Pictured is a “blown” fuse where the wire broke protecting a circuit (credit: modification of work by “Shardayyy”/Flickr).

Circuit breakers are also rated for a maximum current, and open to protect the circuit, but can be reset. Circuit breakers react much faster. The operation of circuit breakers is not within the scope of this chapter and will be discussed in later chapters. Another method of protecting equipment and people is the ground fault circuit interrupter (GFCI), which is common in bathrooms and kitchens. The GFCI outlets respond very quickly to changes in current. These outlets open when there is a change in magnetic field produced by current-carrying conductors, which is also beyond the scope of this chapter and is covered in a later chapter.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = \frac{dE}{dt}$, we see that

$$E = \int P dt$$

is the energy used by a device using power P for a time interval t. If power is delivered at a constant rate, then then the energy can be found by $E = Pt$. For example, the more light bulbs burning, the greater P used; the longer they are on, the greater t is.

The energy unit on electric bills is the kilowatt-hour $(\text{kW} \cdot \text{h})$, consistent with the relationship $E = Pt$. It is easy to estimate the cost of operating electrical appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted into joules. You can prove to yourself that $1 \text{ kW} \cdot \text{h} = 3.6 \times 10^6 \text{ J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This not only reduces the cost but also results in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home’s use of energy goes to lighting, and the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFLs). (See Figure 9.23(b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.)

The heat transfer from these CFLs is less, and they last up to 10 times longer than incandescent bulbs. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last five times longer than CFLs.

EXAMPLE 9.10

Calculating the Cost Effectiveness of LED Bulb

The typical replacement for a 100-W incandescent bulb is a 20-W LED bulb. The 20-W LED bulb can provide the same amount of light output as the 100-W incandescent light bulb. What is the cost savings for using the LED bulb in place of the incandescent bulb for one year, assuming $0.10 per kilowatt-hour is the average energy rate charged by the power company? Assume that the bulb is turned on for three hours a day.

Strategy

(a) Calculate the energy used during the year for each bulb, using $E = Pt$.

(b) Multiply the energy by the cost.

Solution

a. Calculate the power for each bulb.

$$\begin{aligned} E_{\text{Incandescent}} &= Pt = 100\text{ W}\left(\frac{1\text{ kW}}{1000\text{ W}}\right)\left(\frac{3\text{ h}}{\text{day}}\right)(365\text{ days}) = 109.5\text{ kW}\cdot\text{h} \\ E_{\text{LED}} &= Pt = 20\text{ W}\left(\frac{1\text{ kW}}{1000\text{ W}}\right)\left(\frac{3\text{ h}}{\text{day}}\right)(365\text{ days}) = 21.90\text{ kW}\cdot\text{h} \end{aligned}.$$

b. Calculate the cost for each.

$$\begin{aligned} \text{cost}_{\text{Incandescent}} &= 109.5\text{ kW-h}\left(\frac{\$0.10}{\text{kW}\cdot\text{h}}\right) = \$10.95 \\ \text{cost}_{\text{LED}} &= 21.90\text{ kW-h}\left(\frac{\$0.10}{\text{kW}\cdot\text{h}}\right) = \$2.19 \end{aligned}.$$

Significance

A LED bulb uses 80% less energy than the incandescent bulb, saving $8.76 over the incandescent bulb for one year. The LED bulb can cost $20.00 and the 100-W incandescent bulb can cost $0.75, which should be calculated into the computation. A typical lifespan of an incandescent bulb is 1200 hours and is 50,000 hours for the LED bulb. The incandescent bulb would last 1.08 years at 3 hours a day and the LED bulb would last 45.66 years. The initial cost of the LED bulb is high, but the cost to the home owner will be $0.69 for the incandescent bulbs versus $0.44 for the LED bulbs per year. (Note that the LED bulbs are coming down in price.) The cost savings per year is approximately $8.50, and that is just for one bulb.

CHECK YOUR UNDERSTANDING 9.10

Is the efficiency of the various light bulbs the only consideration when comparing the various light bulbs?

Changing light bulbs from incandescent bulbs to CFL or LED bulbs is a simple way to reduce energy consumption in homes and commercial sites. CFL bulbs operate with a much different mechanism than do incandescent lights. The mechanism is complex and beyond the scope of this chapter, but here is a very general description of the mechanism. CFL bulbs contain argon and mercury vapor housed within a spiral-shaped tube. The CFL bulbs use a “ballast” that increases the voltage used by the CFL bulb. The ballast produce an electrical current, which passes through the gas mixture and excites the gas molecules. The excited gas molecules produce ultraviolet (UV) light, which in turn stimulates the fluorescent coating on the inside of the tube. This coating fluoresces in the visible spectrum, emitting visible light. Traditional fluorescent tubes and CFL bulbs had a short time delay of up to a few seconds while the mixture was being “warmed up” and the molecules reached an excited state. It should be noted that these bulbs do contain mercury, which is poisonous, but if the bulb is broken, the mercury is never released. Even if the bulb is broken, the mercury tends to remain in the fluorescent coating. The amount is also quite small and the advantage of the energy saving may outweigh the disadvantage of using mercury.

The CFL light bulbs are being replaced with LED light bulbs, where LED stands for “light-emitting diode.” The diode was briefly discussed as a nonohmic device, made of semiconducting material, which essentially permits current flow in one direction. LEDs are a special type of diode made of semiconducting materials infused with impurities in combinations and concentrations that enable the extra energy from the movement of the electrons during electrical excitation to be converted into visible light. Semiconducting devices will be explained in greater detail in Condensed Matter Physics.

Commercial LEDs are quickly becoming the standard for commercial and residential lighting, replacing incandescent and CFL bulbs. They are designed for the visible spectrum and are constructed from gallium doped with arsenic and phosphorous atoms. The color emitted from an LED depends on the materials used in the semiconductor and the current. In the early years of LED development, small LEDs found on circuit boards were red, green, and yellow, but LED light bulbs can now be programmed to produce millions of colors of light as well as many different hues of white light.

Comparison of Incandescent, CFL, and LED Light Bulbs

The energy savings can be significant when replacing an incandescent light bulb or a CFL light bulb with an LED light. Light bulbs are rated by the amount of power that the bulb consumes, and the amount of light output is measured in lumens. The lumen (lm) is the SI -derived unit of luminous flux and is a measure of the total quantity of visible light emitted by a source. A 60-W incandescent light bulb can be replaced with a 13- to 15-W CFL bulb or a 6- to 8-W LED bulb, all three of which have a light output of approximately 800 lm. A table of light output for some commonly used light bulbs appears in Table 9.2.

The life spans of the three types of bulbs are significantly different. An LED bulb has a life span of 50,000 hours, whereas the CFL has a lifespan of 8000 hours and the incandescent lasts a mere 1200 hours. The LED bulb is the most durable, easily withstanding rough treatment such as jarring and bumping. The incandescent light bulb has little tolerance to the same treatment since the filament and glass can easily break. The CFL bulb is also less durable than the LED bulb because of its glass construction. The amount of heat emitted is 3.4 btu/h for the 8-W LED bulb, 85 btu/h for the 60-W incandescent bulb, and 30 btu/h for the CFL bulb. As mentioned earlier, a major drawback of the CFL bulb is that it contains mercury, a neurotoxin, and must be disposed of as hazardous waste. From these data, it is easy to understand why the LED light bulb is quickly becoming the standard in lighting.

Light Output (lumens)	LED Light Bulbs (watts)	Incandescent Light Bulbs (watts)	CFL Light Bulbs (watts)
450	4–5	40	9–13
800	6–8	60	13–15
1100	9–13	75	18–25
1600	16–20	100	23–30
2600	25–28	150	30–55

Table 9.2 Light Output of LED, Incandescent, and CFL Light Bulbs

Summary of Relationships

In this chapter, we have discussed relationships between voltages, current, resistance, and power. Figure 9.26 shows a summary of the relationships between these measurable quantities for ohmic devices. (Recall that ohmic devices follow Ohm's law $V = IR$.) For example, if you need to calculate the power, use the pink section, which shows that $P = VI$, $P = \frac{V^2}{R}$, and $P = I^2 R$.

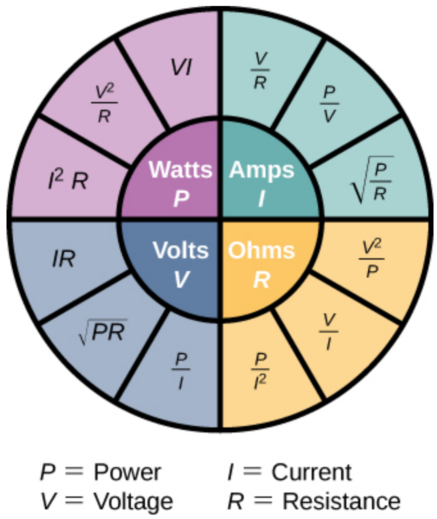


Figure 9.26 This circle shows a summary of the equations for the relationships between power, current, voltage, and resistance.

Which equation you use depends on what values you are given, or you measure. For example if you are given the current and the resistance, use $P = I^2 R$. Although all the possible combinations may seem overwhelming, don't forget that they all are combinations of just two equations, Ohm's law ($V = IR$) and power ($P = IV$).

9.6 Superconductors

Learning Objectives

By the end of this section, you will be able to:

- Describe the phenomenon of superconductivity
- List applications of superconductivity

Touch the power supply of your laptop computer or some other device. It probably feels slightly warm. That heat is an unwanted byproduct of the process of converting household electric power into a current that can be used by your device. Although electric power is reasonably efficient, other losses are associated with it. As discussed in the section on power and energy, transmission of electric power produces $I^2 R$ line losses. These line losses exist whether the power is generated from conventional power plants (using coal, oil, or gas), nuclear plants, solar plants, hydroelectric plants, or wind farms. These losses can be reduced, but not eliminated, by transmitting using a higher voltage. It would be wonderful if these line losses could be eliminated, but that would require transmission lines that have zero resistance. In a world that has a global interest in not wasting energy, the reduction or elimination of this unwanted thermal energy would be a significant achievement. Is this possible?

The Resistance of Mercury

In 1911, Heike Kamerlingh Onnes of Leiden University, a Dutch physicist, was looking at the temperature dependence of the resistance of the element mercury. He cooled the sample of mercury and noticed the familiar behavior of a linear dependence of resistance on temperature; as the temperature decreased, the resistance decreased. Kamerlingh Onnes continued to cool the sample of mercury, using liquid helium. As the temperature approached $4.2\ \text{K}$ ($-269.2\ °\text{C}$), the resistance abruptly went to zero (Figure 9.27). This temperature is known as the **critical temperature** T_c for mercury. The sample of mercury entered into a phase where the resistance was absolutely zero. This phenomenon is known as **superconductivity**. (*Note:* If you connect the leads of a three-digit ohmmeter across a conductor, the reading commonly shows up as

$0.00\ \Omega$. The resistance of the conductor is not actually zero, it is less than $0.01\ \Omega$.) There are various methods to measure very small resistances, such as the four-point method, but an ohmmeter is not an acceptable method to use for testing resistance in superconductivity.

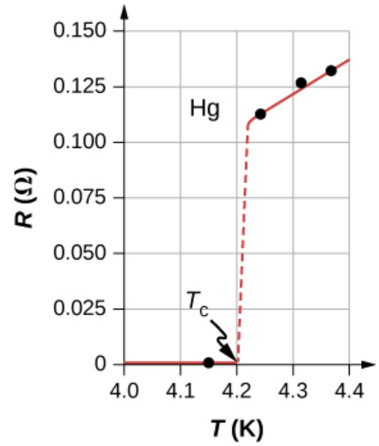


Figure 9.27 The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to the temperature of about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

Other Superconducting Materials

As research continued, several other materials were found to enter a superconducting phase, when the temperature reached near absolute zero. In 1941, an alloy of niobium-nitride was found that could become superconducting at $T_c = 16\ \text{K}\ (-257\ °\text{C})$ and in 1953, vanadium-silicon was found to become superconductive at $T_c = 17.5\ \text{K}(-255.7\ °\text{C})$. The temperatures for the transition into superconductivity were slowly creeping higher. Strangely, many materials that make good conductors, such as copper, silver, and gold, do not exhibit superconductivity. Imagine the energy savings if transmission lines for electric power-generating stations could be made to be superconducting at temperatures near room temperature! A resistance of zero ohms means no $I^2 R$ losses and a great boost to reducing energy consumption. The problem is that $T_c = 17.5\ \text{K}$ is still very cold and in the range of liquid helium temperatures. At this temperature, it is not cost effective to transmit electrical energy because of the cooling requirements.

A large jump was seen in 1986, when a team of researchers, headed by Dr. Ching Wu Chu of Houston University, fabricated a brittle, ceramic compound with a transition temperature of $T_c = 92\ \text{K}\ (-181\ °\text{C})$. The ceramic material, composed of yttrium barium copper oxide (YBCO), was an insulator at room temperature. Although this temperature still seems quite cold, it is near the boiling point of liquid nitrogen, a liquid commonly used in refrigeration. You may have noticed refrigerated trucks traveling down the highway labeled as “Liquid Nitrogen Cooled.”

YBCO ceramic is a material that could be useful for transmitting electrical energy because the cost saving of reducing the $I^2 R$ losses are larger than the cost of cooling the superconducting cable, making it financially feasible. There were and are many engineering problems to overcome. For example, unlike traditional electrical cables, which are flexible and have a decent tensile strength, ceramics are brittle and would break rather than stretch under pressure. Processes that are rather simple with traditional cables, such as making connections, become difficult when working with ceramics. The problems are difficult and complex, and material scientists and engineers are coming up with innovative solutions.

An interesting consequence of the resistance going to zero is that once a current is established in a superconductor, it persists without an applied voltage source. Current loops in a superconductor have been set up and the current loops have been observed to persist for years without decaying.

Zero resistance is not the only interesting phenomenon that occurs as the materials reach their transition temperatures. A second effect is the exclusion of magnetic fields. This is known as the **Meissner effect** (Figure 9.28). A light, permanent magnet placed over a superconducting sample will levitate in a stable position above the superconductor. High-speed trains have been developed that levitate on strong superconducting magnets, eliminating the friction normally experienced between the train and the tracks. In Japan, the Yamanashi Maglev test line opened on April 3, 1997. In April 2015, the MLX01 test vehicle attained a speed of 374 mph (603 km/h).

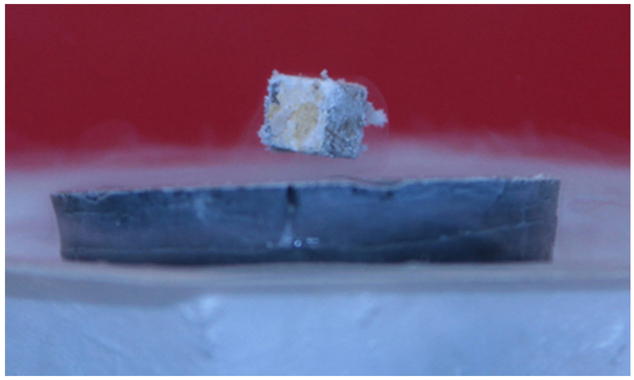


Figure 9.28 A small, strong magnet levitates over a superconductor cooled to liquid nitrogen temperature. The magnet levitates because the superconductor excludes magnetic fields. (credit: Joseph J. Trout)

Table 9.3 shows a select list of elements, compounds, and high-temperature superconductors, along with the critical temperatures for which they become superconducting. Each section is sorted from the highest critical temperature to the lowest. Also listed is the critical magnetic field for some of the materials. This is the strength of the magnetic field that destroys superconductivity. Finally, the type of the superconductor is listed.

There are two types of superconductors. There are 30 pure metals that exhibit zero resistivity below their critical temperature and exhibit the Meissner effect, the property of excluding magnetic fields from the interior of the superconductor while the superconductor is at a temperature below the critical temperature. These metals are called Type I superconductors. The superconductivity exists only below their critical temperatures and below a critical magnetic field strength. Type I superconductors are well described by the BCS theory (described next). Type I superconductors have limited practical applications because the strength of the critical magnetic field needed to destroy the superconductivity is quite low.

Type II superconductors are found to have much higher critical magnetic fields and therefore can carry much higher current densities while remaining in the superconducting state. A collection of various ceramics containing barium-copper-oxide have much higher critical temperatures for the transition into a superconducting state. Superconducting materials that belong to this subcategory of the Type II superconductors are often categorized as high-temperature superconductors.

Introduction to BCS Theory

Type I superconductors, along with some Type II superconductors can be modeled using the BCS theory, proposed by John Bardeen, Leon Cooper, and Robert Schrieffer. Although the theory is beyond the scope of this chapter, a short summary of the theory is provided here. (More detail is provided in Condensed Matter Physics.) The theory considers pairs of electrons and how they are coupled together through lattice-vibration interactions. Through the interactions with the crystalline lattice, electrons near the Fermi energy level feel a small attractive force and form pairs (Cooper pairs), and the coupling is known as a phonon interaction. Single electrons are fermions, which are particles that obey the Pauli exclusion principle. The Pauli exclusion principle in quantum mechanics states that two identical fermions (particles with half-integer spin) cannot occupy the same quantum state simultaneously. Each electron has four quantum numbers (n, l, m_l, m_s). The principal quantum number (n) describes the energy of the electron, the orbital angular momentum quantum number (l) indicates the most probable distance from the nucleus, the magnetic quantum number (m_l) describes the energy levels in the subshell, and the electron spin quantum number (m_s) describes the orientation of the spin of the electron, either up or down. As the material enters a superconducting state, pairs of electrons act more like bosons, which can condense into the same energy level and need not obey the Pauli exclusion principle. The electron pairs have a slightly lower energy and leave an energy gap above them on the order of 0.001 eV. This energy gap inhibits collision interactions that lead to ordinary resistivity. When the material is below the critical temperature, the thermal energy is less than the band gap and the material

CHAPTER REVIEW

Key Terms

ampere (amp) SI unit for current; 1 A = 1 C/s

circuit complete path that an electrical current travels along

conventional current current that flows through a circuit from the positive terminal of a battery through the circuit to the negative terminal of the battery

critical temperature temperature at which a material reaches superconductivity

current density flow of charge through a cross-sectional area divided by the area

diode nonohmic circuit device that allows current flow in only one direction

drift velocity velocity of a charge as it moves nearly randomly through a conductor, experiencing multiple collisions, averaged over a length of a conductor, whose magnitude is the length of conductor traveled divided by the time it takes for the charges to travel the length

electrical conductivity measure of a material's ability to conduct or transmit electricity

electrical current rate at which charge flows, $I = \frac{dQ}{dt}$

electrical power time rate of change of energy in an electric circuit

Josephson junction junction of two pieces of superconducting material separated by a thin layer of insulating material, which can carry a supercurrent

Meissner effect phenomenon that occurs in a superconducting material where all magnetic fields are expelled

nonohmic type of a material for which Ohm's law is not valid

ohm (Ω) unit of electrical resistance, $1\,\Omega = 1$ V/A

Ohm's law empirical relation stating that the current I is proportional to the potential difference V; it is often written as $V = IR$, where R is the resistance

ohmic type of a material for which Ohm's law is valid, that is, the voltage drop across the device is equal to the current times the resistance

resistance electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, $R = V/I$

resistivity intrinsic property of a material, independent of its shape or size, directly proportional to the resistance, denoted by ρ

schematic graphical representation of a circuit using standardized symbols for components and solid lines for the wire connecting the components

SQUID (Superconducting Quantum Interference Device) device that is a very sensitive magnetometer, used to measure extremely subtle magnetic fields

superconductivity phenomenon that occurs in some materials where the resistance goes to exactly zero and all magnetic fields are expelled, which occurs dramatically at some low critical temperature (T_C)

Key Equations

Average electrical current	$I_{\text{ave}} = \frac{\Delta Q}{\Delta t}$
Definition of an ampere	1 A = 1 C/s
Electrical current	$I = \frac{dQ}{dt}$
Drift velocity	$v_d = \frac{I}{nqA}$
Current density	$I = \iint_{\text{area}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$
Resistivity	$\rho = \frac{E}{J}$

Common expression of Ohm's law	$V = IR$
Resistivity as a function of temperature	$\rho = \rho_0 \left[1 + \alpha \left(T - T_0\right)\right]$
Definition of resistance	$R \equiv \frac{V}{I}$
Resistance of a cylinder of material	$R = \rho \frac{L}{A}$
Temperature dependence of resistance	$R = R_0 \left(1 + \alpha \Delta T\right)$
Electric power	$P = IV$
Power dissipated by a resistor	$P = I^2 R = \frac{V^2}{R}$

Summary

9.1 Electrical Current

- The average electrical current I_{ave} is the rate at which charge flows, given by $I_{\text{ave}} = \frac{\Delta Q}{\Delta t}$, where ΔQ is the amount of charge passing through an area in time Δt.
- The instantaneous electrical current, or simply the current *I*, is the rate at which charge flows. Taking the limit as the change in time approaches zero, we have $I = \frac{dQ}{dt}$, where $\frac{dQ}{dt}$ is the time derivative of the charge.
- The direction of conventional current is taken as the direction in which positive charge moves. In a simple direct-current (DC) circuit, this will be from the positive terminal of the battery to the negative terminal.
- The SI unit for current is the ampere, or simply the amp (A), where $1 \text{ A} = 1 \text{ C/s}$.
- Current consists of the flow of free charges, such as electrons, protons, and ions.

9.2 Model of Conduction in Metals

- The current through a conductor depends mainly on the motion of free electrons.
- When an electrical field is applied to a conductor, the free electrons in a conductor do not move through a conductor at a constant speed and direction; instead, the motion is almost random due to collisions with atoms and other free electrons.
- Even though the electrons move in a nearly random fashion, when an electrical field is applied to the conductor, the overall velocity of the electrons can be defined in terms of a drift velocity.
- The current density is a vector quantity defined as the current through an infinitesimal area divided by the area.
- The current can be found from the current density, $I = \iint_{\text{area}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$.
- An incandescent light bulb is a filament of wire enclosed in a glass bulb that is partially evacuated. Current runs through the filament, where the electrical energy is converted to light and heat.

9.3 Resistivity and Resistance

- Resistance has units of ohms (Ω), related to volts and amperes by $1\ \Omega = 1 \text{ V/A}$.
- The resistance *R* of a cylinder of length *L* and cross-sectional area *A* is $R = \frac{\rho L}{A}$, where ρ is the resistivity of the material.
- Values of ρ in Table 9.1 show that materials fall into three groups—conductors, semiconductors, and insulators.
- Temperature affects resistivity; for relatively small temperature changes ΔT, resistivity is $\rho = \rho_0 \left(1 + \alpha \Delta T\right)$, where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity.
- The resistance *R* of an object also varies with temperature: $R = R_0 \left(1 + \alpha \Delta T\right)$, where R_0 is the original resistance, and *R* is the resistance after the temperature change.

9.4 Ohm's Law

- Ohm's law is an empirical relationship for current, voltage, and resistance for some