CHAPTER 7
Electric Potential

Figure 7.1 The energy released in a lightning strike is an excellent illustration of the vast quantities of energy that may be stored and released by an electric potential difference. In this chapter, we calculate just how much energy can be released in a lightning strike and how this varies with the height of the clouds from the ground. (credit: modification of work by Anthony Quintano)

Chapter Outline

INTRODUCTION In Electric Charges and Fields, we just scratched the surface (or at least rubbed it) of electrical phenomena. Two terms commonly used to describe electricity are its energy and *voltage*, which we show in this chapter is directly related to the potential energy in a system.

We know, for example, that great amounts of electrical energy can be stored in batteries, are transmitted cross-country via currents through power lines, and may jump from clouds to explode the sap of trees. In a similar manner, at the molecular level, ions cross cell membranes and transfer information.

We also know about voltages associated with electricity. Batteries are typically a few volts, the outlets in your home frequently produce 120 volts, and power lines can be as high as hundreds of thousands of volts. But energy and voltage are not the same thing. A motorcycle battery, for example, is small and would not be very successful in replacing a much larger car battery, yet each has the same voltage. In this chapter, we examine the relationship between voltage and electrical energy, and begin to explore some of the many applications of electricity.

7.1 Electric Potential Energy

Learning Objectives

By the end of this section, you will be able to:

- Define the work done by an electric force
- Define electric potential energy
- Apply work and potential energy in systems with electric charges

When a free positive charge q is accelerated by an electric field, it is given kinetic energy (Figure 7.2). The process is analogous to an object being accelerated by a gravitational field, as if the charge were going down an electrical hill where its electric potential energy is converted into kinetic energy, although of course the sources of the forces are very different. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.

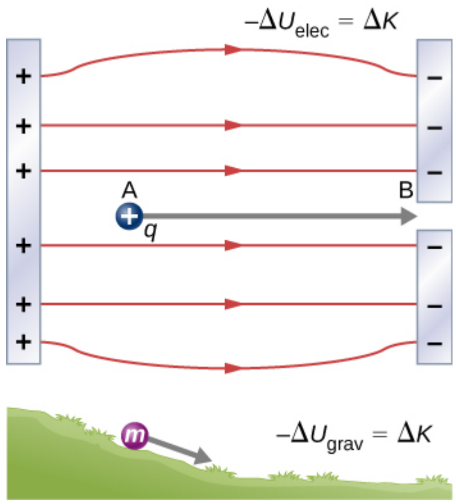


Figure 7.2 A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases, potential energy decreases as kinetic energy increases, $-\Delta U = \Delta K$. Work is done by a force, but since this force is conservative, we can write $W = -\Delta U$.

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken, as we will demonstrate later. This is exactly analogous to the gravitational force. When a force is conservative, it is possible to define a potential energy associated with the force. It is usually easier to work with the potential energy (because it depends only on position) than to calculate the work directly.

To show this explicitly, consider an electric charge $+q$ fixed at the origin and move another charge $+Q$ toward q in such a manner that, at each instant, the applied force $\vec{\mathbf{F}}$ exactly balances the electric force $\vec{\mathbf{F}}_e$ on Q (Figure 7.3). The work done by the applied force $\vec{\mathbf{F}}$ on the charge Q changes the potential energy of Q. We call this potential energy the **electrical potential energy** of Q.

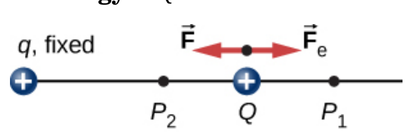


Figure 7.3 Displacement of "test" charge Q in the presence of fixed "source" charge q.

The work W_{12} done by the applied force $\vec{\mathbf{F}}$ when the particle moves from P_1 to P_2 may be calculated by

$$W_{12} = \int_{P_1}^{P_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}.$$

Since the applied force $\vec{\mathbf{F}}$ balances the electric force $\vec{\mathbf{F}}_e$ on Q, the two forces have equal magnitude and opposite directions. Therefore, the applied force is

$$\vec{\mathbf{F}} = -\vec{\mathbf{F}}_e = -\frac{kqQ}{r^2}\hat{\mathbf{r}},$$

where we have defined positive to be pointing away from the origin and r is the distance from the origin. The directions of both the displacement and the applied force in the system in Figure 7.3 are parallel, and thus the work done on the system is positive.

We use the letter U to denote electric potential energy, which has units of joules (J). When a conservative force does negative work, the system gains potential energy. When a conservative force does positive work, the system loses potential energy, $\Delta U = -W$. In the system in Figure 7.3, the Coulomb force acts in the opposite direction to the displacement; therefore, the work is negative. However, we have increased the potential energy in the two-charge system.

EXAMPLE 7.1

Kinetic Energy of a Charged Particle

A +3.0-nC charge Q is initially at rest a distance of 10 cm (r_1) from a +5.0-nC charge q fixed at the origin (Figure 7.4). Naturally, the Coulomb force accelerates Q away from q, eventually reaching 15 cm (r_2).

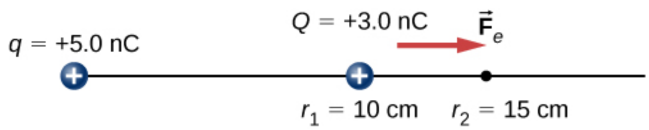


Figure 7.4 The charge Q is repelled by q, thus having work done on it and gaining kinetic energy.

a. What is the work done by the electric field between r_1 and r_2?
b. How much kinetic energy does Q have at r_2?

Strategy

Calculate the work with the usual definition. Since Q started from rest, this is the same as the kinetic energy.

Solution

Integrating force over distance, we obtain

$$\begin{aligned} W_{12} &= \int_{r_1}^{r_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{r_1}^{r_2} \frac{kqQ}{r^2}\, dr = \left[-\frac{kqQ}{r}\right]_{r_1}^{r_2} = kqQ\left[\frac{-1}{r_2} + \frac{1}{r_1}\right] \\ &= \left(8.99 \times 10^9\ \mathrm{Nm^2/C^2}\right)\left(5.0 \times 10^{-9}\ \mathrm{C}\right)\left(3.0 \times 10^{-9}\ \mathrm{C}\right)\left[\frac{-1}{0.15\ \mathrm{m}} + \frac{1}{0.10\ \mathrm{m}}\right] \\ &= 4.5 \times 10^{-7}\ \mathrm{J}. \end{aligned}$$

This is also the value of the kinetic energy at r_2.

Significance

Charge Q was initially at rest; the electric field of q did work on Q, so now Q has kinetic energy equal to the work done by the electric field.

CHECK YOUR UNDERSTANDING 7.1

If Q has a mass of 4.00 μg, what is the speed of Q at r_2?

In this example, the work W done to accelerate a positive charge from rest is positive and results from a loss in U, or a negative ΔU. A value for U can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Electric Potential Energy

Work W done to accelerate a positive charge from rest is positive and results from a loss in U, or a negative ΔU. Mathematically,

$$W = -\Delta U. \tag{7.1}$$

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of electric potential energy than to deal with the Coulomb force directly in real-world applications.

In polar coordinates with q at the origin and Q located at r, the displacement element vector is $d\vec{\mathbf{l}} = \hat{\mathbf{r}}\,dr$ and thus the work becomes

$$W_{12} = kqQ\int_{r_1}^{r_2}\frac{1}{r^2}\hat{\mathrm{r}}\cdot\hat{\mathrm{r}}\,dr = kqQ\frac{1}{r_2} - kqQ\frac{1}{r_1}.$$

Notice that this result only depends on the endpoints and is otherwise independent of the path taken. To explore this further, compare path P_1 to P_2 with path $P_1 P_3 P_4 P_2$ in Figure 7.5.

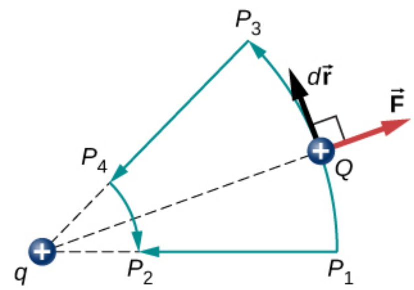


Figure 7.5 Two paths for displacement P_1 to P_2. The work on segments $P_1 P_3$ and $P_4 P_2$ are zero due to the electrical force being perpendicular to the displacement along these paths. Therefore, work on paths $P_1 P_2$ and $P_1 P_3 P_4 P_2$ are equal.

The segments $P_1 P_3$ and $P_4 P_2$ are arcs of circles centered at q. Since the force on Q points either toward or away from q, no work is done by a force balancing the electric force, because it is perpendicular to the displacement along these arcs. Therefore, the only work done is along segment $P_3 P_4$, which is identical to $P_1 P_2$.

One implication of this work calculation is that if we were to go around the path $P_1 P_3 P_4 P_2 P_1$, the net work would be zero (Figure 7.6). Recall that this is how we determine whether a force is conservative or not. Hence, because the electric force is related to the electric field by $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$, the electric field is itself conservative. That is,

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = 0.$$

Note that Q is a constant.

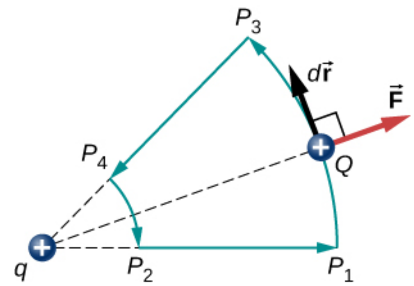


Figure 7.6 A closed path in an electric field. The net work around this path is zero.

Another implication is that we may define an electric potential energy. Recall that the work done by a conservative force is also expressed as the difference in the potential energy corresponding to that force. Therefore, the work W_{ref} to bring a charge from a reference point to a point of interest may be written as

$$W_{\text{ref}} = \int_{r_{\text{ref}}}^{r} \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}$$

and, by Equation 7.1, the difference in potential energy $(U_2 - U_1)$ of the test charge Q between the two points is

$$\Delta U = -\int_{r_{\text{ref}}}^{r} \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}.$$

Therefore, we can write a general expression for the potential energy of two point charges (in spherical coordinates):

$$\Delta U = -\int_{r_{\text{ref}}}^{r} \frac{kqQ}{r^2} dr = -\left[-\frac{kqQ}{r}\right]_{r_{\text{ref}}}^{r} = kqQ\left[\frac{1}{r} - \frac{1}{r_{\text{ref}}}\right].$$

We may take the second term to be an arbitrary constant reference level, which serves as the zero reference:

$$U(r) = k\frac{qQ}{r} - U_{\text{ref}}.$$

A convenient choice of reference that relies on our common sense is that when the two charges are infinitely far apart, there is no interaction between them. (Recall the discussion of reference potential energy in Potential Energy and Conservation of Energy.) Taking the potential energy of this state to be zero removes the term U_{ref} from the equation (just like when we say the ground is zero potential energy in a gravitational potential energy problem), and the potential energy of Q when it is separated from q by a distance r assumes the form

$$U(r) = k\frac{qQ}{r} \left(\text{zero reference at } r = \infty\right). \quad (7.2)$$

This formula is symmetrical with respect to q and Q, so it is best described as the potential energy of the two-charge system.

EXAMPLE 7.2

Potential Energy of a Charged Particle

A $+3.0$-nC charge Q is initially at rest a distance of 10 cm (r_1) from a $+5.0$-nC charge q fixed at the origin (Figure 7.7). Naturally, the Coulomb force accelerates Q away from q, eventually reaching 15 cm (r_2).

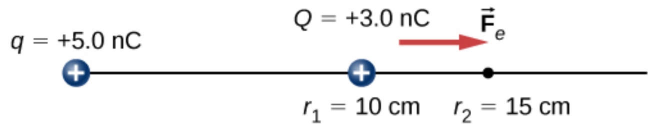


Figure 7.7 The charge Q is repelled by q, thus having work done on it and losing potential energy.

What is the change in the potential energy of the two-charge system from r_1 to r_2?

Strategy

Calculate the potential energy with the definition given above: $\Delta U_{12} = -\int_{r_1}^{r_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$. Since Q started from rest, this is the same as the kinetic energy.

Solution

We have

$$\begin{aligned}\Delta U_{12} &= -\int_{r_1}^{r_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = -\int_{r_1}^{r_2} \frac{kqQ}{r^2} dr = -\left[-\frac{kqQ}{r}\right]_{r_1}^{r_2} = kqQ\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \\ &= \left(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2\right)\left(5.0 \times 10^{-9} \text{ C}\right)\left(3.0 \times 10^{-9} \text{ C}\right)\left[\frac{1}{0.15 \text{ m}} - \frac{1}{0.10 \text{ m}}\right] \\ &= -4.5 \times 10^{-7} \text{ J}.\end{aligned}$$

Significance

The change in the potential energy is negative, as expected, and equal in magnitude to the change in kinetic energy in this system. Recall from Example 7.1 that the change in kinetic energy was positive.

CHECK YOUR UNDERSTANDING 7.2

What is the potential energy of Q relative to the zero reference at infinity at r_2 in the above example?

Due to Coulomb's law, the forces due to multiple charges on a test charge Q superimpose; they may be calculated individually and then added. This implies that the work integrals and hence the resulting potential energies exhibit the same behavior. To demonstrate this, we consider an example of assembling a system of four charges.

EXAMPLE 7.3

Assembling Four Positive Charges

Find the amount of work an external agent must do in assembling four charges $+2.0\ \mu$C,$+3.0\ \mu$C, $+4.0\ \mu$C, and $+5.0\ \mu$C at the vertices of a square of side 1.0 cm, starting each charge from infinity (Figure 7.8).

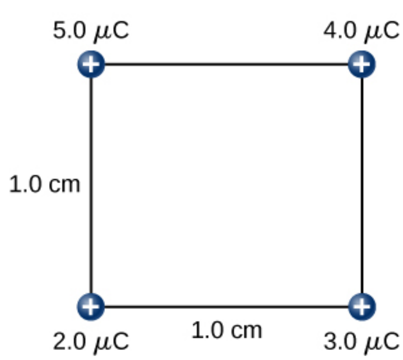


Figure 7.8 How much work is needed to assemble this charge configuration?

The factor of 1/2 accounts for adding each pair of charges twice.

7.2 Electric Potential and Potential Difference

Learning Objectives

By the end of this section, you will be able to:

- Define electric potential, voltage, and potential difference
- Define the electron-volt
- Calculate electric potential and potential difference from potential energy and electric field
- Describe systems in which the electron-volt is a useful unit
- Apply conservation of energy to electric systems

Recall that earlier we defined electric field to be a quantity independent of the test charge in a given system, which would nonetheless allow us to calculate the force that would result on an arbitrary test charge. (The default assumption in the absence of other information is that the test charge is positive.) We briefly defined a field for gravity, but gravity is always attractive, whereas the electric force can be either attractive or repulsive. Therefore, although potential energy is perfectly adequate in a gravitational system, it is convenient to define a quantity that allows us to calculate the work on a charge independent of the magnitude of the charge. Calculating the work directly may be difficult, since $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$ and the direction and magnitude of $\vec{\mathbf{F}}$ can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that because $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$, the work, and hence ΔU, is proportional to the test charge q. To have a physical quantity that is independent of test charge, we define **electric potential** V (or simply potential, since electric is understood) to be the potential energy per unit charge:

Electric Potential

The electric potential energy per unit charge is

$$V = \frac{U}{q}. \tag{7.4}$$

Since U is proportional to q, the dependence on q cancels. Thus, V does not depend on q. The change in potential energy ΔU is crucial, so we are concerned with the difference in potential or potential difference ΔV between two points, where

$$\Delta V = V_B - V_A = \frac{\Delta U}{q}.$$

Electric Potential Difference

The **electric potential difference** between points A and B, $V_B - V_A$, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1 \text{ V} = 1 \text{ J/C}$$

The familiar term **voltage** is the common name for electric potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor. It is worthwhile to emphasize the distinction between potential difference and electrical potential energy.

Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta U}{q} \text{ or } \Delta U = q\Delta V. \tag{7.5}$$

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus, a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other because $\Delta U = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12-V batteries.

EXAMPLE 7.4

Calculating Energy

You have a 12.0-V motorcycle battery that can move 5000 C of charge, and a 12.0-V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy

To say we have a 12.0-V battery means that its terminals have a 12.0-V potential difference. When such a battery moves charge, it puts the charge through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta U = q\Delta V$. To find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000 \text{ C}$ and $\Delta V = 12.0 \text{ V}$. The total energy delivered by the motorcycle battery is

$$\Delta U_{\text{cycle}} = (5000 \text{ C})(12.0 \text{ V}) = (5000 \text{ C})(12.0 \text{ J/C}) = 6.00 \times 10^4 \text{ J}.$$

Similarly, for the car battery, $q = 60{,}000 \text{ C}$ and

$$\Delta U_{\text{car}} = (60{,}000 \text{ C})(12.0 \text{ V}) = 7.20 \times 10^5 \text{ J}.$$

Significance

Voltage and energy are related, but they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. A car battery has a much larger engine to start than a motorcycle. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a depleted car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

CHECK YOUR UNDERSTANDING 7.4

How much energy does a 1.5-V AAA battery have that can move 100 C?

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (*A*) through whatever circuitry is involved and attract them to their positive terminals (*B*), as shown in Figure 7.12. The change in potential is $\Delta V = V_B - V_A = +12 \text{ V}$ and the charge q is negative, so that $\Delta U = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.

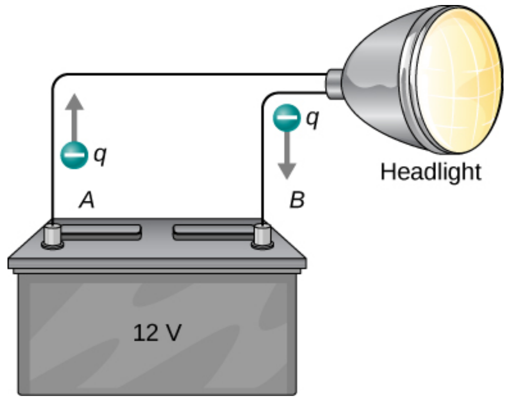


Figure 7.12 A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative terminal. Inside the battery, both positive and negative charges move.

EXAMPLE 7.5

How Many Electrons Move through a Headlight Each Second?

When a 12.0-V car battery powers a single 30.0-W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moves in 1.00 s. The charge moved is related to voltage and energy through the equations $\Delta U = q\Delta V$. A 30.0-W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta U = -30\,\text{J}$ and, since the electrons are going from the negative terminal to the positive, we see that $\Delta V = +12.0\,\text{V}$.

Solution

To find the charge q moved, we solve the equation $\Delta U = q\Delta V$:

$$q = \frac{\Delta U}{\Delta V}.$$

Entering the values for ΔU and ΔV, we get

$$q = \frac{-30.0\,\text{J}}{+12.0\,\text{V}} = \frac{-30.0\,\text{J}}{+12.0\,\text{J/C}} = -2.50\,\text{C}.$$

The number of electrons n_e is the total charge divided by the charge per electron. That is,

$$n_e = \frac{-2.50\,\text{C}}{-1.60 \times 10^{-19}\,\text{C/e}^-} = 1.56 \times 10^{19}\,\text{electrons}.$$

Significance

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

CHECK YOUR UNDERSTANDING 7.5

How many electrons would go through a 24.0-W lamp?

The Electron-Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful X-rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects.

Figure 7.13 shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates, as it might be in an old-model television tube or oscilloscope. The electron gains kinetic energy that is later converted into another form—light in the television tube, for example. (Note that in terms of energy, "downhill" for the electron is "uphill" for a positive charge.) Since energy is related to voltage by $\Delta U = q\Delta V$, we can think of the joule as a coulomb-volt.

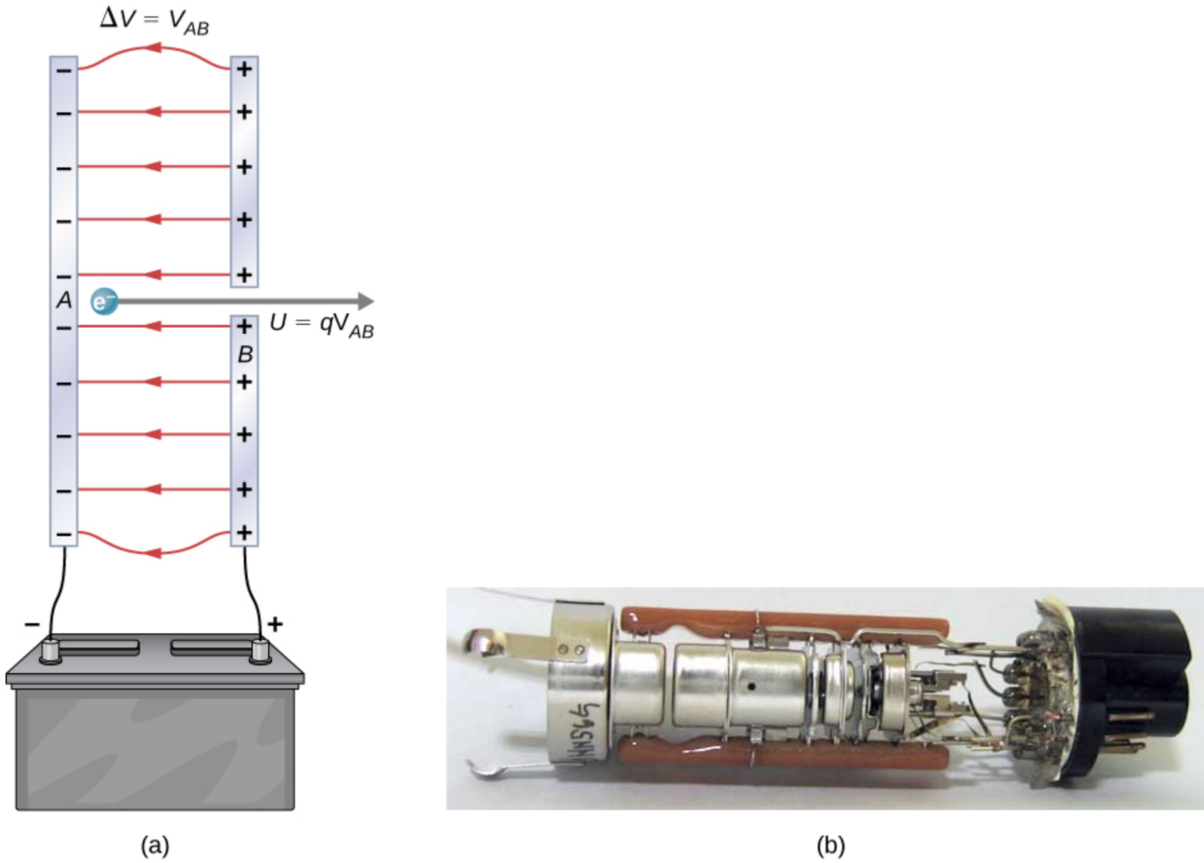


Figure 7.13 A typical electron gun accelerates electrons using a potential difference between two separated metal plates. By conservation of energy, the kinetic energy has to equal the change in potential energy, so $KE = qV$. The energy of the electron in electron-volts is numerically the same as the voltage between the plates. For example, a 5000-V potential difference produces 5000-eV electrons. The conceptual construct, namely two parallel plates with a hole in one, is shown in (a), while a real electron gun is shown in (b).

Electron-Volt

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron-volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.60 \times 10^{-19} \text{ J}.$$

An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V gains 50 eV. A potential difference of 100,000 V (100 kV) gives an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V gains 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron-volt a simple and convenient energy unit in such circumstances.

The electron-volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron-volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it acquires an energy of 30 keV (30,000 eV) and can break up as many as 6000 of these molecules ($30{,}000 \text{ eV} \div 5 \text{ eV per molecule} = 6000 \text{ molecules}$). Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can thus produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) due to work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $K + U = \text{constant}$. A loss of U for a charged particle becomes an increase in its K. Conservation of energy is stated in equation form as

$$K + U = \text{constant}$$

or

$$K_\mathrm{i} + U_\mathrm{i} = K_\mathrm{f} + U_\mathrm{f}$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

EXAMPLE 7.6

Electrical Potential Energy Converted into Kinetic Energy

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $K_\mathrm{i} = 0, K_\mathrm{f} = \frac{1}{2}mv^2, U_\mathrm{i} = qV, U_\mathrm{f} = 0$.

Solution

Conservation of energy states that

$$K_\mathrm{i} + U_\mathrm{i} = K_\mathrm{f} + U_\mathrm{f}.$$

Entering the forms identified above, we obtain

$$qV = \frac{mv^2}{2}.$$

We solve this for v:

$$v = \sqrt{\frac{2qV}{m}}.$$

Entering values for q, V, and m gives

$$v = \sqrt{\frac{2(-1.60 \times 10^{-19}\ \text{C})(-100\ \text{J/C})}{9.11 \times 10^{-31}\ \text{kg}}} = 5.93 \times 10^6\ \text{m/s}.$$

Significance

Note that both the charge and the initial voltage are negative, as in Figure 7.13. From the discussion of electric charge and electric field, we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. These higher voltages produce electron speeds so great that effects from special relativity must be taken into account and hence are reserved for a later chapter (Relativity). That is why we consider a low voltage (accurately) in this example.

CHECK YOUR UNDERSTANDING 7.6

How would this example change with a positron? A positron is identical to an electron except the charge is positive.

Voltage and Electric Field

So far, we have explored the relationship between voltage and energy. Now we want to explore the relationship between voltage and electric field. We will start with the general case for a non-uniform $\vec{\mathbf{E}}$ field. Recall that our general formula for the potential energy of a test charge q at point P relative to reference point R is

$$U_P = -\int_R^P \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}.$$

When we substitute in the definition of electric field ($\vec{\mathbf{E}} = \vec{\mathbf{F}}/q$), this becomes

$$U_P = -q\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}.$$

Applying our definition of potential ($V = U/q$) to this potential energy, we find that, in general,

$$V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}. \tag{7.6}$$

From our previous discussion of the potential energy of a charge in an electric field, the result is independent of the path chosen, and hence we can pick the integral path that is most convenient.

Consider the special case of a positive point charge q at the origin. To calculate the potential caused by q at a distance r from the origin relative to a reference of 0 at infinity (recall that we did the same for potential energy), let $P = r$ and $R = \infty$, with $d\vec{\mathbf{l}} = d\vec{\mathbf{r}} = \hat{\mathbf{r}}dr$ and use $\vec{\mathbf{E}} = \frac{kq}{r^2}\hat{\mathbf{r}}$. When we evaluate the integral

$$V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

for this system, we have

$$V_r = -\int_\infty^r \frac{kq}{r^2}\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}dr,$$

which simplifies to

$$V_r = -\int_{\infty}^{r} \frac{kq}{r^2}\,dr = \frac{kq}{r} - \frac{kq}{\infty} = \frac{kq}{r}.$$

This result,

$$V_r = \frac{kq}{r}$$

is the standard form of the potential of a point charge. This will be explored further in the next section.

To examine another interesting special case, suppose a uniform electric field $\vec{\mathbf{E}}$ is produced by placing a potential difference (or voltage) ΔV across two parallel metal plates, labeled A and B (Figure 7.14). Examining this situation will tell us what voltage is needed to produce a certain electric field strength. It will also reveal a more fundamental relationship between electric potential and electric field.

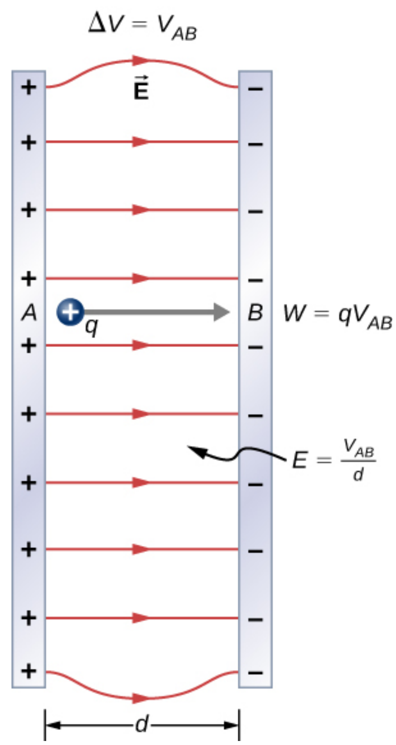


Figure 7.14 The relationship between V and E for parallel conducting plates is $E = V/d$. (Note that $\Delta V = V_{AB}$ in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows: $-\Delta V = V_A - V_B = V_{AB}$.)

From a physicist's point of view, either ΔV or $\vec{\mathbf{E}}$ can be used to describe any interaction between charges. However, ΔV is a scalar quantity and has no direction, whereas $\vec{\mathbf{E}}$ is a vector quantity, having both magnitude and direction. (Note that the magnitude of the electric field, a scalar quantity, is represented by E.) The relationship between ΔV and $\vec{\mathbf{E}}$ is revealed by calculating the work done by the electric force in moving a charge from point A to point B. But, as noted earlier, arbitrary charge distributions require calculus. We therefore look at a uniform electric field as an interesting special case.

The work done by the electric field in Figure 7.14 to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

$$W = -\Delta U = -q\Delta V.$$

The potential difference between points A and B is

$$-\Delta V = -(V_B - V_A) = V_A - V_B = V_{AB}.$$

Entering this into the expression for work yields

$$W = qV_{AB}.$$

Work is $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = Fd\cos\theta$; here $\cos\theta = 1$, since the path is parallel to the field. Thus, $W = Fd$. Since $F = qE$, we see that $W = qEd$.

Substituting this expression for work into the previous equation gives

$$qEd = qV_{AB}.$$

The charge cancels, so we obtain for the voltage between points A and B

$$\left.\begin{aligned} V_{AB} &= Ed \\ E &= \frac{V_{AB}}{d} \end{aligned}\right\}\text{(uniform } E\text{-field only)}$$

where d is the distance from A to B, or the distance between the plates in Figure 7.14. Note that this equation implies that the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus, the following relation among units is valid:

$$1\ \mathrm{N/C} = 1\ \mathrm{V/m}.$$

Furthermore, we may extend this to the integral form. Substituting Equation 7.5 into our definition for the potential difference between points A and B, we obtain

$$V_{BA} = V_B - V_A = -\int_R^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} + \int_R^A \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

which simplifies to

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}.$$

As a demonstration, from this we may calculate the potential difference between two points (A and B) equidistant from a point charge q at the origin, as shown in Figure 7.15.

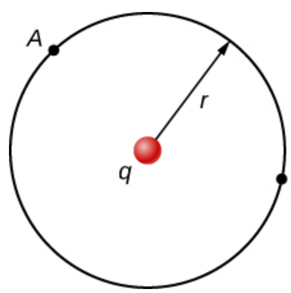


Figure 7.15 The arc for calculating the potential difference between two points that are equidistant from a point charge at the origin.

To do this, we integrate around an arc of the circle of constant radius r between A and B, which means we let $d\vec{\mathbf{l}} = r\hat{\boldsymbol{\varphi}}\,d\varphi$, while using $\vec{\mathbf{E}} = \frac{kq}{r^2}\hat{\mathbf{r}}$. Thus,

$$\Delta V_{BA} = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} \tag{7.7}$$

for this system becomes

$$V_B - V_A = -\int_A^B \frac{kq}{r^2}\hat{\mathbf{r}} \cdot r\hat{\boldsymbol{\varphi}}\,d\varphi.$$

However, $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\varphi}} = 0$ and therefore

$$V_B - V_A = 0.$$

This result, that there is no difference in potential along a constant radius from a point charge, will come in handy when we map potentials.

EXAMPLE 7.7

What Is the Highest Voltage Possible between Two Plates?

Dry air can support a maximum electric field strength of about 3.0×10^6 V/m. Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

Strategy

We are given the maximum electric field E between the plates and the distance d between them. We can use the equation $V_{AB} = Ed$ to calculate the maximum voltage.

Solution

The potential difference or voltage between the plates is

$$V_{AB} = Ed.$$

Entering the given values for E and d gives

$$V_{AB} = (3.0 \times 10^6 \text{ V/m})(0.025 \text{ m}) = 7.5 \times 10^4 \text{ V}$$

or

$$V_{AB} = 75 \text{ kV}.$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

Significance

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5-cm (1-in.) gap, or 150 kV for a 5-cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage can cause a spark if there are spines on the surface, since sharp points have larger field strengths than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up with static electricity on dry days (Figure 7.16).

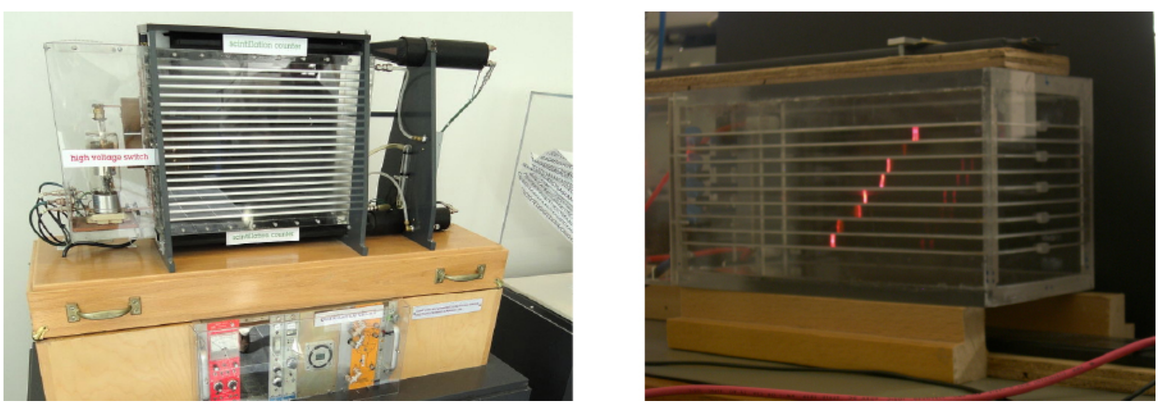


Figure 7.16 A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays). This form of detector is now archaic and no longer in use except for demonstration purposes. (credit b: modification of work by Jack Collins)

EXAMPLE 7.8

Field and Force inside an Electron Gun

An electron gun (Figure 7.13) has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. (a) What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a $0.500\text{-}\mu\text{C}$ charge that gets between the plates?

Strategy

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression $E = \frac{V_{AB}}{d}$. Once we know the electric field strength, we can find the force on a charge by using $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$. Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, $F = qE$.

Solution

a. The expression for the magnitude of the electric field between two uniform metal plates is

$$E = \frac{V_{AB}}{d}.$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for V_{AB} and the plate separation of 0.0400 m, we obtain

$$E = \frac{25.0\,\text{kV}}{0.0400\,\text{m}} = 6.25 \times 10^5\ \text{V/m}.$$

b. The magnitude of the force on a charge in an electric field is obtained from the equation

$$F = qE.$$

Substituting known values gives

$$F = (0.500 \times 10^{-6}\ \text{C})(6.25 \times 10^5\ \text{V/m}) = 0.313\,\text{N}.$$

Significance

Note that the units are newtons, since $1\ \text{V/m} = 1\ \text{N/C}$. Because the electric field is uniform between the plates, the force on the charge is the same no matter where the charge is located between the plates.

EXAMPLE 7.9

Calculating Potential of a Point Charge

Given a point charge $q = +2.0\,\text{nC}$ at the origin, calculate the potential difference between point P_1 a distance $a = 4.0\,\text{cm}$ from q, and P_2 a distance $b = 12.0\,\text{cm}$ from q, where the two points have an angle of $\varphi = 24°$ between them (Figure 7.17).

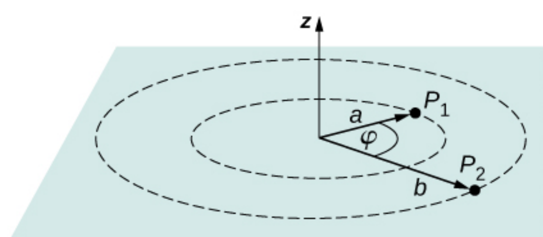


Figure 7.17 Find the difference in potential between P_1 and P_2.

Strategy

Do this in two steps. The first step is to use $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$ and let $A = a = 4.0\,\text{cm}$ and

$B = b = 12.0\text{ cm}$, with $d\vec{\mathbf{l}} = d\vec{\mathbf{r}} = \hat{\mathbf{r}}dr$ and $\vec{\mathbf{E}} = \frac{kq}{r^2}\hat{\mathbf{r}}$. Then perform the integral. The second step is to integrate $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$ around an arc of constant radius r, which means we let $d\vec{\mathbf{l}} = r\hat{\varphi}\,d\varphi$ with limits $0 \le \varphi \le 24°$, still using $\vec{\mathbf{E}} = \frac{kq}{r^2}\hat{\mathbf{r}}$. Then add the two results together.

Solution

For the first part, $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$ for this system becomes $V_b - V_a = -\int_a^b \frac{kq}{r^2}\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}dr$ which computes to

$$\begin{aligned}\Delta V &= -\int_a^b \frac{kq}{r^2}dr = kq\left[\frac{1}{a} - \frac{1}{b}\right] \\ &= \left(8.99 \times 10^9\ \text{Nm}^2/\text{C}^2\right)\left(2.0 \times 10^{-9}\ \text{C}\right)\left[\frac{1}{0.040\ \text{m}} - \frac{1}{0.12\ \text{m}}\right] = 300\ \text{V}.\end{aligned}$$

For the second step, $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$ becomes $\Delta V = -\int_0^{24°} \frac{kq}{r^2}\hat{\mathbf{r}} \cdot r\hat{\boldsymbol{\varphi}}\,d\varphi$, but $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\varphi}} = 0$ and therefore $\Delta V = 0$. Adding the two parts together, we get 300 V.

Significance

We have demonstrated the use of the integral form of the potential difference to obtain a numerical result. Notice that, in this particular system, we could have also used the formula for the potential due to a point charge at the two points and simply taken the difference.

CHECK YOUR UNDERSTANDING 7.7

From the examples, how does the energy of a lightning strike vary with the height of the clouds from the ground? Consider the cloud-ground system to be two parallel plates.

Before presenting problems involving electrostatics, we suggest a problem-solving strategy to follow for this topic.

PROBLEM-SOLVING STRATEGY

Electrostatics

1. Examine the situation to determine if static electricity is involved; this may concern separated stationary charges, the forces among them, and the electric fields they create.
2. Identify the system of interest. This includes noting the number, locations, and types of charges involved.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful. Determine whether the Coulomb force is to be considered directly—if so, it may be useful to draw a free-body diagram, using electric field lines.
4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is important to distinguish the Coulomb force F from the electric field E, for example.
5. Solve the appropriate equation for the quantity to be determined (the unknown) or draw the field lines as requested.
6. Examine the answer to see if it is reasonable: Does it make sense? Are units correct and the numbers involved reasonable?

7.3 Calculations of Electric Potential

Learning Objectives

By the end of this section, you will be able to:

- Calculate the potential due to a point charge
- Calculate the potential of a system of multiple point charges
- Describe an electric dipole
- Define dipole moment
- Calculate the potential of a continuous charge distribution

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (such as charge on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider.

We can use calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge q. Noting the connection between work and potential $W = -q\Delta V$, as in the last section, we can obtain the following result.

Electric Potential V of a Point Charge

The electric potential V of a point charge is given by

$$V = \frac{kq}{r} \text{(point charge)} \tag{7.8}$$

where k is a constant equal to $8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

The potential at infinity is chosen to be zero. Thus, V for a point charge decreases with distance, whereas $\vec{\mathbf{E}}$ for a point charge decreases with distance squared:

$$E = \frac{F}{q_t} = \frac{kq}{r^2}.$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field $\vec{\mathbf{E}}$ is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as vectors, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas $\vec{\mathbf{E}}$ is closely associated with force, a vector.

EXAMPLE 7.10

What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb (μC) range. What is the voltage 5.00 cm away from the center of a 1-cm-diameter solid metal sphere that has a –3.00-nC static charge?

Strategy

As we discussed in Electric Charges and Fields, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus, we can find the voltage using the equation $V = \frac{kq}{r}$.

Solution

Entering known values into the expression for the potential of a point charge, we obtain

$$V = k\frac{q}{r} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(\frac{-3.00 \times 10^{-9} \text{ C}}{5.00 \times 10^{-2} \text{ m}}\right) = -539 \text{ V}.$$

Significance

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger distances. Conversely, a negative charge would be repelled, as expected.

EXAMPLE 7.11

What Is the Excess Charge on a Van de Graaff Generator?

A demonstration Van de Graaff generator has a 25.0-cm-diameter metal sphere that produces a voltage of 100 kV near its surface (Figure 7.18). What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)

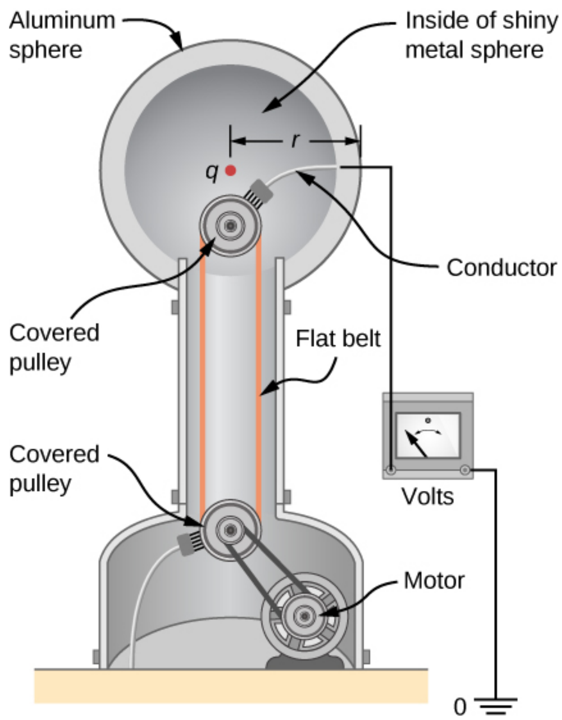


Figure 7.18 The voltage of this demonstration Van de Graaff generator is measured between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

Strategy

The potential on the surface is the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.) We can thus determine the excess charge using the equation

$$V = \frac{kq}{r}.$$

Solution

Solving for q and entering known values gives

$$q = \frac{rV}{k} = \frac{(0.125\text{ m})\left(100 \times 10^3\text{ V}\right)}{8.99 \times 10^9\text{ N} \cdot \text{m}^2/\text{C}^2} = 1.39 \times 10^{-6}\text{ C} = 1.39\ \mu\text{C}.$$

Significance

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

CHECK YOUR UNDERSTANDING 7.8

What is the potential inside the metal sphere in Example 7.10?

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted earlier, this is analogous to taking sea level as $h = 0$ when considering gravitational potential energy $U_g = mgh$.

Systems of Multiple Point Charges

Just as the electric field obeys a superposition principle, so does the electric potential. Consider a system consisting of N charges $q_1, q_2, \ldots, q_N$. What is the net electric potential V at a space point P from these charges? Each of these charges is a source charge that produces its own electric potential at point P, independent of whatever other changes may be doing. Let $V_1, V_2, \ldots, V_N$ be the electric potentials at P produced by the charges $q_1, q_2, \ldots, q_N$, respectively. Then, the net electric potential V_P at that point is equal to the sum of these individual electric potentials. You can easily show this by calculating the potential energy of a test charge when you bring the test charge from the reference point at infinity to point P:

$$V_P = V_1 + V_2 + \cdots + V_N = \sum_1^N V_i.$$

Note that electric potential follows the same principle of superposition as electric field and electric potential energy. To show this more explicitly, note that a test charge q_i at the point P in space has distances of $r_1, r_2, \ldots, r_N$ from the N charges fixed in space above, as shown in Figure 7.19. Using our formula for the potential of a point charge for each of these (assumed to be point) charges, we find that

$$V_P = \sum_1^N k\frac{q_i}{r_i} = k\sum_1^N \frac{q_i}{r_i}. \tag{7.9}$$

Therefore, the electric potential energy of the test charge is

$$U_P = q_t V_P = q_t k \sum_1^N \frac{q_i}{r_i},$$

which is the same as the work to bring the test charge into the system, as found in the first section of the chapter.

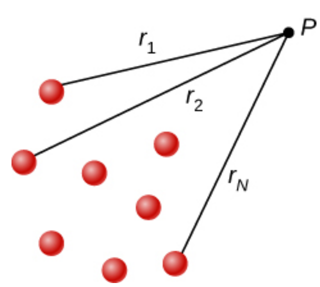


Figure 7.19 Notation for direct distances from charges to a space point P.

The Electric Dipole

An **electric dipole** is a system of two equal but opposite charges a fixed distance apart. This system is used to model many real-world systems, including atomic and molecular interactions. One of these systems is the water molecule, under certain circumstances. These circumstances are met inside a microwave oven, where

However, this limit does not exist because the argument of the logarithm becomes [2/0] as $L \to \infty$, so this way of finding V of an infinite wire does not work. The reason for this problem may be traced to the fact that the charges are not localized in some space but continue to infinity in the direction of the wire. Hence, our (unspoken) assumption that zero potential must be an infinite distance from the wire is no longer valid.

To avoid this difficulty in calculating limits, let us use the definition of potential by integrating over the electric field from the previous section, and the value of the electric field from this charge configuration from the previous chapter.

Solution

We use the integral

$$V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

where R is a finite distance from the line of charge, as shown in Figure 7.26.

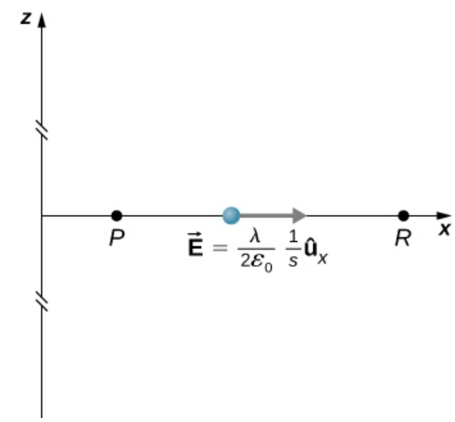


Figure 7.26 Points of interest for calculating the potential of an infinite line of charge.

With this setup, we use $\vec{\mathbf{E}}_P = 2k\lambda\frac{1}{s}\hat{\mathbf{s}}$ and $d\vec{\mathbf{l}} = d\vec{\mathbf{s}}$ to obtain

$$V_P - V_R = -\int_R^P 2k\lambda\frac{1}{s}ds = -2k\lambda \ln\frac{s_P}{s_R}.$$

Now, if we define the reference potential $V_R = 0$ at $s_R = 1$ m, this simplifies to

$$V_P = -2k\lambda \ln s_P.$$

Note that this form of the potential is quite usable; it is 0 at 1 m and is undefined at infinity, which is why we could not use the latter as a reference.

Significance

Although calculating potential directly can be quite convenient, we just found a system for which this strategy does not work well. In such cases, going back to the definition of potential in terms of the electric field may offer a way forward.

CHECK YOUR UNDERSTANDING 7.10

What is the potential on the axis of a nonuniform ring of charge, where the charge density is $\lambda(\theta) = \lambda \cos\theta$?

7.4 Determining Field from Potential

Learning Objectives

By the end of this section, you will be able to:

- Explain how to calculate the electric field in a system from the given potential
- Calculate the electric field in a given direction from a given potential
- Calculate the electric field throughout space from a given potential

Recall that we were able, in certain systems, to calculate the potential by integrating over the electric field. As you may already suspect, this means that we may calculate the electric field by taking derivatives of the potential, although going from a scalar to a vector quantity introduces some interesting wrinkles. We frequently need $\vec{\mathbf{E}}$ to calculate the force in a system; since it is often simpler to calculate the potential directly, there are systems in which it is useful to calculate V and then derive $\vec{\mathbf{E}}$ from it.

In general, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of $\vec{\mathbf{E}}$ and also in the direction of lower potential V. Furthermore, the magnitude of $\vec{\mathbf{E}}$ equals the rate of decrease of V with distance. The faster V decreases over distance, the greater the electric field. This gives us the following result.

Relationship between Voltage and Uniform Electric Field

In equation form, the relationship between voltage and uniform electric field is

$$E = -\frac{\Delta V}{\Delta s}$$

where Δs is the distance over which the change in potential ΔV takes place. The minus sign tells us that E points in the direction of decreasing potential. The electric field is said to be the gradient (as in grade or slope) of the electric potential.

For continually changing potentials, ΔV and Δs become infinitesimals, and we need differential calculus to determine the electric field. As shown in Figure 7.27, if we treat the distance Δs as very small so that the electric field is essentially constant over it, we find that

$$E_s = -\frac{dV}{ds}.$$

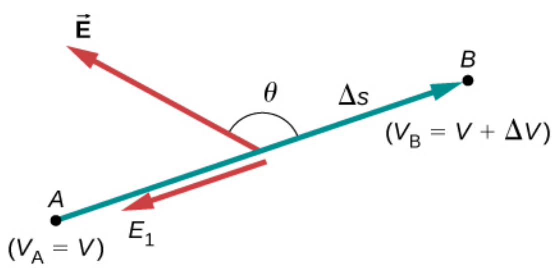


Figure 7.27 The electric field component along the displacement Δs is given by $E = -\frac{\Delta V}{\Delta s}$. Note that *A* and *B* are assumed to be so close together that the field is constant along Δs.

Therefore, the electric field components in the Cartesian directions are given by

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}. \qquad (7.13)$$

This allows us to define the “grad” or “del” vector operator, which allows us to compute the gradient in one step. In Cartesian coordinates, it takes the form

$$\vec{\nabla} = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}. \tag{7.14}$$

With this notation, we can calculate the electric field from the potential with

$$\vec{\mathbf{E}} = -\vec{\nabla} V, \tag{7.15}$$

a process we call calculating the gradient of the potential.

If we have a system with either cylindrical or spherical symmetry, we only need to use the del operator in the appropriate coordinates:

$$\text{Cylindrical: } \vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial \varphi} + \hat{\mathbf{z}}\frac{\partial}{\partial z} \tag{7.16}$$

$$\text{Spherical: } \vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi} \tag{7.17}$$

EXAMPLE 7.17

Electric Field of a Point Charge

Calculate the electric field of a point charge from the potential.

Strategy

The potential is known to be $V = k\frac{q}{r}$, which has a spherical symmetry. Therefore, we use the spherical del operator in the formula $\vec{\mathbf{E}} = -\vec{\nabla} V$.

Solution

Performing this calculation gives us

$$\vec{\mathbf{E}} = -\left(\hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\right)k\frac{q}{r} = -kq\left(\hat{\mathbf{r}}\frac{\partial}{\partial r}\frac{1}{r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}\frac{1}{r} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\frac{1}{r}\right).$$

This equation simplifies to

$$\vec{\mathbf{E}} = -kq\left(\hat{\mathbf{r}}\frac{-1}{r^2} + \hat{\theta}0 + \hat{\varphi}0\right) = k\frac{q}{r^2}\hat{\mathbf{r}}$$

as expected.

Significance

We not only obtained the equation for the electric field of a point particle that we've seen before, we also have a demonstration that $\vec{\mathbf{E}}$ points in the direction of decreasing potential, as shown in Figure 7.28.

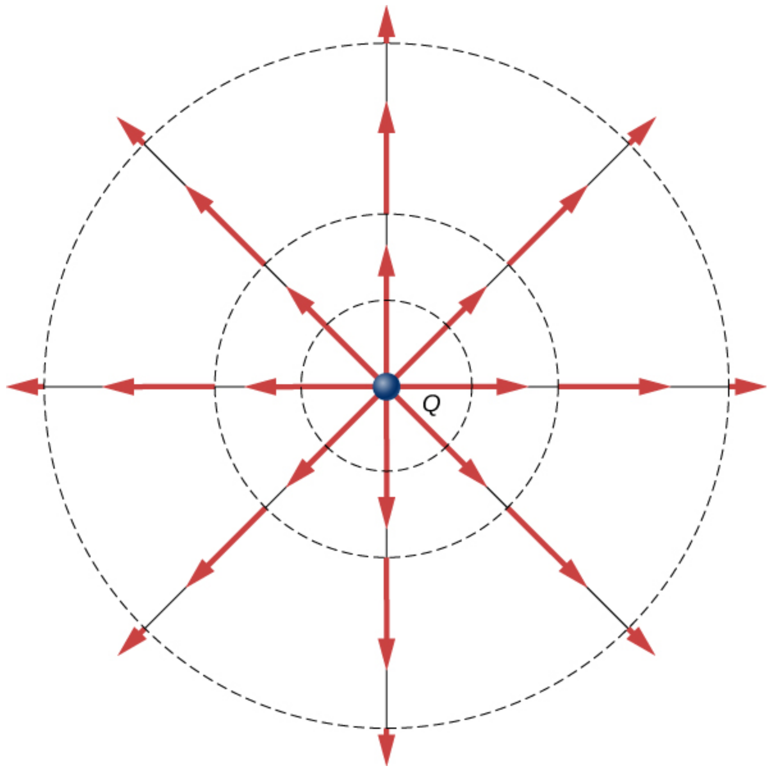


Figure 7.28 Electric field vectors inside and outside a uniformly charged sphere.

EXAMPLE 7.18

Electric Field of a Ring of Charge

Use the potential found in Example 7.8 to calculate the electric field along the axis of a ring of charge (Figure 7.29).

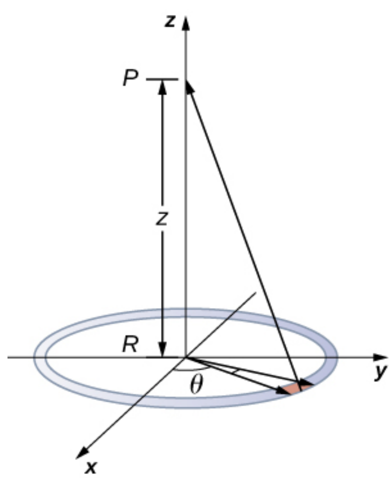


Figure 7.29 We want to calculate the electric field from the electric potential due to a ring charge.

Strategy

In this case, we are only interested in one dimension, the z-axis. Therefore, we use $E_z = -\frac{\partial V}{\partial z}$ with the potential $V = k\frac{q_{\text{tot}}}{\sqrt{z^2+R^2}}$ found previously.

Solution

Taking the derivative of the potential yields

$$E_z = -\frac{\partial}{\partial z}\frac{kq_{\text{tot}}}{\sqrt{z^2 + R^2}} = k\frac{q_{\text{tot}}\, z}{\left(z^2 + R^2\right)^{3/2}}.$$

Significance

Again, this matches the equation for the electric field found previously. It also demonstrates a system in which using the full del operator is not necessary.

CHECK YOUR UNDERSTANDING 7.11

Which coordinate system would you use to calculate the electric field of a dipole?

7.5 Equipotential Surfaces and Conductors

Learning Objectives

By the end of this section, you will be able to:

- Define equipotential surfaces and equipotential lines
- Explain the relationship between equipotential lines and electric field lines
- Map equipotential lines for one or two point charges
- Describe the potential of a conductor
- Compare and contrast equipotential lines and elevation lines on topographic maps

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. This is not surprising, since the two concepts are related. Consider Figure 7.30, which shows an isolated positive point charge and its electric field lines, which radiate out from a positive charge and terminate on negative charges. We use red arrows to represent the magnitude and direction of the electric field, and we use black lines to represent places where the electric potential is constant. These are called **equipotential surfaces** in three dimensions, or **equipotential lines** in two dimensions. The term *equipotential* is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius r surrounding the charge. This is true because the potential for a point charge is given by $V = kq/r$ and thus has the same value at any point that is a given distance r from the charge. An equipotential sphere is a circle in the two-dimensional view of Figure 7.30. Because the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.

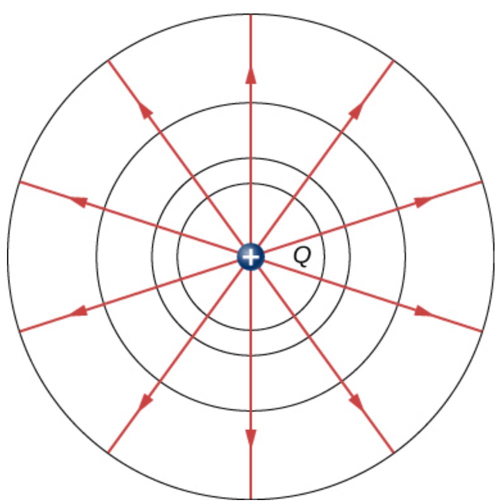


Figure 7.30 An isolated point charge Q with its electric field lines in red and equipotential lines in black. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case. For a three-

dimensional version, explore the first media link.

It is important to note that *equipotential lines are always perpendicular to electric field lines*. No work is required to move a charge along an equipotential, since $\Delta V = 0$. Thus, the work is

$$W = -\Delta U = -q\Delta V = 0.$$

Work is zero if the direction of the force is perpendicular to the displacement. Force is in the same direction as *E*, so motion along an equipotential must be perpendicular to *E*. More precisely, work is related to the electric field by

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = q\vec{\mathbf{E}} \cdot \vec{\mathbf{d}} = qEd\cos\theta = 0.$$

Note that in this equation, E and F symbolize the magnitudes of the electric field and force, respectively. Neither q nor E is zero; d is also not zero. So cos θ must be 0, meaning θ must be 90°. In other words, motion along an equipotential is perpendicular to *E*.

One of the rules for static electric fields and conductors is that the electric field must be perpendicular to the surface of any conductor. This implies that a *conductor is an equipotential surface in static situations*. There can be no voltage difference across the surface of a conductor, or charges will flow. One of the uses of this fact is that a conductor can be fixed at what we consider zero volts by connecting it to the earth with a good conductor—a process called **grounding**. Grounding can be a useful safety tool. For example, grounding the metal case of an electrical appliance ensures that it is at zero volts relative to Earth.

Because a conductor is an equipotential, it can replace any equipotential surface. For example, in Figure 7.30, a charged spherical conductor can replace the point charge, and the electric field and potential surfaces outside of it will be unchanged, confirming the contention that a spherical charge distribution is equivalent to a point charge at its center.

Figure 7.31 shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines. Conversely, given the equipotential lines, as in Figure 7.32(a), the electric field lines can be drawn by making them perpendicular to the equipotentials, as in Figure 7.32(b).

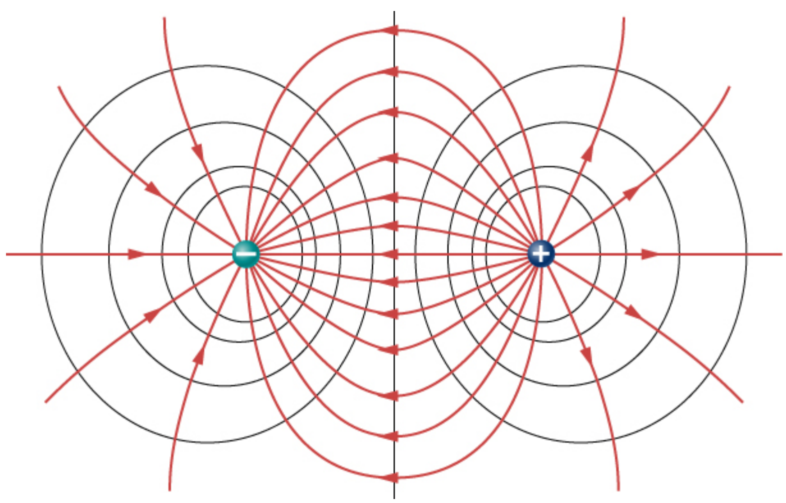


Figure 7.31 The electric field lines and equipotential lines for two equal but opposite charges. The equipotential lines can be drawn by making them perpendicular to the electric field lines, if those are known. Note that the potential is greatest (most positive) near the positive charge and least (most negative) near the negative charge. For a three-dimensional version, explore the first media link.

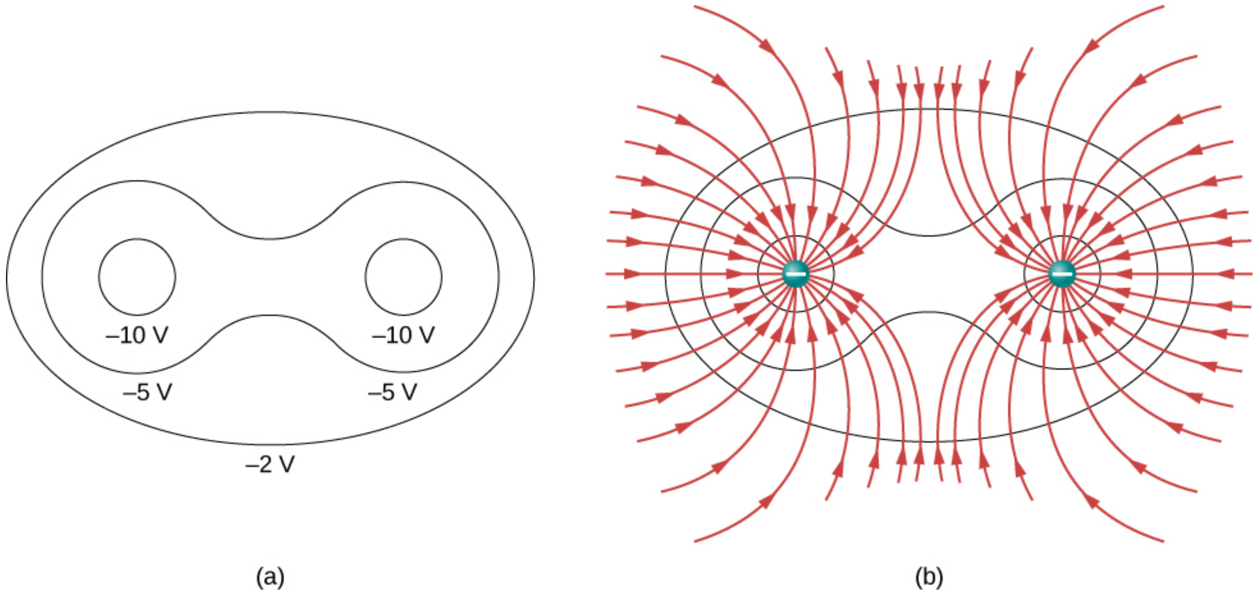


Figure 7.32 (a) These equipotential lines might be measured with a voltmeter in a laboratory experiment. (b) The corresponding electric field lines are found by drawing them perpendicular to the equipotentials. Note that these fields are consistent with two equal negative charges. For a three-dimensional version, play with the first media link.

To improve your intuition, we show a three-dimensional variant of the potential in a system with two opposing charges. Figure 7.33 displays a three-dimensional map of electric potential, where lines on the map are for equipotential surfaces. The hill is at the positive charge, and the trough is at the negative charge. The potential is zero far away from the charges. Note that the cut off at a particular potential implies that the charges are on conducting spheres with a finite radius.

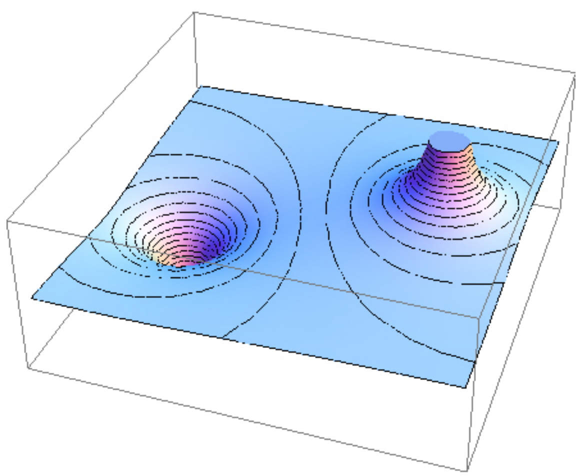


Figure 7.33 Electric potential map of two opposite charges of equal magnitude on conducting spheres. The potential is negative near the negative charge and positive near the positive charge.

A two-dimensional map of the cross-sectional plane that contains both charges is shown in Figure 7.34. The line that is equidistant from the two opposite charges corresponds to zero potential, since at the points on the line, the positive potential from the positive charge cancels the negative potential from the negative charge. Equipotential lines in the cross-sectional plane are closed loops, which are not necessarily circles, since at each point, the net potential is the sum of the potentials from each charge.

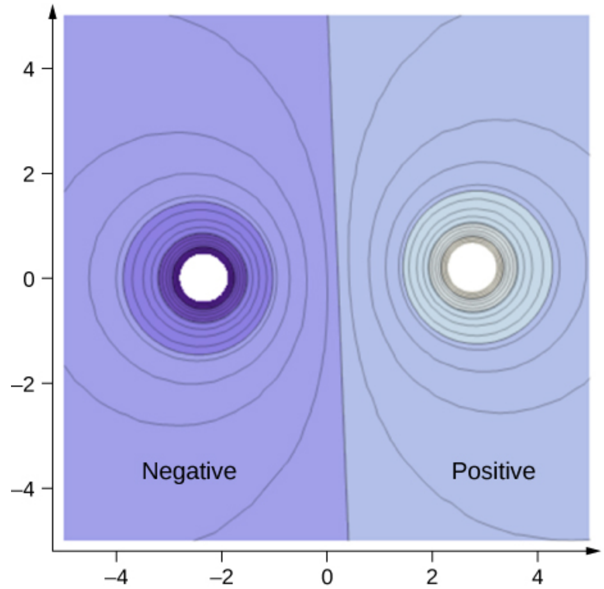


Figure 7.34 A cross-section of the electric potential map of two opposite charges of equal magnitude. The potential is negative near the negative charge and positive near the positive charge.

INTERACTIVE

View this simulation (https://openstax.org/l/21equipsurelec) to observe and modify the equipotential surfaces and electric fields for many standard charge configurations. There's a lot to explore.

One of the most important cases is that of the familiar parallel conducting plates shown in Figure 7.35. Between the plates, the equipotentials are evenly spaced and parallel. The same field could be maintained by placing conducting plates at the equipotential lines at the potentials shown.

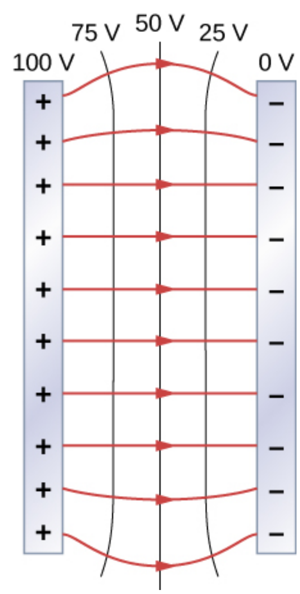
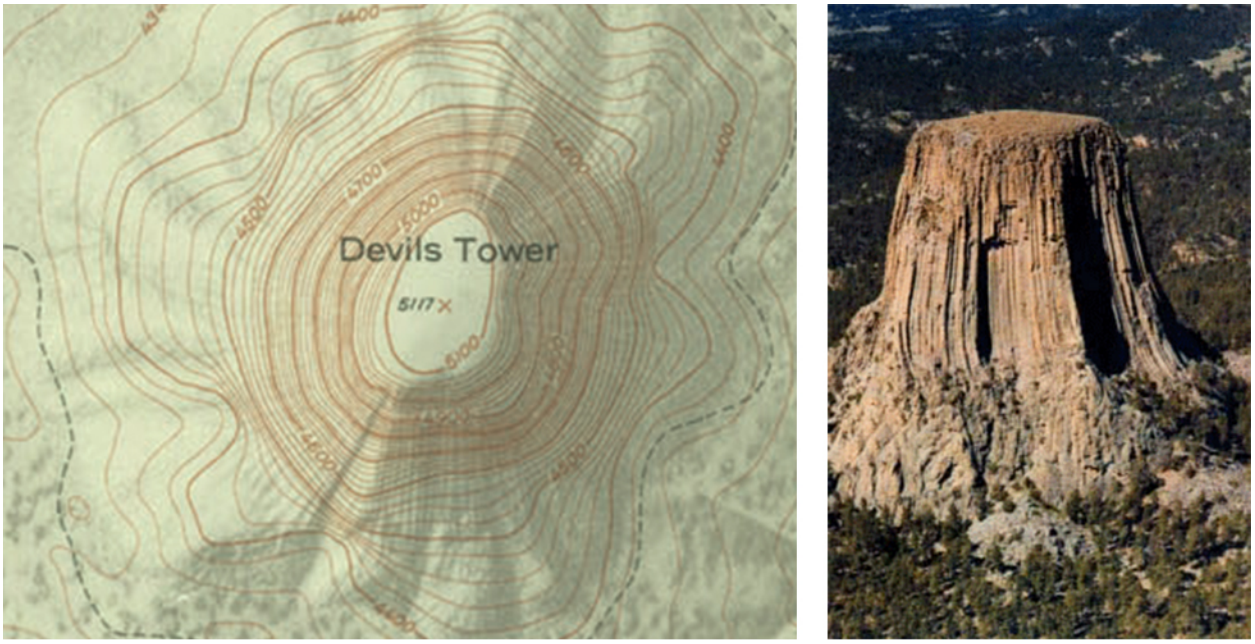


Figure 7.35 The electric field and equipotential lines between two metal plates. Note that the electric field is perpendicular to the equipotentials and hence normal to the plates at their surface as well as in the center of the region between them.

Consider the parallel plates in Figure 7.2. These have equipotential lines that are parallel to the plates in the space between and evenly spaced. An example of this (with sample values) is given in Figure 7.35. We could draw a similar set of equipotential isolines for gravity on the hill shown in Figure 7.2. If the hill has any extent at the same slope, the isolines along that extent would be parallel to each other. Furthermore, in regions of constant slope, the isolines would be evenly spaced. An example of real topographic lines is shown in Figure 7.36.



(a) (b)

Figure 7.36 A topographical map along a ridge has roughly parallel elevation lines, similar to the equipotential lines in Figure 7.35. (a) A topographical map of Devil's Tower, Wyoming. Lines that are close together indicate very steep terrain. (b) A perspective photo of Devil's Tower shows just how steep its sides are. Notice the top of the tower has the same shape as the center of the topographical map.

EXAMPLE 7.19

Calculating Equipotential Lines

You have seen the equipotential lines of a point charge in Figure 7.30. How do we calculate them? For example, if we have a +10-nC charge at the origin, what are the equipotential surfaces at which the potential is (a) 100 V, (b) 50 V, (c) 20 V, and (d) 10 V?

Strategy

Set the equation for the potential of a point charge equal to a constant and solve for the remaining variable(s). Then calculate values as needed.

Solution

In $V = k\frac{q}{r}$, let V be a constant. The only remaining variable is r; hence, $r = k\frac{q}{V} = \text{constant}$. Thus, the equipotential surfaces are spheres about the origin. Their locations are:

a. $r = k\frac{q}{V} = \left(8.99 \times 10^9\ \text{Nm}^2/\text{C}^2\right)\frac{\left(10 \times 10^{-9}\ \text{C}\right)}{100\ \text{V}} = 0.90\ \text{m};$

b. $r = k\frac{q}{V} = \left(8.99 \times 10^9\ \text{Nm}^2/\text{C}^2\right)\frac{\left(10 \times 10^{-9}\ \text{C}\right)}{50\ \text{V}} = 1.8\ \text{m};$

c. $r = k\frac{q}{V} = \left(8.99 \times 10^9\ \text{Nm}^2/\text{C}^2\right)\frac{\left(10 \times 10^{-9}\ \text{C}\right)}{20\ \text{V}} = 4.5\ \text{m};$

d. $r = k\frac{q}{V} = \left(8.99 \times 10^9\ \text{Nm}^2/\text{C}^2\right)\frac{\left(10 \times 10^{-9}\ \text{C}\right)}{10\ \text{V}} = 9.0\ \text{m}.$

Significance

This means that equipotential surfaces around a point charge are spheres of constant radius, as shown earlier, with well-defined locations.

EXAMPLE 7.20

Potential Difference between Oppositely Charged Parallel Plates

Two large conducting plates carry equal and opposite charges, with a surface charge density σ of magnitude $6.81 \times 10^{-7}\ \mathrm{C/m^2}$, as shown in Figure 7.37. The separation between the plates is $l = 6.50$ mm. (a) What is the electric field between the plates? (b) What is the potential difference between the plates? (c) What is the distance between equipotential planes which differ by 100 V?

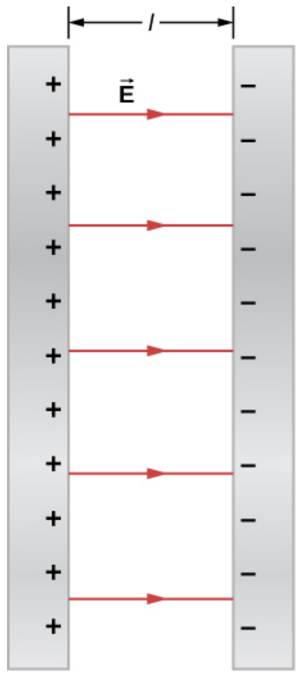


Figure 7.37 The electric field between oppositely charged parallel plates. A portion is released at the positive plate.

Strategy

(a) Since the plates are described as "large" and the distance between them is not, we will approximate each of them as an infinite plane, and apply the result from Gauss's law in the previous chapter.

(b) Use $\Delta V_{AB} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$.

(c) Since the electric field is constant, find the ratio of 100 V to the total potential difference; then calculate this fraction of the distance.

Solution

a. The electric field is directed from the positive to the negative plate as shown in the figure, and its magnitude is given by

$$E = \frac{\sigma}{\varepsilon_0} = \frac{6.81 \times 10^{-7}\ \mathrm{C/m^2}}{8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}} = 7.69 \times 10^4\ \mathrm{V/m}.$$

b. To find the potential difference ΔV between the plates, we use a path from the negative to the positive plate that is directed against the field. The displacement vector $d\vec{\mathbf{l}}$ and the electric field $\vec{\mathbf{E}}$ are antiparallel so $\vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -E\,dl$. The potential difference between the positive plate and the negative plate is then

$$\Delta V = -\int E \cdot dl = E \int dl = El = (7.69 \times 10^4\ \mathrm{V/m})(6.50 \times 10^{-3}\ \mathrm{m}) = 500\ \mathrm{V}.$$

c. The total potential difference is 500 V, so 1/5 of the distance between the plates will be the distance between 100-V potential differences. The distance between the plates is 6.5 mm, so there will be 1.3 mm between 100-V potential differences.

Significance

You have now seen a numerical calculation of the locations of equipotentials between two charged parallel plates.

CHECK YOUR UNDERSTANDING 7.12

What are the equipotential surfaces for an infinite line charge?

Distribution of Charges on Conductors

In Example 7.19 with a point charge, we found that the equipotential surfaces were in the form of spheres, with the point charge at the center. Given that a conducting sphere in electrostatic equilibrium is a spherical equipotential surface, we should expect that we could replace one of the surfaces in Example 7.19 with a conducting sphere and have an identical solution outside the sphere. Inside will be rather different, however.

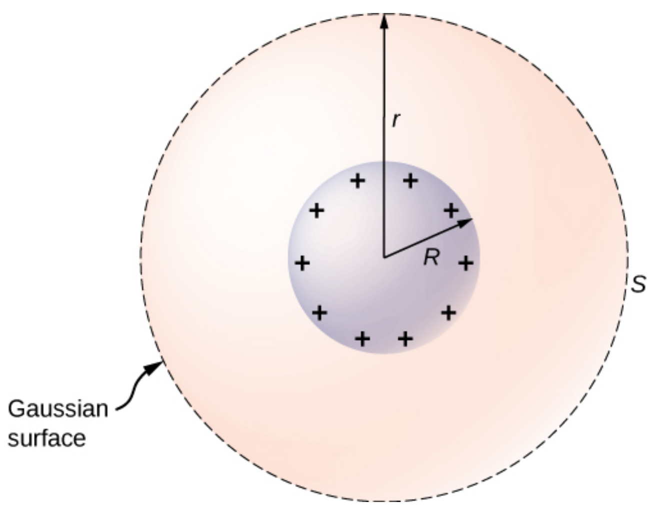


Figure 7.38 An isolated conducting sphere.

To investigate this, consider the isolated conducting sphere of Figure 7.38 that has a radius R and an excess charge q. To find the electric field both inside and outside the sphere, note that the sphere is isolated, so its surface change distribution and the electric field of that distribution are spherically symmetric. We can therefore represent the field as $\vec{\mathbf{E}} = E(r)\hat{\mathbf{r}}$. To calculate $E(r)$, we apply Gauss's law over a closed spherical surface S of radius r that is concentric with the conducting sphere. Since r is constant and $\hat{\mathbf{n}} = \hat{\mathbf{r}}$ on the sphere,

$$\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, da = E(r) \oint da = E(r)\, 4\pi r^2.$$

For $r < R$, S is within the conductor, so recall from our previous study of Gauss's law that $q_{\text{enc}} = 0$ and Gauss's law gives $E(r) = 0$, as expected inside a conductor at equilibrium. If $r > R$, S encloses the conductor so $q_{\text{enc}} = q$. From Gauss's law,

$$E(r)\, 4\pi r^2 = \frac{q}{\varepsilon_0}.$$

The electric field of the sphere may therefore be written as

$$\begin{aligned} E &= 0 & (r < R), \\ E &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} & (r \geq R). \end{aligned}$$

As expected, in the region $r \geq R$, the electric field due to a charge q placed on an isolated conducting sphere of radius R is identical to the electric field of a point charge q located at the center of the sphere.

To find the electric potential inside and outside the sphere, note that for $r \geq R$, the potential must be the same as that of an isolated point charge q located at $r = 0$,

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\,(r \geq R)$$

simply due to the similarity of the electric field.

For $r < R$, $E = 0$, so $V(r)$ is constant in this region. Since $V(R) = q/4\pi\varepsilon_0 R$,

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}\,(r < R).$$

We will use this result to show that

$$\sigma_1 R_1 = \sigma_2 R_2,$$

for two conducting spheres of radii R_1 and R_2, with surface charge densities σ_1 and σ_2 respectively, that are connected by a thin wire, as shown in Figure 7.39. The spheres are sufficiently separated so that each can be treated as if it were isolated (aside from the wire). Note that the connection by the wire means that this entire system must be an equipotential.

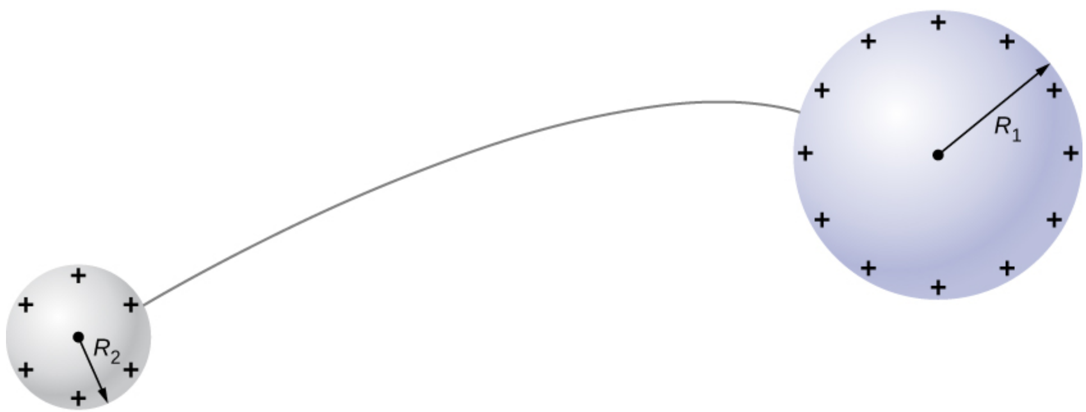


Figure 7.39 Two conducting spheres are connected by a thin conducting wire.

We have just seen that the electrical potential at the surface of an isolated, charged conducting sphere of radius R is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}.$$

Now, the spheres are connected by a conductor and are therefore at the same potential; hence

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_2},$$

and

$$\frac{q_1}{R_1} = \frac{q_2}{R_2}.$$

The net charge on a conducting sphere and its surface charge density are related by $q = \sigma(4\pi R^2)$. Substituting this equation into the previous one, we find

$$\sigma_1 R_1 = \sigma_2 R_2.$$

Obviously, two spheres connected by a thin wire do not constitute a typical conductor with a variable radius of curvature. Nevertheless, this result does at least provide a qualitative idea of how charge density varies over the surface of a conductor. The equation indicates that where the radius of curvature is large (points B and D in

Figure 7.40), σ and E are small.

Similarly, the charges tend to be denser where the curvature of the surface is greater, as demonstrated by the charge distribution on oddly shaped metal (Figure 7.40). The surface charge density is higher at locations with a small radius of curvature than at locations with a large radius of curvature.

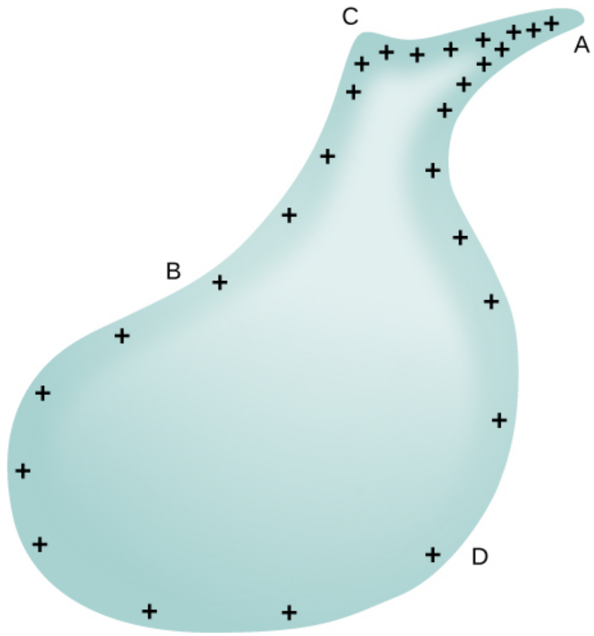


Figure 7.40 The surface charge density and the electric field of a conductor are greater at regions with smaller radii of curvature.

A practical application of this phenomenon is the lightning rod, which is simply a grounded metal rod with a sharp end pointing upward. As positive charge accumulates in the ground due to a negatively charged cloud overhead, the electric field around the sharp point gets very large. When the field reaches a value of approximately 3.0×10^6 N/C (the *dielectric strength* of the air), the free ions in the air are accelerated to such high energies that their collisions with air molecules actually ionize the molecules. The resulting free electrons in the air then flow through the rod to Earth, thereby neutralizing some of the positive charge. This keeps the electric field between the cloud and the ground from getting large enough to produce a lightning bolt in the region around the rod.

An important application of electric fields and equipotential lines involves the heart. The heart relies on electrical signals to maintain its rhythm. The movement of electrical signals causes the chambers of the heart to contract and relax. When a person has a heart attack, the movement of these electrical signals may be disturbed. An artificial pacemaker and a defibrillator can be used to initiate the rhythm of electrical signals. The equipotential lines around the heart, the thoracic region, and the axis of the heart are useful ways of monitoring the structure and functions of the heart. An electrocardiogram (ECG) measures the small electric signals being generated during the activity of the heart.

INTERACTIVE

Play around with this simulation (https://openstax.org/l/21pointcharsim) to move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more.

7.6 Applications of Electrostatics

Learning Objectives

By the end of this section, you will be able to:

- Describe some of the many practical applications of electrostatics, including several printing technologies
- Relate these applications to Newton's second law and the electric force

The study of electrostatics has proven useful in many areas. This module covers just a few of the many applications of electrostatics.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. Figure 7.41 shows a schematic of a large research version. Van de Graaffs use both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.

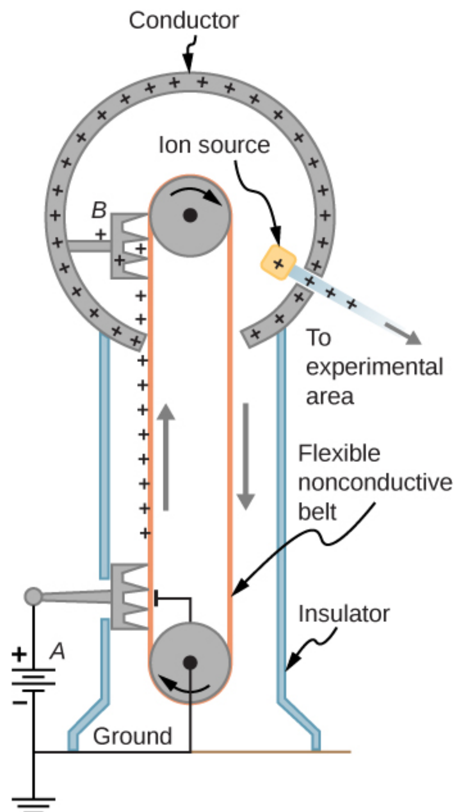


Figure 7.41 Schematic of Van de Graaff generator. A battery (*A*) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (*B*) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

Xerography

Most copy machines use an electrostatic process called **xerography**—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in Figure 7.42.

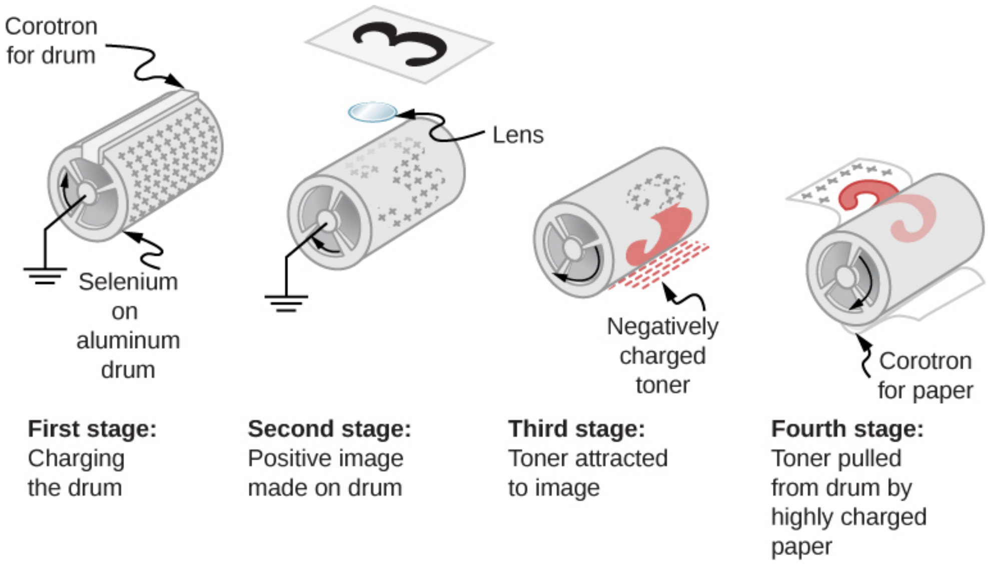


Figure 7.42 Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image, creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a **photoconductor**. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is grounded so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. In locations where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, so the image has been transferred to the drum.

The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it is attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner to the fibers of the paper.

Laser Printers

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in Figure 7.43. In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and in the past may have contained a computer more powerful than the one giving them the raw data to be printed.

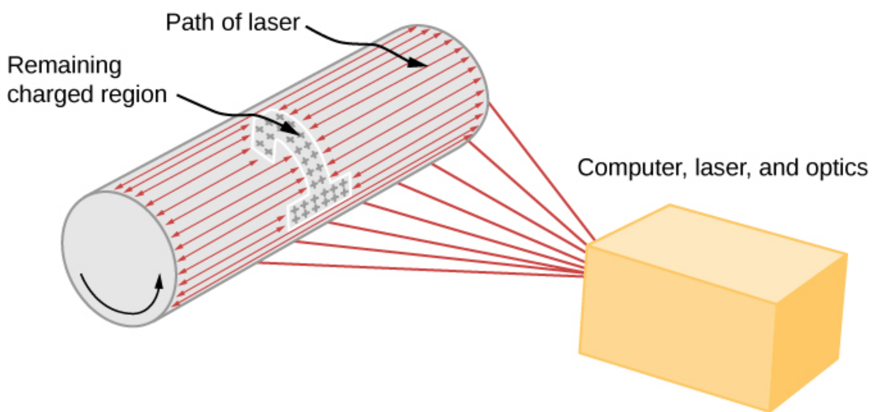


Figure 7.43 In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positively charged image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.

Ink Jet Printers and Electrostatic Painting

The **ink jet printer**, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge (Figure 7.44).

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)

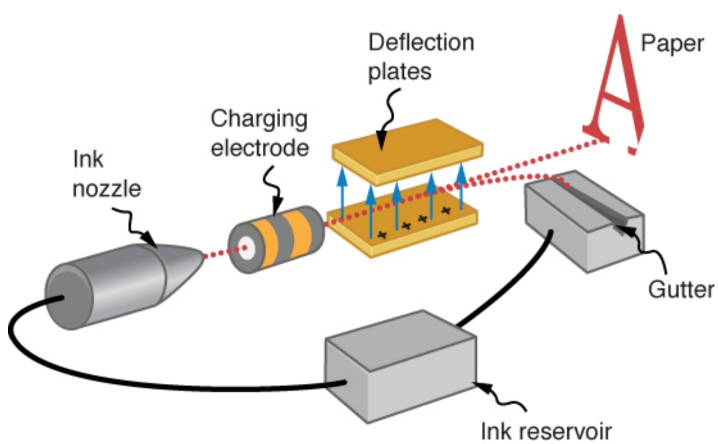


Figure 7.44 The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page.

Electrostatic painting employs electrostatic charge to spray paint onto oddly shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach hard-to-get-to places, applying an even coat in a controlled manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

Smoke Precipitators and Electrostatic Air Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles (Figure 7.45)

Large **electrostatic precipitators** are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.

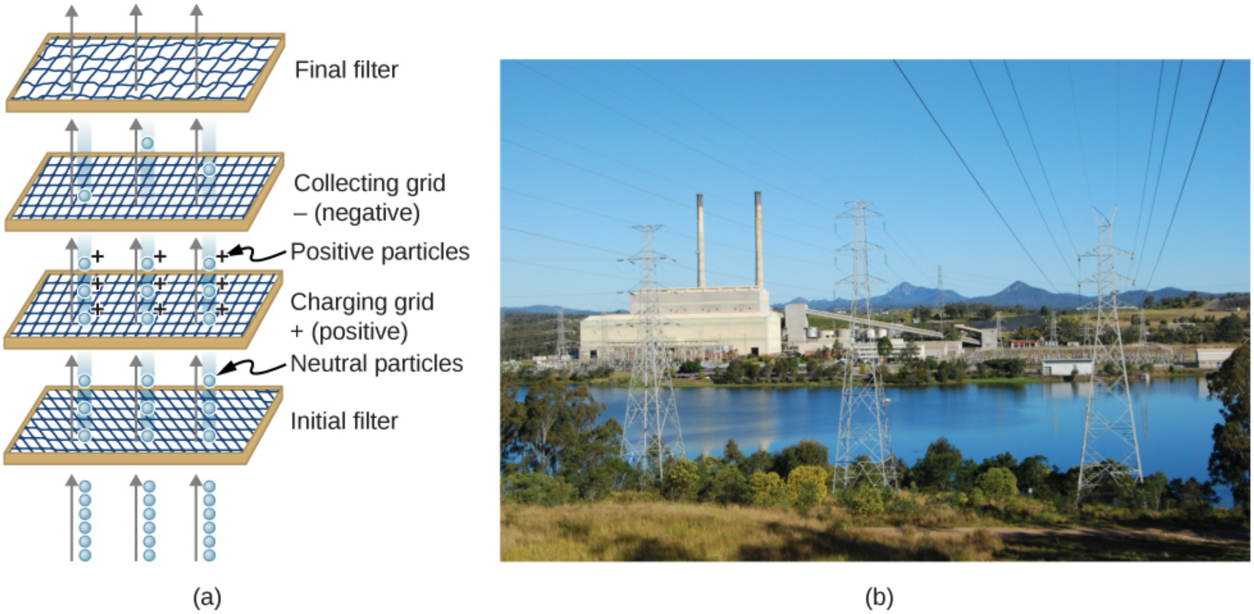


Figure 7.45 (a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of electrostatic precipitators is seen by the absence of smoke from this power plant. (credit b: modification of work by “Cmdalgleish”/Wikimedia Commons)

CHAPTER REVIEW

Key Terms

electric dipole system of two equal but opposite charges a fixed distance apart

electric dipole moment quantity defined as $\vec{\mathbf{p}} = q\vec{\mathbf{d}}$ for all dipoles, where the vector points from the negative to positive charge

electric potential potential energy per unit charge

electric potential difference the change in potential energy of a charge q moved between two points, divided by the charge.

electric potential energy potential energy stored in a system of charged objects due to the charges

electron-volt energy given to a fundamental charge accelerated through a potential difference of one volt

electrostatic precipitators filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

equipotential line two-dimensional representation of an equipotential surface

equipotential surface surface (usually in three dimensions) on which all points are at the same potential

grounding process of attaching a conductor to the earth to ensure that there is no potential difference between it and Earth

ink jet printer small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

photoconductor substance that is an insulator until it is exposed to light, when it becomes a conductor

Van de Graaff generator machine that produces a large amount of excess charge, used for experiments with high voltage

voltage change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

xerography dry copying process based on electrostatics

Key Equations

Potential energy of a two-charge system	$U(r) = k\frac{qQ}{r}$
Work done to assemble a system of charges	$W_{12\cdots N} = \frac{k}{2}\sum_{i}^{N}\sum_{j}^{N}\frac{q_i q_j}{r_{ij}}$ for $i \neq j$
Potential difference	$\Delta V = \frac{\Delta U}{q}$ or $\Delta U = q\Delta V$
Electric potential	$V = \frac{U}{q} = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$
Potential difference between two points	$\Delta V_{BA} = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$
Electric potential of a point charge	$V = \frac{kq}{r}$
Electric potential of a system of point charges	$V_P = k\sum_1^N \frac{q_i}{r_i}$
Electric dipole moment	$\vec{\mathbf{p}} = q\vec{\mathbf{d}}$
Electric potential due to a dipole	$V_P = k\frac{\vec{\mathbf{p}}\cdot\hat{\mathbf{r}}}{r^2}$

Electric potential of a continuous charge distribution	$V_P = k\int \frac{dq}{r}$
Electric field components	$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$
Del operator in Cartesian coordinates	$\vec{\nabla} = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}$
Electric field as gradient of potential	$\vec{\mathbf{E}} = -\vec{\nabla}V$
Del operator in cylindrical coordinates	$\vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial \varphi} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$
Del operator in spherical coordinates	$\vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}$

Summary

7.1 Electric Potential Energy

- The work done to move a charge from point A to B in an electric field is path independent, and the work around a closed path is zero. Therefore, the electric field and electric force are conservative.
- We can define an electric potential energy, which between point charges is $U(r) = k\frac{qQ}{r}$, with the zero reference taken to be at infinity.
- The superposition principle holds for electric potential energy; the potential energy of a system of multiple charges is the sum of the potential energies of the individual pairs.

7.2 Electric Potential and Potential Difference

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_B - V_A$, that is, the change in potential of a charge q moved from A to B, is equal to the change in potential energy divided by the charge.
- Potential difference is commonly called voltage, represented by the symbol ΔV: $\Delta V = \frac{\Delta U}{q}$ or $\Delta U = q\Delta V$.
- An electron-volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form, $1\text{ eV} = \left(1.60 \times 10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60 \times 10^{-19}\text{ C}\right)(1\text{ J/C}) = 1.60 \times 10^{-19}\text{ J}$.

7.3 Calculations of Electric Potential

- Electric potential is a scalar whereas electric field is a vector.
- Addition of voltages as numbers gives the voltage due to a combination of point charges, allowing us to use the principle of superposition: $V_P = k\sum_1^N \frac{q_i}{r_i}$.
- An electric dipole consists of two equal and opposite charges a fixed distance apart, with a dipole moment $\vec{\mathbf{p}} = q\vec{\mathbf{d}}$.
- Continuous charge distributions may be calculated with $V_P = k\int \frac{dq}{r}$.

7.4 Determining Field from Potential

- Just as we may integrate over the electric field to calculate the potential, we may take the derivative of the potential to calculate the electric field.
- This may be done for individual components of the electric field, or we may calculate the entire electric field vector with the gradient operator.

7.5 Equipotential Surfaces and Conductors

- An equipotential surface is the collection of points in space that are all at the same potential. Equipotential lines are the two-dimensional representation of equipotential surfaces.
- Equipotential surfaces are always perpendicular to electric field lines.
- Conductors in static equilibrium are equipotential surfaces.
- Topographic maps may be thought of as showing gravitational equipotential lines.