CHAPTER 8
Capacitance

Figure 8.1 The tree-like branch patterns in this clear Plexiglas® block are known as a Lichtenberg figure, named for the German physicist Georg Christof Lichtenberg (1742–1799), who was the first to study these patterns. The "branches" are created by the dielectric breakdown produced by a strong electric field. (credit: modification of work by Bert Hickman)

Chapter Outline

INTRODUCTION Capacitors are important components of electrical circuits in many electronic devices, including pacemakers, cell phones, and computers. In this chapter, we study their properties, and, over the next few chapters, we examine their function in combination with other circuit elements. By themselves, capacitors are often used to store electrical energy and release it when needed; with other circuit components, capacitors often act as part of a filter that allows some electrical signals to pass while blocking others. You can see why capacitors are considered one of the fundamental components of electrical circuits.

8.1 Capacitors and Capacitance

Learning Objectives

By the end of this section, you will be able to:

- Explain the concepts of a capacitor and its capacitance
- Describe how to evaluate the capacitance of a system of conductors

A **capacitor** is a device used to store electrical charge and electrical energy. Capacitors are generally with two electrical conductors separated by a distance. (Note that such electrical conductors are sometimes referred to as "electrodes," but more correctly, they are "capacitor plates.") The space between capacitors may simply be a vacuum, and, in that case, a capacitor is then known as a "vacuum capacitor." However, the space is usually filled with an insulating material known as a **dielectric**. (You will learn more about dielectrics in the sections on dielectrics later in this chapter.) The amount of storage in a capacitor is determined by a property called *capacitance*, which you will learn more about a bit later in this section.

Capacitors have applications ranging from filtering static from radio reception to energy storage in heart defibrillators. Typically, commercial capacitors have two conducting parts close to one another but not touching, such as those in Figure 8.2. Most of the time, a dielectric is used between the two plates. When battery terminals are connected to an initially uncharged capacitor, the battery potential moves a small amount of charge of magnitude Q from the positive plate to the negative plate. The capacitor remains neutral overall, but with charges $+Q$ and $-Q$ residing on opposite plates.

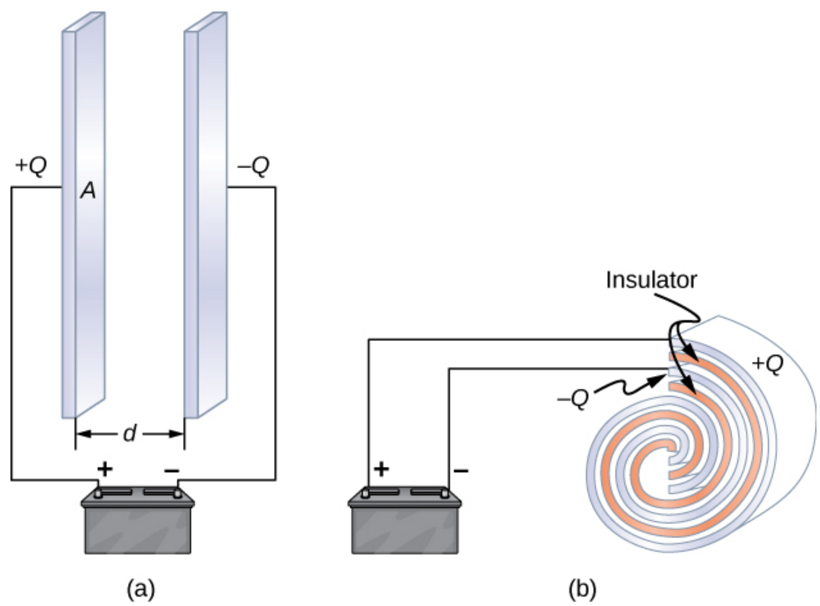


Figure 8.2 Both capacitors shown here were initially uncharged before being connected to a battery. They now have charges of $+Q$ and $-Q$ (respectively) on their plates. (a) A parallel-plate capacitor consists of two plates of opposite charge with area *A* separated by distance *d*. (b) A rolled capacitor has a dielectric material between its two conducting sheets (plates).

A system composed of two identical parallel-conducting plates separated by a distance is called a **parallel-plate capacitor** (Figure 8.3). The magnitude of the electrical field in the space between the parallel plates is $E = \sigma/\varepsilon_0$, where σ denotes the surface charge density on one plate (recall that σ is the charge Q per the surface area A). Thus, the magnitude of the field is directly proportional to Q.

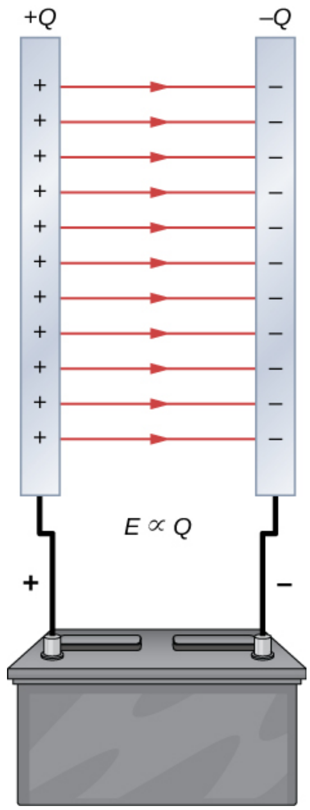


Figure 8.3 The charge separation in a capacitor shows that the charges remain on the surfaces of the capacitor plates. Electrical field lines in a parallel-plate capacitor begin with positive charges and end with negative charges. The magnitude of the electrical field in the space between the plates is in direct proportion to the amount of charge on the capacitor.

Capacitors with different physical characteristics (such as shape and size of their plates) store different amounts of charge for the same applied voltage V across their plates. The **capacitance** C of a capacitor is defined as the ratio of the maximum charge Q that can be stored in a capacitor to the applied voltage V across its plates. In other words, capacitance is the largest amount of charge per volt that can be stored on the device:

$$C = \frac{Q}{V}. \tag{8.1}$$

The SI unit of capacitance is the farad (F), named after Michael Faraday (1791–1867). Since capacitance is the charge per unit voltage, one farad is one coulomb per one volt, or

$$1\text{ F} = \frac{1\text{ C}}{1\text{ V}}.$$

By definition, a 1.0-F capacitor is able to store 1.0 C of charge (a very large amount of charge) when the potential difference between its plates is only 1.0 V. One farad is therefore a very large capacitance. Typical capacitance values range from picofarads ($1\text{ pF} = 10^{-12}\text{ F}$) to millifarads ($1\text{ mF} = 10^{-3}\text{ F}$), which also includes microfarads ($1\ \mu\text{F} = 10^{-6}\text{ F}$). Capacitors can be produced in various shapes and sizes (Figure 8.4).

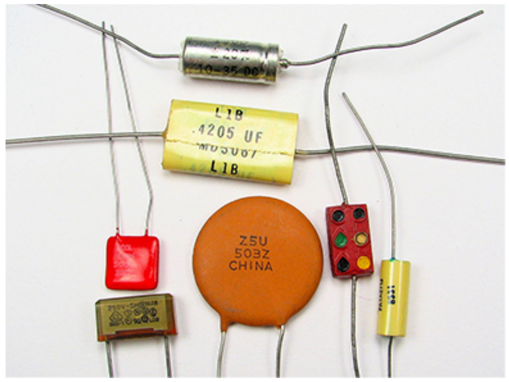


Figure 8.4 These are some typical capacitors used in electronic devices. A capacitor's size is not necessarily related to its capacitance value. (credit: Windell Oskay)

Calculation of Capacitance

We can calculate the capacitance of a pair of conductors with the standard approach that follows.

PROBLEM-SOLVING STRATEGY

Calculating Capacitance

1. Assume that the capacitor has a charge Q.
2. Determine the electrical field $\vec{\mathbf{E}}$ between the conductors. If symmetry is present in the arrangement of conductors, you may be able to use Gauss's law for this calculation.
3. Find the potential difference between the conductors from

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}, \tag{8.2}$$

where the path of integration leads from one conductor to the other. The magnitude of the potential difference is then $V = |V_B - V_A|$.
4. With V known, obtain the capacitance directly from Equation 8.1.

To show how this procedure works, we now calculate the capacitances of parallel-plate, spherical, and cylindrical capacitors. In all cases, we assume vacuum capacitors (empty capacitors) with no dielectric substance in the space between conductors.

Parallel-Plate Capacitor

The parallel-plate capacitor (Figure 8.5) has two identical conducting plates, each having a surface area A, separated by a distance d. When a voltage V is applied to the capacitor, it stores a charge Q, as shown. We can see how its capacitance may depend on A and d by considering characteristics of the Coulomb force. We know that force between the charges increases with charge values and decreases with the distance between them. We should expect that the bigger the plates are, the more charge they can store. Thus, C should be greater for a larger value of A. Similarly, the closer the plates are together, the greater the attraction of the opposite charges on them. Therefore, C should be greater for a smaller d.

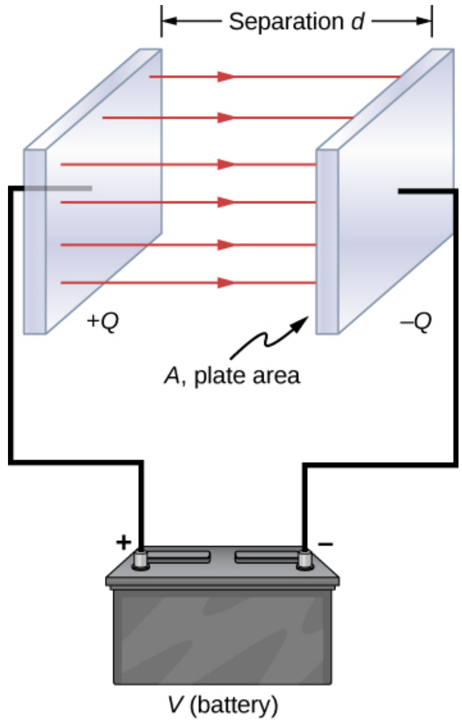


Figure 8.5 In a parallel-plate capacitor with plates separated by a distance *d*, each plate has the same surface area *A*.

We define the surface charge density σ on the plates as

$$\sigma = \frac{Q}{A}.$$

We know from previous chapters that when *d* is small, the electrical field between the plates is fairly uniform (ignoring edge effects) and that its magnitude is given by

$$E = \frac{\sigma}{\varepsilon_0},$$

where the constant ε_0 is the permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m. The SI unit of F/m is equivalent to $C^2/N \cdot m^2$. Since the electrical field $\vec{\mathbf{E}}$ between the plates is uniform, the potential difference between the plates is

$$V = Ed = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{\varepsilon_0 A}.$$

Therefore Equation 8.1 gives the capacitance of a parallel-plate capacitor as

$$C = \frac{Q}{V} = \frac{Q}{Qd/\varepsilon_0 A} = \varepsilon_0 \frac{A}{d}. \tag{8.3}$$

Notice from this equation that capacitance is a function *only of the geometry* and what material fills the space between the plates (in this case, vacuum) of this capacitor. In fact, this is true not only for a parallel-plate capacitor, but for all capacitors: The capacitance is independent of *Q* or *V*. If the charge changes, the potential changes correspondingly so that *Q*/*V* remains constant.

EXAMPLE 8.1

Capacitance and Charge Stored in a Parallel-Plate Capacitor

(a) What is the capacitance of an empty parallel-plate capacitor with metal plates that each have an area of $1.00\ m^2$, separated by 1.00 mm? (b) How much charge is stored in this capacitor if a voltage of 3.00×10^3 V is applied to it?

Strategy

Finding the capacitance C is a straightforward application of Equation 8.3. Once we find C, we can find the charge stored by using Equation 8.1.

Solution

a. Entering the given values into Equation 8.3 yields

$$C = \varepsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12} \frac{\text{F}}{\text{m}}\right) \frac{1.00\,\text{m}^2}{1.00 \times 10^{-3}\,\text{m}} = 8.85 \times 10^{-9}\,\text{F} = 8.85\,\text{nF}.$$

This small capacitance value indicates how difficult it is to make a device with a large capacitance.

b. Inverting Equation 8.1 and entering the known values into this equation gives

$$Q = CV = (8.85 \times 10^{-9}\,\text{F})(3.00 \times 10^3\,\text{V}) = 26.6\,\mu\text{C}.$$

Significance

This charge is only slightly greater than those found in typical static electricity applications. Since air breaks down (becomes conductive) at an electrical field strength of about 3.0 MV/m, no more charge can be stored on this capacitor by increasing the voltage.

EXAMPLE 8.2

A 1-F Parallel-Plate Capacitor

Suppose you wish to construct a parallel-plate capacitor with a capacitance of 1.0 F. What area must you use for each plate if the plates are separated by 1.0 mm?

Solution

Rearranging Equation 8.3, we obtain

$$A = \frac{Cd}{\varepsilon_0} = \frac{(1.0\,\text{F})(1.0 \times 10^{-3}\,\text{m})}{8.85 \times 10^{-12}\,\text{F/m}} = 1.1 \times 10^8\,\text{m}^2.$$

Each square plate would have to be 10 km across. It used to be a common prank to ask a student to go to the laboratory stockroom and request a 1-F parallel-plate capacitor, until stockroom attendants got tired of the joke.

CHECK YOUR UNDERSTANDING 8.1

The capacitance of a parallel-plate capacitor is 2.0 pF. If the area of each plate is 2.4 cm^2, what is the plate separation?

CHECK YOUR UNDERSTANDING 8.2

Verify that σ/V and ε_0/d have the same physical units.

Spherical Capacitor

A spherical capacitor is another set of conductors whose capacitance can be easily determined (Figure 8.6). It consists of two concentric conducting spherical shells of radii R_1 (inner shell) and R_2 (outer shell). The shells are given equal and opposite charges $+Q$ and $-Q$, respectively. From symmetry, the electrical field between the shells is directed radially outward. We can obtain the magnitude of the field by applying Gauss's law over a spherical Gaussian surface of radius r concentric with the shells. The enclosed charge is $+Q$; therefore we have

$$\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = E(4\pi r^2) = \frac{Q}{\varepsilon_0}.$$

Thus, the electrical field between the conductors is

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{\mathbf{r}}.$$

We substitute this $\vec{\mathbf{E}}$ into Equation 8.2 and integrate along a radial path between the shells:

$$V = \int_{R_1}^{R_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \int_{R_1}^{R_2} \left(\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{\mathbf{r}} \right) \cdot (\hat{\mathbf{r}} \quad dr) = \frac{Q}{4\pi\varepsilon_0} \int_{R_1}^{R_2} \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

In this equation, the potential difference between the plates is $V = -(V_2 - V_1) = V_1 - V_2$. We substitute this result into Equation 8.1 to find the capacitance of a spherical capacitor:

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}. \qquad 8.4$$

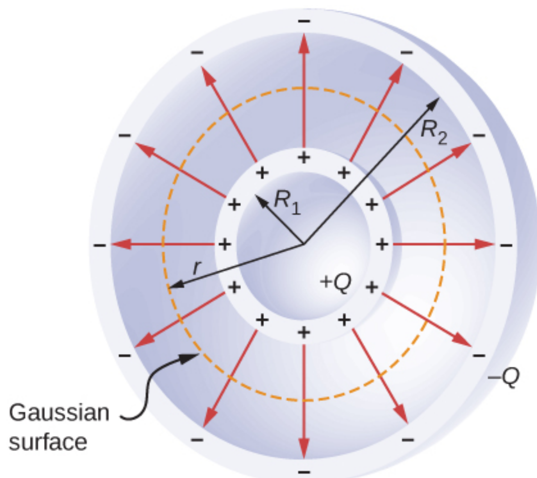


Figure 8.6 A spherical capacitor consists of two concentric conducting spheres. Note that the charges on a conductor reside on its surface.

EXAMPLE 8.3

Capacitance of an Isolated Sphere

Calculate the capacitance of a single isolated conducting sphere of radius R_1 and compare it with Equation 8.4 in the limit as $R_2 \to \infty$.

Strategy

We assume that the charge on the sphere is Q, and so we follow the four steps outlined earlier. We also assume the other conductor to be a concentric hollow sphere of infinite radius.

Solution

On the outside of an isolated conducting sphere, the electrical field is given by Equation 8.2. The magnitude of the potential difference between the surface of an isolated sphere and infinity is

$$V = \int_{R_1}^{+\infty} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \frac{Q}{4\pi\varepsilon_0} \int_{R_1}^{+\infty} \frac{1}{r^2} \hat{\mathbf{r}} \cdot (\hat{\mathbf{r}}\, dr) = \frac{Q}{4\pi\varepsilon_0} \int_{R_1}^{+\infty} \frac{dr}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R_1}.$$

The capacitance of an isolated sphere is therefore

$$C = \frac{Q}{V} = Q\frac{4\pi\varepsilon_0 R_1}{Q} = 4\pi\varepsilon_0 R_1.$$

Significance

The same result can be obtained by taking the limit of Equation 8.4 as $R_2 \to \infty$. A single isolated sphere is therefore equivalent to a spherical capacitor whose outer shell has an infinitely large radius.

CHECK YOUR UNDERSTANDING 8.3

The radius of the outer sphere of a spherical capacitor is five times the radius of its inner shell. What are the dimensions of this capacitor if its capacitance is 5.00 pF?

Cylindrical Capacitor

A cylindrical capacitor consists of two concentric, conducting cylinders (Figure 8.7). The inner cylinder, of radius R_1, may either be a shell or be completely solid. The outer cylinder is a shell of inner radius R_2. We assume that the length of each cylinder is l and that the excess charges $+Q$ and $-Q$ reside on the inner and outer cylinders, respectively.

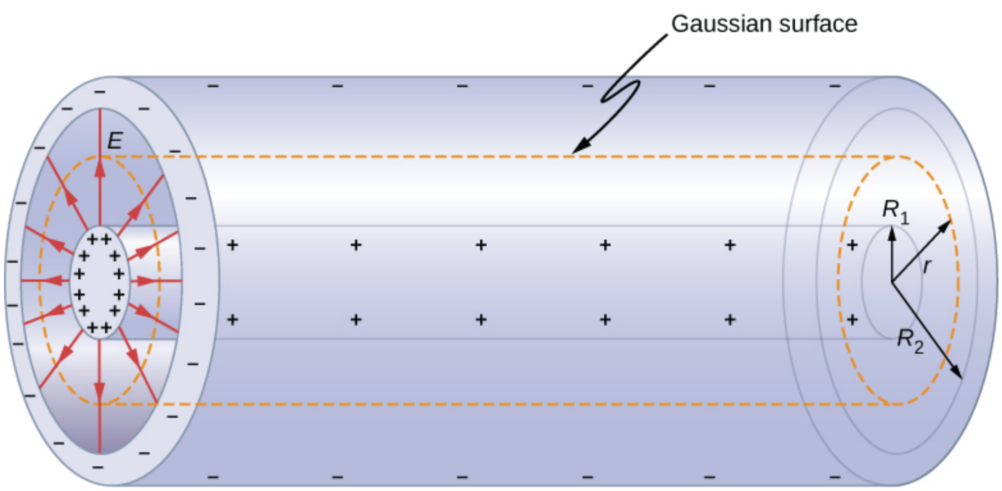


Figure 8.7 A cylindrical capacitor consists of two concentric, conducting cylinders. Here, the charge on the outer surface of the inner cylinder is positive (indicated by +) and the charge on the inner surface of the outer cylinder is negative (indicated by −).

With edge effects ignored, the electrical field between the conductors is directed radially outward from the common axis of the cylinders. Using the Gaussian surface shown in Figure 8.7, we have

$$\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = E(2\pi r l) = \frac{Q}{\varepsilon_0}.$$

Therefore, the electrical field between the cylinders is

$$\vec{\mathbf{E}} = \frac{1}{2\pi\varepsilon_0}\frac{Q}{r\,l}\hat{\mathbf{r}}. \quad \textbf{8.5}$$

Here $\hat{\mathbf{r}}$ is the unit radial vector along the radius of the cylinder. We can substitute into Equation 8.2 and find the potential difference between the cylinders:

$$V = \int_{R_1}^{R_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}_p = \frac{Q}{2\pi\varepsilon_0\, l}\int_{R_1}^{R_2} \frac{1}{r}\hat{\mathbf{r}} \cdot (\hat{\mathbf{r}}\, dr) = \frac{Q}{2\pi\varepsilon_0\, l}\int_{R_1}^{R_2} \frac{dr}{r} = \frac{Q}{2\pi\varepsilon_0\, l}\ln r\Big|_{R_1}^{R_2} = \frac{Q}{2\pi\varepsilon_0\, l}\ln\frac{R_2}{R_1}.$$

Thus, the capacitance of a cylindrical capacitor is

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0\, l}{\ln(R_2/R_1)}. \qquad 8.6$$

As in other cases, this capacitance depends only on the geometry of the conductor arrangement. An important application of Equation 8.6 is the determination of the capacitance per unit length of a *coaxial cable*, which is commonly used to transmit time-varying electrical signals. A coaxial cable consists of two concentric, cylindrical conductors separated by an insulating material. (Here, we assume a vacuum between the conductors, but the physics is qualitatively almost the same when the space between the conductors is filled by a dielectric.) This configuration shields the electrical signal propagating down the inner conductor from stray electrical fields external to the cable. Current flows in opposite directions in the inner and the outer conductors, with the outer conductor usually grounded. Now, from Equation 8.6, the capacitance per unit length of the coaxial cable is given by

$$\frac{C}{l} = \frac{2\pi\varepsilon_0}{\ln(R_2/R_1)}.$$

In practical applications, it is important to select specific values of C/l. This can be accomplished with appropriate choices of radii of the conductors and of the insulating material between them.

CHECK YOUR UNDERSTANDING 8.4

When a cylindrical capacitor is given a charge of 0.500 nC, a potential difference of 20.0 V is measured between the cylinders. (a) What is the capacitance of this system? (b) If the cylinders are 1.0 m long, what is the ratio of their radii?

Several types of practical capacitors are shown in Figure 8.4. Common capacitors are often made of two small pieces of metal foil separated by two small pieces of insulation (see Figure 8.2(b)). The metal foil and insulation are encased in a protective coating, and two metal leads are used for connecting the foils to an external circuit. Some common insulating materials are mica, ceramic, paper, and Teflon™ non-stick coating.

Another popular type of capacitor is an electrolytic capacitor. It consists of an oxidized metal in a conducting paste. The main advantage of an electrolytic capacitor is its high capacitance relative to other common types of capacitors. For example, capacitance of one type of aluminum electrolytic capacitor can be as high as 1.0 F. However, you must be careful when using an electrolytic capacitor in a circuit, because it only functions correctly when the metal foil is at a higher potential than the conducting paste. When reverse polarization occurs, electrolytic action destroys the oxide film. This type of capacitor cannot be connected across an alternating current source, because half of the time, ac voltage would have the wrong polarity, as an alternating current reverses its polarity (see Alternating-Current Circuits on alternating-current circuits).

A variable air capacitor (Figure 8.8) has two sets of parallel plates. One set of plates is fixed (indicated as "stator"), and the other set of plates is attached to a shaft that can be rotated (indicated as "rotor"). By turning the shaft, the cross-sectional area in the overlap of the plates can be changed; therefore, the capacitance of this system can be tuned to a desired value. Capacitor tuning has applications in any type of radio transmission and in receiving radio signals from electronic devices. Any time you tune your car radio to your favorite station, think of capacitance.

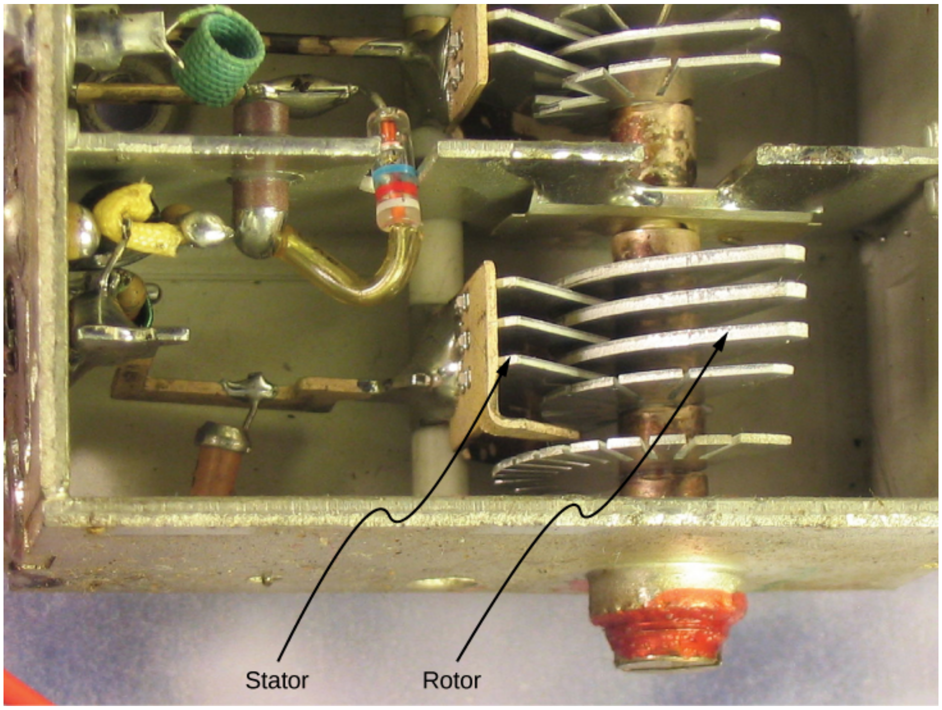


Figure 8.8 In a variable air capacitor, capacitance can be tuned by changing the effective area of the plates. (credit: modification of work by Robbie Sproule)

The symbols shown in Figure 8.9 are circuit representations of various types of capacitors. We generally use the symbol shown in Figure 8.9(a). The symbol in Figure 8.9(c) represents a variable-capacitance capacitor. Notice the similarity of these symbols to the symmetry of a parallel-plate capacitor. An electrolytic capacitor is represented by the symbol in part Figure 8.9(b), where the curved plate indicates the negative terminal.

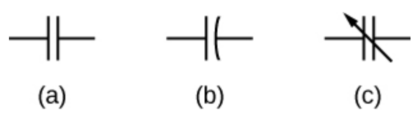


Figure 8.9 This shows three different circuit representations of capacitors. The symbol in (a) is the most commonly used one. The symbol in (b) represents an electrolytic capacitor. The symbol in (c) represents a variable-capacitance capacitor.

An interesting applied example of a capacitor model comes from cell biology and deals with the electrical potential in the plasma membrane of a living cell (Figure 8.10). Cell membranes separate cells from their surroundings but allow some selected ions to pass in or out of the cell. The potential difference across a membrane is about 70 mV. The cell membrane may be 7 to 10 nm thick. Treating the cell membrane as a nano-sized capacitor, the estimate of the smallest electrical field strength across its 'plates' yields the value

$$E = \frac{V}{d} = \frac{70 \times 10^{-3}\,\text{V}}{10 \times 10^{-9}\,\text{m}} = 7 \times 10^{6}\ \text{V/m} > 3\,\text{MV/m}.$$

This magnitude of electrical field is great enough to create an electrical spark in the air.

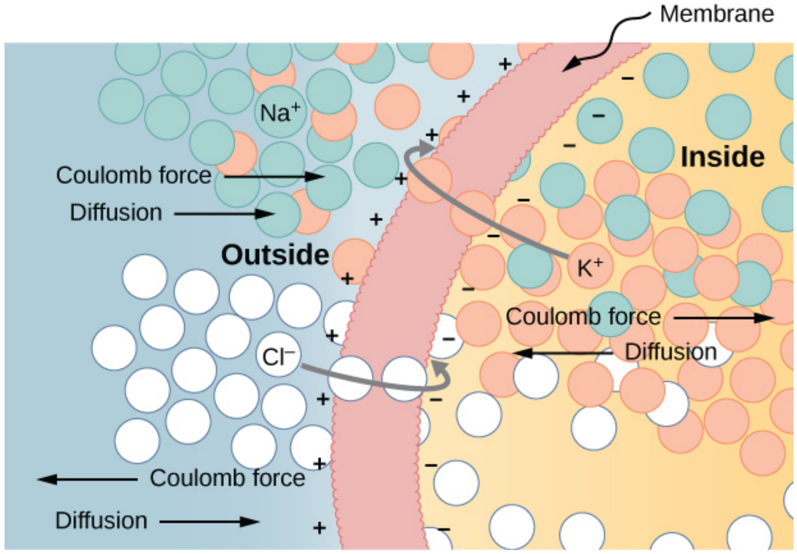


Figure 8.10 The semipermeable membrane of a biological cell has different concentrations of ions on its interior surface than on its exterior. Diffusion moves the K^+ (potassium) and Cl^- (chloride) ions in the directions shown, until the Coulomb force halts further transfer. In this way, the exterior of the membrane acquires a positive charge and its interior surface acquires a negative charge, creating a potential difference across the membrane. The membrane is normally impermeable to Na+ (sodium ions).

INTERACTIVE

Visit the PhET Explorations: Capacitor Lab (https://openstax.org/l/21phetcapacitor) to explore how a capacitor works. Change the size of the plates and add a dielectric to see the effect on capacitance. Change the voltage and see charges built up on the plates. Observe the electrical field in the capacitor. Measure the voltage and the electrical field.

8.2 Capacitors in Series and in Parallel

Learning Objectives

By the end of this section, you will be able to:

- Explain how to determine the equivalent capacitance of capacitors in series and in parallel combinations
- Compute the potential difference across the plates and the charge on the plates for a capacitor in a network and determine the net capacitance of a network of capacitors

Several capacitors can be connected together to be used in a variety of applications. Multiple connections of capacitors behave as a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. Capacitors can be arranged in two simple and common types of connections, known as *series* and *parallel*, for which we can easily calculate the total capacitance. These two basic combinations, series and parallel, can also be used as part of more complex connections.

The Series Combination of Capacitors

Figure 8.11 illustrates a series combination of three capacitors, arranged in a row within the circuit. As for any capacitor, the capacitance of the combination is related to the charge and voltage by using Equation 8.1. When this series combination is connected to a battery with voltage V, each of the capacitors acquires an identical charge Q. To explain, first note that the charge on the plate connected to the positive terminal of the battery is $+Q$ and the charge on the plate connected to the negative terminal is $-Q$. Charges are then induced on the other plates so that the sum of the charges on all plates, and the sum of charges on any pair of capacitor plates, is zero. However, the potential drop $V_1 = Q/C_1$ on one capacitor may be different from the potential drop $V_2 = Q/C_2$ on another capacitor, because, generally, the capacitors may have different capacitances. The series combination of two or three capacitors resembles a single capacitor with a smaller capacitance.

Generally, any number of capacitors connected in series is equivalent to one capacitor whose capacitance (called the *equivalent capacitance*) is smaller than the smallest of the capacitances in the series combination. Charge on this equivalent capacitor is the same as the charge on any capacitor in a series combination: That is, *all capacitors of a series combination have the same charge*. This occurs due to the conservation of charge in the circuit. When a charge Q in a series circuit is removed from a plate of the first capacitor (which we denote as $-Q$), it must be placed on a plate of the second capacitor (which we denote as $+Q$), and so on.

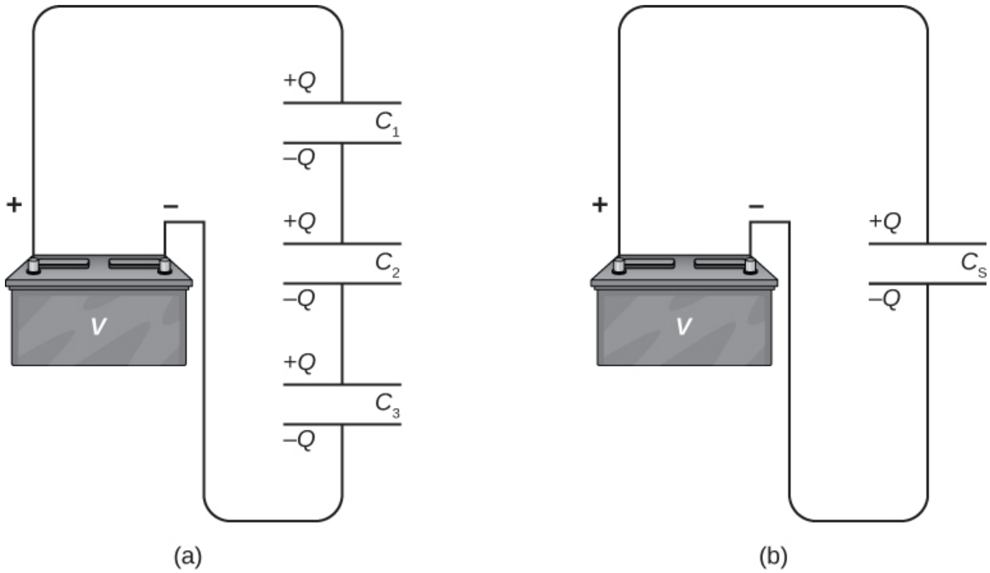


Figure 8.11 (a) Three capacitors are connected in series. The magnitude of the charge on each plate is Q. (b) The network of capacitors in (a) is equivalent to one capacitor that has a smaller capacitance than any of the individual capacitances in (a), and the charge on its plates is Q.

We can find an expression for the total (equivalent) capacitance by considering the voltages across the individual capacitors. The potentials across capacitors 1, 2, and 3 are, respectively, $V_1 = Q/C_1$, $V_2 = Q/C_2$, and $V_3 = Q/C_3$. These potentials must sum up to the voltage of the battery, giving the following potential balance:

$$V = V_1 + V_2 + V_3.$$

Potential V is measured across an equivalent capacitor that holds charge Q and has an equivalent capacitance C_S. Entering the expressions for V_1, V_2, and V_3, we get

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

Canceling the charge Q, we obtain an expression containing the equivalent capacitance, C_S, of three capacitors connected in series:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

This expression can be generalized to any number of capacitors in a series network.

Series Combination

For capacitors connected in a **series combination**, the reciprocal of the equivalent capacitance is the sum of reciprocals of individual capacitances:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots. \tag{8.7}$$

EXAMPLE 8.4

Equivalent Capacitance of a Series Network

Find the total capacitance for three capacitors connected in series, given their individual capacitances are $1.000\ \mu\text{F}$, $5.000\ \mu\text{F}$, and $8.000\ \mu\text{F}$.

Strategy

Because there are only three capacitors in this network, we can find the equivalent capacitance by using Equation 8.7 with three terms.

Solution

We enter the given capacitances into Equation 8.7:

$$\begin{aligned}\frac{1}{C_S} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ &= \frac{1}{1.000\ \mu\text{F}} + \frac{1}{5.000\ \mu\text{F}} + \frac{1}{8.000\ \mu\text{F}} \\ \frac{1}{C_S} &= \frac{1.325}{\mu\text{F}}.\end{aligned}$$

Now we invert this result and obtain $C_S = \frac{\mu\text{F}}{1.325} = 0.755\ \mu\text{F}$.

Significance

Note that in a series network of capacitors, the equivalent capacitance is always less than the smallest individual capacitance in the network.

The Parallel Combination of Capacitors

A parallel combination of three capacitors, with one plate of each capacitor connected to one side of the circuit and the other plate connected to the other side, is illustrated in Figure 8.12(a). Since the capacitors are connected in parallel, *they all have the same voltage V across their plates*. However, each capacitor in the parallel network may store a different charge. To find the equivalent capacitance C_P of the parallel network, we note that the total charge Q stored by the network is the sum of all the individual charges:

$$Q = Q_1 + Q_2 + Q_3.$$

On the left-hand side of this equation, we use the relation $Q = C_P V$, which holds for the entire network. On the right-hand side of the equation, we use the relations $Q_1 = C_1 V$, $Q_2 = C_2 V$, and $Q_3 = C_3 V$ for the three capacitors in the network. In this way we obtain

$$C_P V = C_1 V + C_2 V + C_3 V.$$

This equation, when simplified, is the expression for the equivalent capacitance of the parallel network of three capacitors:

$$C_P = C_1 + C_2 + C_3.$$

This expression is easily generalized to any number of capacitors connected in parallel in the network.

Parallel Combination

For capacitors connected in a **parallel combination**, the equivalent (net) capacitance is the sum of all individual capacitances in the network,

$$C_P = C_1 + C_2 + C_3 + \cdots. \qquad 8.8$$

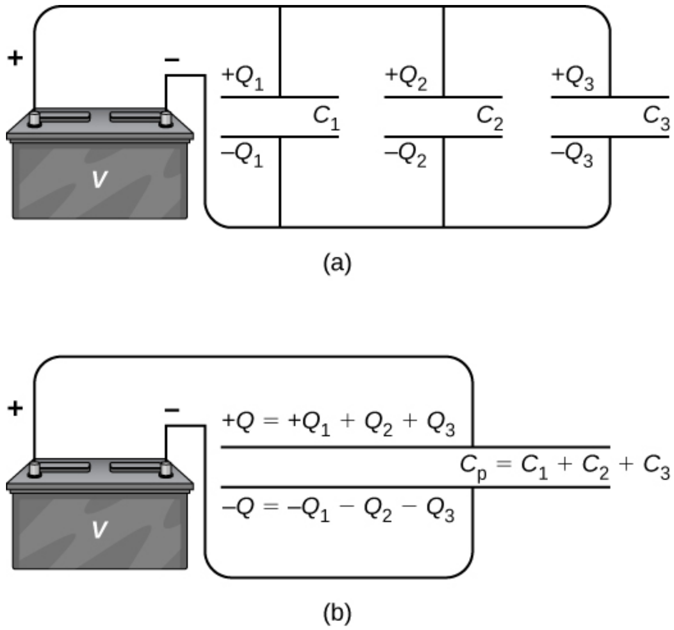


Figure 8.12 (a) Three capacitors are connected in parallel. Each capacitor is connected directly to the battery. (b) The charge on the equivalent capacitor is the sum of the charges on the individual capacitors.

EXAMPLE 8.5

Equivalent Capacitance of a Parallel Network

Find the net capacitance for three capacitors connected in parallel, given their individual capacitances are $1.0\ \mu\text{F}, 5.0\ \mu\text{F},$ and $8.0\ \mu\text{F}.$

Strategy

Because there are only three capacitors in this network, we can find the equivalent capacitance by using Equation 8.8 with three terms.

Solution

Entering the given capacitances into Equation 8.8 yields

$$\begin{aligned} C_\text{P} &= C_1 + C_2 + C_3 = 1.0\ \mu\text{F} + 5.0\ \mu\text{F} + 8.0\ \mu\text{F} \\ C_\text{P} &= 14.0\ \mu\text{F}. \end{aligned}$$

Significance

Note that in a parallel network of capacitors, the equivalent capacitance is always larger than any of the individual capacitances in the network.

Capacitor networks are usually some combination of series and parallel connections, as shown in Figure 8.13. To find the net capacitance of such combinations, we identify parts that contain only series or only parallel connections, and find their equivalent capacitances. We repeat this process until we can determine the equivalent capacitance of the entire network. The following example illustrates this process.

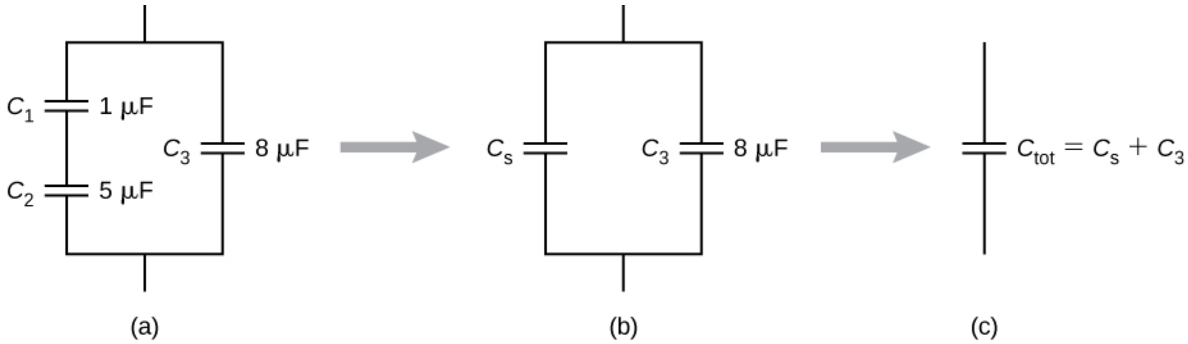


Figure 8.13 (a) This circuit contains both series and parallel connections of capacitors. (b) C_1 and C_2 are in series; their equivalent capacitance is C_S. (c) The equivalent capacitance C_S is connected in parallel with C_3. Thus, the equivalent capacitance of the entire network is the sum of C_S and C_3.

EXAMPLE 8.6

Equivalent Capacitance of a Network

Find the total capacitance of the combination of capacitors shown in Figure 8.13. Assume the capacitances are known to three decimal places ($C_1 = 1.000\ \mu\text{F}, C_2 = 5.000\ \mu\text{F}, C_3 = 8.000\ \mu\text{F}$). Round your answer to three decimal places.

Strategy

We first identify which capacitors are in series and which are in parallel. Capacitors C_1 and C_2 are in series. Their combination, labeled C_S, is in parallel with C_3.

Solution

Since C_1 and C_2 are in series, their equivalent capacitance C_S is obtained with Equation 8.7:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1.000\ \mu\text{F}} + \frac{1}{5.000\ \mu\text{F}} = \frac{1.200}{\mu\text{F}} \Rightarrow C_S = 0.833\ \mu\text{F}.$$

Capacitance C_S is connected in parallel with the third capacitance C_3, so we use Equation 8.8 to find the equivalent capacitance C of the entire network:

$$C = C_S + C_3 = 0.833\ \mu\text{F} + 8.000\ \mu\text{F} = 8.833\ \mu\text{F}.$$

EXAMPLE 8.7

Network of Capacitors

Determine the net capacitance C of the capacitor combination shown in Figure 8.14 when the capacitances are $C_1 = 12.0\ \mu\text{F}, C_2 = 2.0\ \mu\text{F}$, and $C_3 = 4.0\ \mu\text{F}$. When a 12.0-V potential difference is maintained across the combination, find the charge and the voltage across each capacitor.

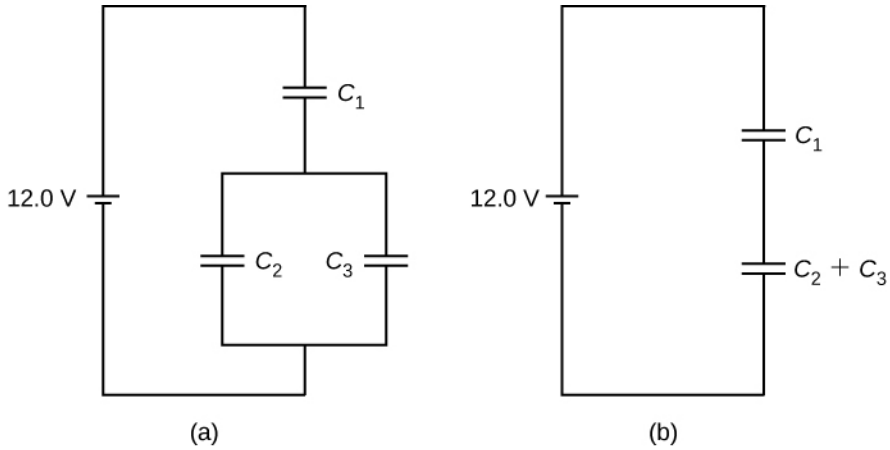


Figure 8.14 (a) A capacitor combination. (b) An equivalent two-capacitor combination.

Strategy

We first compute the net capacitance C_{23} of the parallel connection C_2 and C_3. Then C is the net capacitance of the series connection C_1 and C_{23}. We use the relation $C = Q/V$ to find the charges Q_1, Q_2, and Q_3, and the voltages V_1, V_2, and V_3, across capacitors 1, 2, and 3, respectively.

Solution

The equivalent capacitance for C_2 and C_3 is

$$C_{23} = C_2 + C_3 = 2.0\ \mu\text{F} + 4.0\ \mu\text{F} = 6.0\ \mu\text{F}.$$

The entire three-capacitor combination is equivalent to two capacitors in series,

$$\frac{1}{C} = \frac{1}{12.0\ \mu\text{F}} + \frac{1}{6.0\ \mu\text{F}} = \frac{1}{4.0\ \mu\text{F}} \Rightarrow C = 4.0\ \mu\text{F}.$$

Consider the equivalent two-capacitor combination in Figure 8.14(b). Since the capacitors are in series, they have the same charge, $Q_1 = Q_{23}$. Also, the capacitors share the 12.0-V potential difference, so

$$12.0\ \text{V} = V_1 + V_{23} = \frac{Q_1}{C_1} + \frac{Q_{23}}{C_{23}} = \frac{Q_1}{12.0\ \mu\text{F}} + \frac{Q_1}{6.0\ \mu\text{F}} \Rightarrow Q_1 = 48.0\ \mu\text{C}.$$

Now the potential difference across capacitor 1 is

$$V_1 = \frac{Q_1}{C_1} = \frac{48.0\ \mu\text{C}}{12.0\ \mu\text{F}} = 4.0\ \text{V}.$$

Because capacitors 2 and 3 are connected in parallel, they are at the same potential difference:

$$V_2 = V_3 = 12.0\ \text{V} - 4.0\ \text{V} = 8.0\ \text{V}.$$

Hence, the charges on these two capacitors are, respectively,

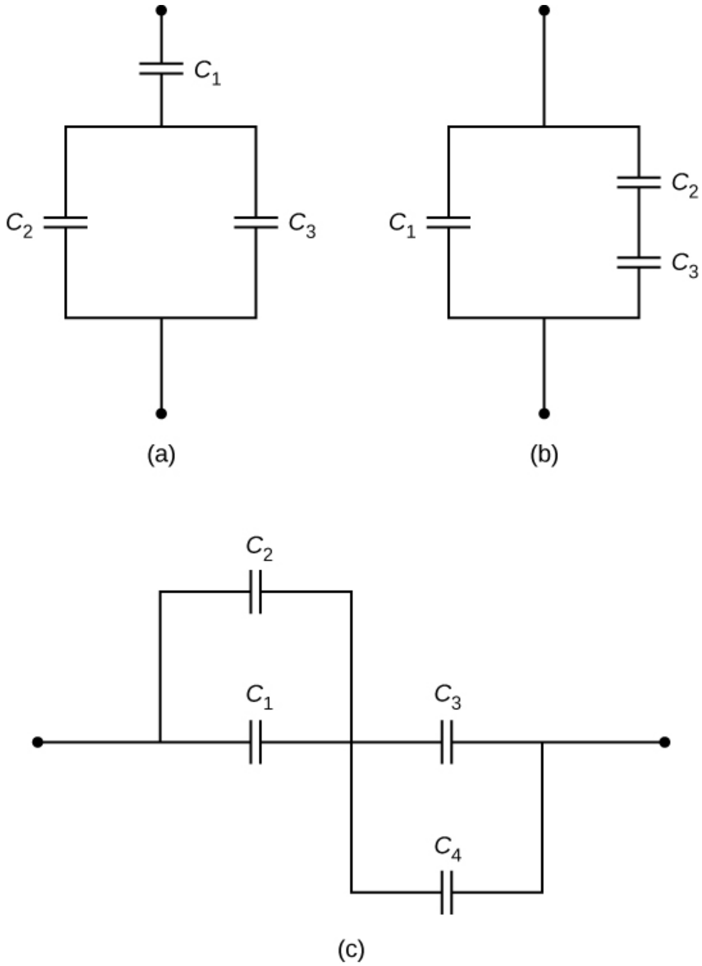
$$Q_2 = C_2 V_2 = (2.0\ \mu\text{F})(8.0\ \text{V}) = 16.0\ \mu\text{C},$$
$$Q_3 = C_3 V_3 = (4.0\ \mu\text{F})(8.0\ \text{V}) = 32.0\ \mu\text{C}.$$

Significance

As expected, the net charge on the parallel combination of C_2 and C_3 is $Q_{23} = Q_2 + Q_3 = 48.0\ \mu\text{C}$.

CHECK YOUR UNDERSTANDING 8.5

Determine the net capacitance C of each network of capacitors shown below. Assume that $C_1 = 1.0$ pF, $C_2 = 2.0$ pF, $C_3 = 4.0$ pF, and $C_4 = 5.0$ pF. Find the charge on each capacitor, assuming there is a potential difference of 12.0 V across each network.



8.3 Energy Stored in a Capacitor

Learning Objectives

By the end of this section, you will be able to:

- Explain how energy is stored in a capacitor
- Use energy relations to determine the energy stored in a capacitor network

Most of us have seen dramatizations of medical personnel using a defibrillator to pass an electrical current through a patient's heart to get it to beat normally. Often realistic in detail, the person applying the shock directs another person to "make it 400 joules this time." The energy delivered by the defibrillator is stored in a capacitor and can be adjusted to fit the situation. SI units of joules are often employed. Less dramatic is the use of capacitors in microelectronics to supply energy when batteries are charged (Figure 8.15). Capacitors are also used to supply energy for flash lamps on cameras.

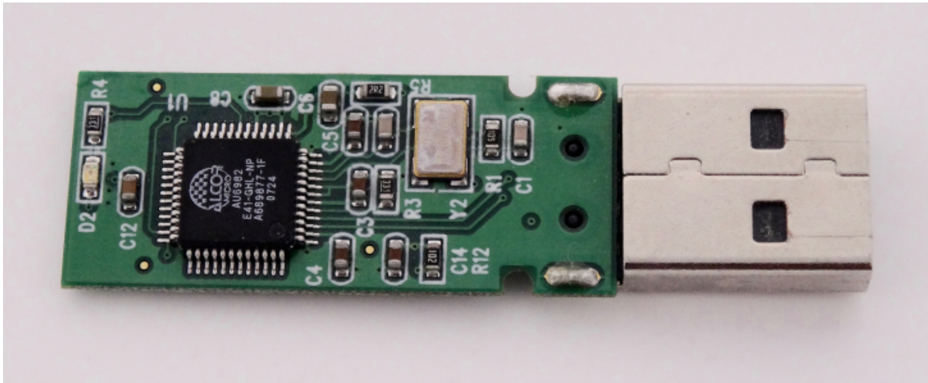


Figure 8.15 The capacitors on the circuit board for an electronic device follow a labeling convention that identifies each one with a code that begins with the letter "C." (credit: Windell Oskay)

The energy U_C stored in a capacitor is electrostatic potential energy and is thus related to the charge Q and voltage V between the capacitor plates. A charged capacitor stores energy in the electrical field between its plates. As the capacitor is being charged, the electrical field builds up. When a charged capacitor is disconnected from a battery, its energy remains in the field in the space between its plates.

To gain insight into how this energy may be expressed (in terms of Q and V), consider a charged, empty, parallel-plate capacitor; that is, a capacitor without a dielectric but with a vacuum between its plates. The space between its plates has a volume Ad, and it is filled with a uniform electrostatic field E. The total energy U_C of the capacitor is contained within this space. The **energy density** u_E in this space is simply U_C divided by the volume Ad. If we know the energy density, the energy can be found as $U_C = u_E(Ad)$. We will learn in Electromagnetic Waves (after completing the study of Maxwell's equations) that the energy density u_E in a region of free space occupied by an electrical field E depends only on the magnitude of the field and is

$$u_E = \frac{1}{2}\varepsilon_0 E^2. \tag{8.9}$$

If we multiply the energy density by the volume between the plates, we obtain the amount of energy stored between the plates of a parallel-plate capacitor: $U_C = u_E(Ad) = \frac{1}{2}\varepsilon_0 E^2 Ad = \frac{1}{2}\varepsilon_0 \frac{V^2}{d^2} Ad = \frac{1}{2}V^2 \varepsilon_0 \frac{A}{d} = \frac{1}{2}V^2 C.$

In this derivation, we used the fact that the electrical field between the plates is uniform so that $E = V/d$ and $C = \varepsilon_0 A/d$. Because $C = Q/V$, we can express this result in other equivalent forms:

$$U_C = \frac{1}{2}V^2 C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV. \tag{8.10}$$

The expression in Equation 8.10 for the energy stored in a parallel-plate capacitor is generally valid for all types of capacitors. To see this, consider any uncharged capacitor (not necessarily a parallel-plate type). At some instant, we connect it across a battery, giving it a potential difference $V = q/C$ between its plates. Initially, the charge on the plates is $Q = 0$. As the capacitor is being charged, the charge gradually builds up on its plates, and after some time, it reaches the value Q. To move an infinitesimal charge dq from the negative plate to the positive plate (from a lower to a higher potential), the amount of work dW that must be done on dq is $dW = V dq = \frac{q}{C} dq$.

This work becomes the energy stored in the electrical field of the capacitor. In order to charge the capacitor to a charge Q, the total work required is

$$W = \int_0^{W(Q)} dW = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C}.$$

Since the geometry of the capacitor has not been specified, this equation holds for any type of capacitor. The total work W needed to charge a capacitor is the electrical potential energy U_C stored in it, or $U_C = W$. When the charge is expressed in coulombs, potential is expressed in volts, and the capacitance is expressed in farads, this relation gives the energy in joules.

Knowing that the energy stored in a capacitor is $U_C = Q^2/(2C)$, we can now find the energy density u_E stored in a vacuum between the plates of a charged parallel-plate capacitor. We just have to divide U_C by the volume Ad of space between its plates and take into account that for a parallel-plate capacitor, we have $E = \sigma/\varepsilon_0$ and $C = \varepsilon_0 A/d$. Therefore, we obtain

$$u_E = \frac{U_C}{Ad} = \frac{1}{2}\frac{Q^2}{C}\frac{1}{Ad} = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A/d}\frac{1}{Ad} = \frac{1}{2}\frac{1}{\varepsilon_0}\left(\frac{Q}{A}\right)^2 = \frac{\sigma^2}{2\varepsilon_0} = \frac{(E\varepsilon_0)^2}{2\varepsilon_0} = \frac{\varepsilon_0}{2}E^2.$$

We see that this expression for the density of energy stored in a parallel-plate capacitor is in accordance with the general relation expressed in Equation 8.9. We could repeat this calculation for either a spherical capacitor or a cylindrical capacitor—or other capacitors—and in all cases, we would end up with the general relation given by Equation 8.9.

EXAMPLE 8.8

Energy Stored in a Capacitor

Calculate the energy stored in the capacitor network in Figure 8.14(a) when the capacitors are fully charged and when the capacitances are $C_1 = 12.0\ \mu\text{F}$, $C_2 = 2.0\ \mu\text{F}$, and $C_3 = 4.0\ \mu\text{F}$, respectively.

Strategy

We use Equation 8.10 to find the energy U_1, U_2, and U_3 stored in capacitors 1, 2, and 3, respectively. The total energy is the sum of all these energies.

Solution

We identify $C_1 = 12.0\ \mu\text{F}$ and $V_1 = 4.0\ \text{V}$, $C_2 = 2.0\ \mu\text{F}$ and $V_2 = 8.0\ \text{V}$, $C_3 = 4.0\ \mu\text{F}$ and $V_3 = 8.0\ \text{V}$. The energies stored in these capacitors are

$$\begin{aligned} U_1 &= \tfrac{1}{2}C_1V_1^2 = \tfrac{1}{2}(12.0\ \mu\text{F})(4.0\ \text{V})^2 = 96\ \mu\text{J}, \\ U_2 &= \tfrac{1}{2}C_2V_2^2 = \tfrac{1}{2}(2.0\ \mu\text{F})(8.0\ \text{V})^2 = 64\ \mu\text{J}, \\ U_3 &= \tfrac{1}{2}C_3V_3^2 = \tfrac{1}{2}(4.0\ \mu\text{F})(8.0\ \text{V})^2 = 130\ \mu\text{J}. \end{aligned}$$

The total energy stored in this network is

$$U_C = U_1 + U_2 + U_3 = 96\ \mu\text{J} + 64\ \mu\text{J} + 130\ \mu\text{J} = 0.29\ \text{mJ}.$$

Significance

We can verify this result by calculating the energy stored in the single 4.0-μF capacitor, which is found to be equivalent to the entire network. The voltage across the network is 12.0 V. The total energy obtained in this way agrees with our previously obtained result, $U_C = \frac{1}{2}CV^2 = \frac{1}{2}(4.0\ \mu\text{F})(12.0\ \text{V})^2 = 0.29\ \text{mJ}$.

CHECK YOUR UNDERSTANDING 8.6

The potential difference across a 5.0-pF capacitor is 0.40 V. (a) What is the energy stored in this capacitor? (b) The potential difference is now increased to 1.20 V. By what factor is the stored energy increased?

In a cardiac emergency, a portable electronic device known as an automated external defibrillator (AED) can be a lifesaver. A **defibrillator** (Figure 8.16) delivers a large charge in a short burst, or a shock, to a person's heart to correct abnormal heart rhythm (an arrhythmia). A heart attack can arise from the onset of fast, irregular beating of the heart—called cardiac or ventricular fibrillation. Applying a large shock of electrical energy can terminate the arrhythmia and allow the body's natural pacemaker to resume its normal rhythm. Today, it is common for ambulances to carry AEDs. AEDs are also found in many public places. These are designed to be used by lay persons. The device automatically diagnoses the patient's heart rhythm and then applies the shock with appropriate energy and waveform. CPR (cardiopulmonary resuscitation) is recommended in many cases before using a defibrillator.

Figure 8.16 Automated external defibrillators are found in many public places. These portable units provide verbal instructions for use in the important first few minutes for a person suffering a cardiac attack. (credit: Owain Davies)

EXAMPLE 8.9

Capacitance of a Heart Defibrillator

A heart defibrillator delivers $4.00 \times 10^2 \text{ J}$ of energy by discharging a capacitor initially at $1.00 \times 10^4 \text{ V}$. What is its capacitance?

Strategy

We are given U_C and V, and we are asked to find the capacitance C. We solve Equation 8.10 for C and substitute.

Solution

Solving this expression for C and entering the given values yields $C = 2\frac{U_C}{V^2} = 2\frac{4.00 \times 10^2 \text{ J}}{(1.00 \times 10^4 \text{ V})^2} = 8.00 \, \mu\text{F}$.

8.4 Capacitor with a Dielectric

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects a dielectric in a capacitor has on capacitance and other properties
- Calculate the capacitance of a capacitor containing a dielectric

As we discussed earlier, an insulating material placed between the plates of a capacitor is called a dielectric. Inserting a dielectric between the plates of a capacitor affects its capacitance. To see why, let's consider an experiment described in Figure 8.17. Initially, a capacitor with capacitance C_0 when there is air between its plates is charged by a battery to voltage V_0. When the capacitor is fully charged, the battery is disconnected. A charge Q_0 then resides on the plates, and the potential difference between the plates is measured to be V_0. Now, suppose we insert a dielectric that *totally* fills the gap between the plates. If we monitor the voltage, we find that the voltmeter reading has dropped to a *smaller* value V. We write this new voltage value as a fraction of the original voltage V_0, with a positive number κ, $\kappa > 1$:

$$V = \frac{1}{\kappa} V_0.$$

The constant κ in this equation is called the **dielectric constant** of the material between the plates, and its value is characteristic for the material. A detailed explanation for why the dielectric reduces the voltage is given in the next section. Different materials have different dielectric constants (a table of values for typical materials is provided in the next section). Once the battery becomes disconnected, there is no path for a charge to flow to the battery from the capacitor plates. Hence, the insertion of the dielectric has no effect on the charge on the plate, which remains at a value of Q_0. Therefore, we find that the capacitance of the capacitor with a dielectric is

$$C = \frac{Q_0}{V} = \frac{Q_0}{V_0/\kappa} = \kappa \frac{Q_0}{V_0} = \kappa C_0. \quad \textbf{8.11}$$

This equation tells us that the *capacitance C_0 of an empty (vacuum) capacitor can be increased by a factor of κ when we insert a dielectric material to completely fill the space between its plates*. Note that Equation 8.11 can also be used for an empty capacitor by setting $\kappa = 1$. In other words, we can say that the dielectric constant of the vacuum is 1, which is a reference value.

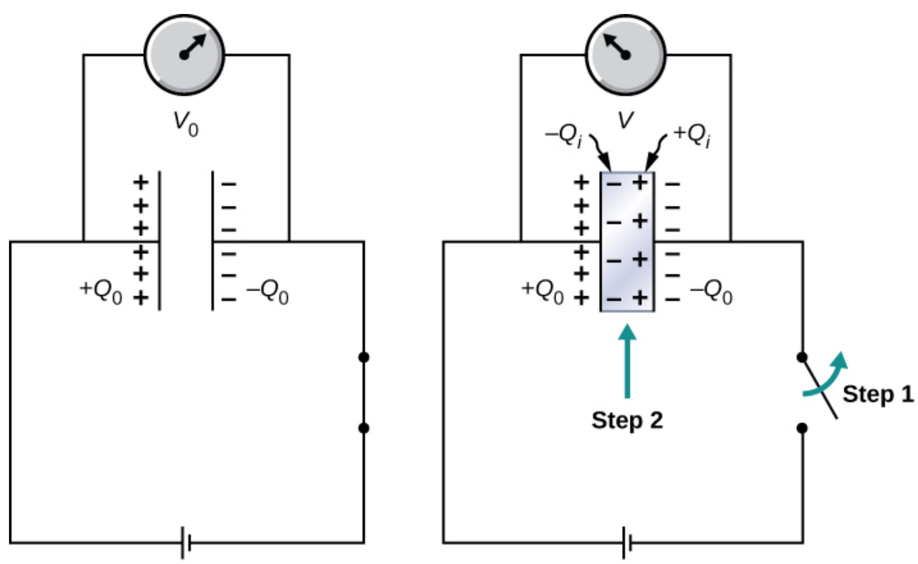


Figure 8.17 (a) When fully charged, a vacuum capacitor has a voltage V_0 and charge Q_0 (the charges remain on plate's inner surfaces; the schematic indicates the sign of charge on each plate). (b) In step 1, the battery is disconnected. Then, in step 2, a dielectric (that is electrically neutral) is inserted into the charged capacitor. When the voltage across the capacitor is now measured, it is found that the voltage value has decreased to $V = V_0/\kappa$. The schematic indicates the sign of the induced charge that is now present on the surfaces of the dielectric material between the plates.

The principle expressed by Equation 8.11 is widely used in the construction industry (Figure 8.18). Metal plates in an electronic stud finder act effectively as a capacitor. You place a stud finder with its flat side on the wall and move it continually in the horizontal direction. When the finder moves over a wooden stud, the capacitance of its plates changes, because wood has a different dielectric constant than a gypsum wall. This change triggers a signal in a circuit, and thus the stud is detected.

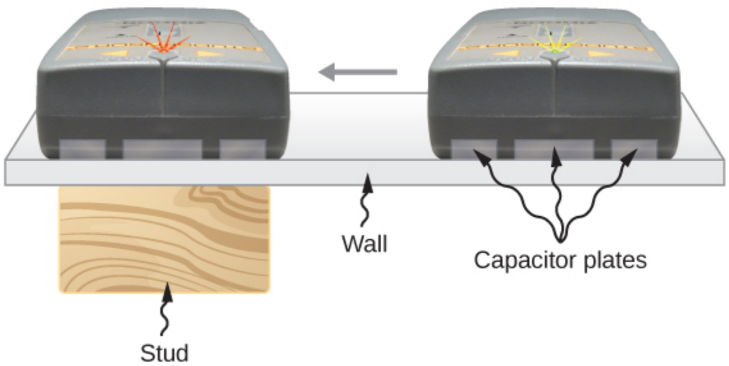


Figure 8.18 An electronic stud finder is used to detect wooden studs behind drywall. (credit top: modification of work by Jane Whitney)

The electrical energy stored by a capacitor is also affected by the presence of a dielectric. When the energy stored in an empty capacitor is U_0, the energy U stored in a capacitor with a dielectric is smaller by a factor of κ,

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{\kappa C_0} = \frac{1}{\kappa}U_0. \tag{8.12}$$

As a dielectric material sample is brought near an empty charged capacitor, the sample reacts to the electrical field of the charges on the capacitor plates. Just as we learned in Electric Charges and Fields on electrostatics, there will be the induced charges on the surface of the sample; however, they are not free charges like in a conductor, because a perfect insulator does not have freely moving charges. These induced charges on the dielectric surface are of an opposite sign to the free charges on the plates of the capacitor, and so they are attracted by the free charges on the plates. Consequently, the dielectric is “pulled” into the gap, and the work to polarize the dielectric material between the plates is done at the expense of the stored electrical energy, which is reduced, in accordance with Equation 8.12.

EXAMPLE 8.10

Inserting a Dielectric into an Isolated Capacitor

An empty 20.0-pF capacitor is charged to a potential difference of 40.0 V. The charging battery is then disconnected, and a piece of Teflon™ with a dielectric constant of 2.1 is inserted to completely fill the space between the capacitor plates (see Figure 8.17). What are the values of (a) the capacitance, (b) the charge of the plate, (c) the potential difference between the plates, and (d) the energy stored in the capacitor with and without dielectric?

Strategy

We identify the original capacitance $C_0 = 20.0\text{ pF}$ and the original potential difference $V_0 = 40.0\text{ V}$ between the plates. We combine Equation 8.11 with other relations involving capacitance and substitute.

Solution

a. The capacitance increases to
$$C = \kappa C_0 = 2.1(20.0\text{ pF}) = 42.0\text{ pF}.$$

b. Without dielectric, the charge on the plates is
$$Q_0 = C_0 V_0 = (20.0\text{ pF})(40.0\text{ V}) = 0.8\text{ nC}.$$

Since the battery is disconnected before the dielectric is inserted, the plate charge is unaffected by the dielectric and remains at 0.8 nC.

c. With the dielectric, the potential difference becomes
$$V = \frac{1}{\kappa}V_0 = \frac{1}{2.1}40.0\text{ V} = 19.0\text{ V}.$$

d. The stored energy without the dielectric is
$$U_0 = \frac{1}{2}C_0 V_0^2 = \frac{1}{2}(20.0\text{ pF})(40.0\text{ V})^2 = 16.0\text{ nJ}.$$

With the dielectric inserted, we use Equation 8.12 to find that the stored energy decreases to
$$U = \frac{1}{\kappa}U_0 = \frac{1}{2.1}16.0\text{ nJ} = 7.6\text{ nJ}.$$

Significance

Notice that the effect of a dielectric on the capacitance of a capacitor is a drastic increase of its capacitance. This effect is far more profound than a mere change in the geometry of a capacitor.

CHECK YOUR UNDERSTANDING 8.7

When a dielectric is inserted into an isolated and charged capacitor, the stored energy decreases to 33% of its original value. (a) What is the dielectric constant? (b) How does the capacitance change?

8.5 Molecular Model of a Dielectric

Learning Objectives

By the end of this section, you will be able to:

- Explain the polarization of a dielectric in a uniform electrical field
- Describe the effect of a polarized dielectric on the electrical field between capacitor plates
- Explain dielectric breakdown

We can understand the effect of a dielectric on capacitance by looking at its behavior at the molecular level. As we have seen in earlier chapters, in general, all molecules can be classified as either *polar* or *nonpolar*. There is a net separation of positive and negative charges in an isolated polar molecule, whereas there is no charge separation in an isolated nonpolar molecule (Figure 8.19). In other words, polar molecules have permanent *electric-dipole moments* and nonpolar molecules do not. For example, a molecule of water is polar, and a molecule of oxygen is nonpolar. Nonpolar molecules can become polar in the presence of an external electrical field, which is called *induced polarization*.

CHAPTER REVIEW

Key Terms

capacitance amount of charge stored per unit volt

capacitor device that stores electrical charge and electrical energy

dielectric insulating material used to fill the space between two plates

dielectric breakdown phenomenon that occurs when an insulator becomes a conductor in a strong electrical field

dielectric constant factor by which capacitance increases when a dielectric is inserted between the plates of a capacitor

dielectric strength critical electrical field strength above which molecules in insulator begin to break down and the insulator starts to conduct

energy density energy stored in a capacitor divided by the volume between the plates

induced electric-dipole moment dipole moment that a nonpolar molecule may acquire when it is placed in an electrical field

induced electrical field electrical field in the dielectric due to the presence of induced charges

induced surface charges charges that occur on a dielectric surface due to its polarization

parallel combination components in a circuit arranged with one side of each component connected to one side of the circuit and the other sides of the components connected to the other side of the circuit

parallel-plate capacitor system of two identical parallel conducting plates separated by a distance

series combination components in a circuit arranged in a row one after the other in a circuit

Key Equations

Capacitance	$C = \frac{Q}{V}$
Capacitance of a parallel-plate capacitor	$C = \varepsilon_0 \frac{A}{d}$
Capacitance of a vacuum spherical capacitor	$C = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$
Capacitance of a vacuum cylindrical capacitor	$C = \frac{2\pi\varepsilon_0 l}{\ln(R_2/R_1)}$
Capacitance of a series combination	$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$
Capacitance of a parallel combination	$C_P = C_1 + C_2 + C_3 + \cdots$
Energy density	$u_E = \frac{1}{2}\varepsilon_0 E^2$
Energy stored in a capacitor	$U_C = \frac{1}{2}V^2 C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$
Capacitance of a capacitor with dielectric	$C = \kappa C_0$
Energy stored in an isolated capacitor with dielectric	$U = \frac{1}{\kappa}U_0$
Dielectric constant	$\kappa = \frac{E_0}{E}$
Induced electrical field in a dielectric	$\vec{\mathbf{E}}_i = \left(\frac{1}{\kappa} - 1\right)\vec{\mathbf{E}}_0$

Summary

8.1 Capacitors and Capacitance

- A capacitor is a device that stores an electrical charge and electrical energy. The amount of charge a vacuum capacitor can store depends on two major factors: the voltage applied and the capacitor's physical characteristics, such as its size and geometry.
- The capacitance of a capacitor is a parameter that tells us how much charge can be stored in the capacitor per unit potential difference between its plates. Capacitance of a system of conductors depends only on the geometry of their arrangement and physical properties of the insulating material that fills the space between the conductors. The unit of capacitance is the farad, where $1 \text{ F} = 1 \text{ C}/1 \text{ V}$.

8.2 Capacitors in Series and in Parallel

- When several capacitors are connected in a series combination, the reciprocal of the equivalent capacitance is the sum of the reciprocals of the individual capacitances.
- When several capacitors are connected in a parallel combination, the equivalent capacitance is the sum of the individual capacitances.
- When a network of capacitors contains a combination of series and parallel connections, we identify the series and parallel networks, and compute their equivalent capacitances step by step until the entire network becomes reduced to one equivalent capacitance.

8.3 Energy Stored in a Capacitor

- Capacitors are used to supply energy to a variety of devices, including defibrillators, microelectronics such as calculators, and flash lamps.
- The energy stored in a capacitor is the work required to charge the capacitor, beginning with no charge on its plates. The energy is stored in the electrical field in the space between the capacitor plates. It depends on the amount of electrical charge on the plates and on the potential difference between the plates.
- The energy stored in a capacitor network is the sum of the energies stored on individual capacitors in the network. It can be computed as the energy stored in the equivalent capacitor of the network.

8.4 Capacitor with a Dielectric

- The capacitance of an empty capacitor is increased by a factor of κ when the space between its plates is completely filled by a dielectric with dielectric constant κ.
- Each dielectric material has its specific dielectric constant.
- The energy stored in an empty isolated capacitor is decreased by a factor of κ when the space between its plates is completely filled with a dielectric with dielectric constant κ while disconnecting the battery and keeping the charge on the capacitor constant.

8.5 Molecular Model of a Dielectric

- When a dielectric is inserted between the plates of a capacitor, equal and opposite surface charge is induced on the two faces of the dielectric. The induced surface charge produces an induced electrical field that opposes the field of the free charge on the capacitor plates.
- The dielectric constant of a material is the ratio of the electrical field in vacuum to the net electrical field in the material. A capacitor filled with dielectric has a larger capacitance than an empty capacitor.
- The dielectric strength of an insulator represents a critical value of electrical field at which the molecules in an insulating material start to become ionized. When this happens, the material can conduct and dielectric breakdown is observed.

Conceptual Questions

8.1 Capacitors and Capacitance

1. Does the capacitance of a device depend on the applied voltage? Does the capacitance of a device depend on the charge residing on it?

2. Would you place the plates of a parallel-plate capacitor closer together or farther apart to increase their capacitance?

3. The value of the capacitance is zero if the plates are not charged. True or false?

4. If the plates of a capacitor have different areas, will they acquire the same charge when the capacitor is connected across a battery?