CHAPTER 6
Gauss's Law

Figure 6.1 This chapter introduces the concept of flux, which relates a physical quantity and the area through which it is flowing. Although we introduce this concept with the electric field, the concept may be used for many other quantities, such as fluid flow. (credit: modification of work by "Alessandro"/Flickr)

Chapter Outline

INTRODUCTION Flux is a general and broadly applicable concept in physics. However, in this chapter, we concentrate on the flux of the electric field. This allows us to introduce Gauss's law, which is particularly useful for finding the electric fields of charge distributions exhibiting spatial symmetry. The main topics discussed here are

1. **Electric flux.** We define electric flux for both open and closed surfaces.
2. **Gauss's law.** We derive Gauss's law for an arbitrary charge distribution and examine the role of electric flux in Gauss's law.
3. **Calculating electric fields with Gauss's law.** The main focus of this chapter is to explain how to use Gauss's law to find the electric fields of spatially symmetrical charge distributions. We discuss the importance of choosing a Gaussian surface and provide examples involving the applications of Gauss's law.

4. **Electric fields in conductors.** Gauss's law provides useful insight into the absence of electric fields in conducting materials.

So far, we have found that the electrostatic field begins and ends at point charges and that the field of a point charge varies inversely with the square of the distance from that charge. These characteristics of the electrostatic field lead to an important mathematical relationship known as Gauss's law. This law is named in honor of the extraordinary German mathematician and scientist Karl Friedrich Gauss (Figure 6.2). Gauss's law gives us an elegantly simple way of finding the electric field, and, as you will see, it can be much easier to use than the integration method described in the previous chapter. However, there is a catch—Gauss's law has a limitation in that, while always true, it can be readily applied only for charge distributions with certain symmetries.

Figure 6.2 Karl Friedrich Gauss (1777–1855) was a legendary mathematician of the nineteenth century. Although his major contributions were to the field of mathematics, he also did important work in physics and astronomy.

6.1 Electric Flux

Learning Objectives

By the end of this section, you will be able to:

- Define the concept of flux
- Describe electric flux
- Calculate electric flux for a given situation

The concept of **flux** describes how much of something goes through a given area. More formally, it is the dot product of a vector field (in this chapter, the electric field) with an area. You may conceptualize the flux of an electric field as a measure of the number of electric field lines passing through an area (Figure 6.3). The larger the area, the more field lines go through it and, hence, the greater the flux; similarly, the stronger the electric field is (represented by a greater density of lines), the greater the flux. On the other hand, if the area rotated so that the plane is aligned with the field lines, none will pass through and there will be no flux.

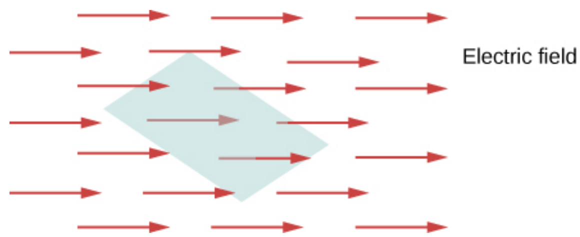

Figure 6.3 The flux of an electric field through the shaded area captures information about the “number” of electric field lines passing through the area. The numerical value of the electric flux depends on the magnitudes of the electric field and the area, as well as the relative orientation of the area with respect to the direction of the electric field.

A macroscopic analogy that might help you imagine this is to put a hula hoop in a flowing river. As you change the angle of the hoop relative to the direction of the current, more or less of the flow will go through the hoop. Similarly, the amount of flow through the hoop depends on the strength of the current and the size of the hoop. Again, flux is a general concept; we can also use it to describe the amount of sunlight hitting a solar panel or the amount of energy a telescope receives from a distant star, for example.

To quantify this idea, Figure 6.4(a) shows a planar surface S_1 of area A_1 that is perpendicular to the uniform electric field $\vec{\mathbf{E}} = E\hat{\mathbf{y}}$. If N field lines pass through S_1, then we know from the definition of electric field lines (Electric Charges and Fields) that $N/A_1 \propto E$, or $N \propto EA_1$.

The quantity EA_1 is the **electric flux** through S_1. We represent the electric flux through an open surface like S_1 by the symbol Φ. Electric flux is a scalar quantity and has an SI unit of newton-meters squared per coulomb ($\text{N} \cdot \text{m}^2/\text{C}$). Notice that $N \propto EA_1$ may also be written as $N \propto \Phi$, demonstrating that *electric flux is a measure of the number of field lines crossing a surface*.

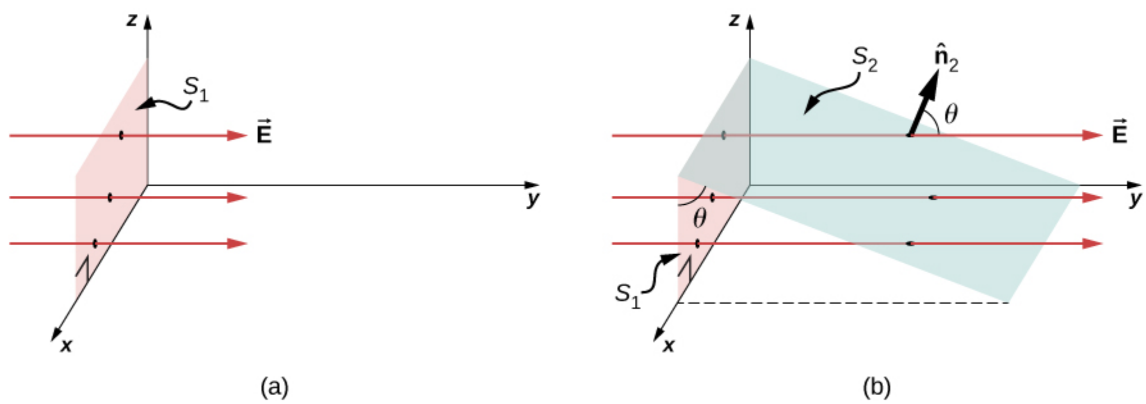

Figure 6.4 (a) A planar surface S_1 of area A_1 is perpendicular to the electric field $E\hat{\mathbf{j}}$. *N* field lines cross surface S_1. (b) A surface S_2 of area A_2 whose projection onto the *xz*-plane is S_1.The same number of field lines cross each surface.

Now consider a planar surface that is not perpendicular to the field. How would we represent the electric flux? Figure 6.4(b) shows a surface S_2 of area A_2 that is inclined at an angle θ to the *xz*-plane and whose projection in that plane is S_1 (area A_1). The areas are related by $A_2 \cos\theta = A_1$. Because the same number of field lines crosses both S_1 and S_2, the fluxes through both surfaces must be the same. The flux through S_2 is therefore $\Phi = EA_1 = EA_2 \cos\theta$. Designating $\hat{\mathbf{n}}_2$ as a unit vector normal to S_2 (see Figure 6.4(b)), we obtain

$$\Phi = \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}_2 A_2.$$

INTERACTIVE

Check out this video (https://openstax.org/l/21fluxsizeangl) to observe what happens to the flux as the area changes in size and angle, or the electric field changes in strength.

Area Vector

For discussing the flux of a vector field, it is helpful to introduce an area vector $\vec{\mathbf{A}}$. This allows us to write the last equation in a more compact form. What should the magnitude of the area vector be? What should the direction of the area vector be? What are the implications of how you answer the previous question?

The **area vector** of a flat surface of area A has the following magnitude and direction:

- Magnitude is equal to area (A)
- Direction is along the normal to the surface ($\hat{\mathbf{n}}$); that is, perpendicular to the surface.

Since the normal to a flat surface can point in either direction from the surface, the direction of the area vector of an open surface needs to be chosen, as shown in Figure 6.5.

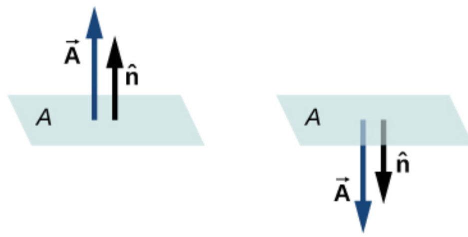

Figure 6.5 The direction of the area vector of an open surface needs to be chosen; it could be either of the two cases displayed here. The area vector of a part of a closed surface is defined to point from the inside of the closed space to the outside. This rule gives a unique direction.

Since $\hat{\mathbf{n}}$ is a unit normal to a surface, it has two possible directions at every point on that surface (Figure 6.6(a)). For an open surface, we can use either direction, as long as we are consistent over the entire surface. Part (c) of the figure shows several cases.

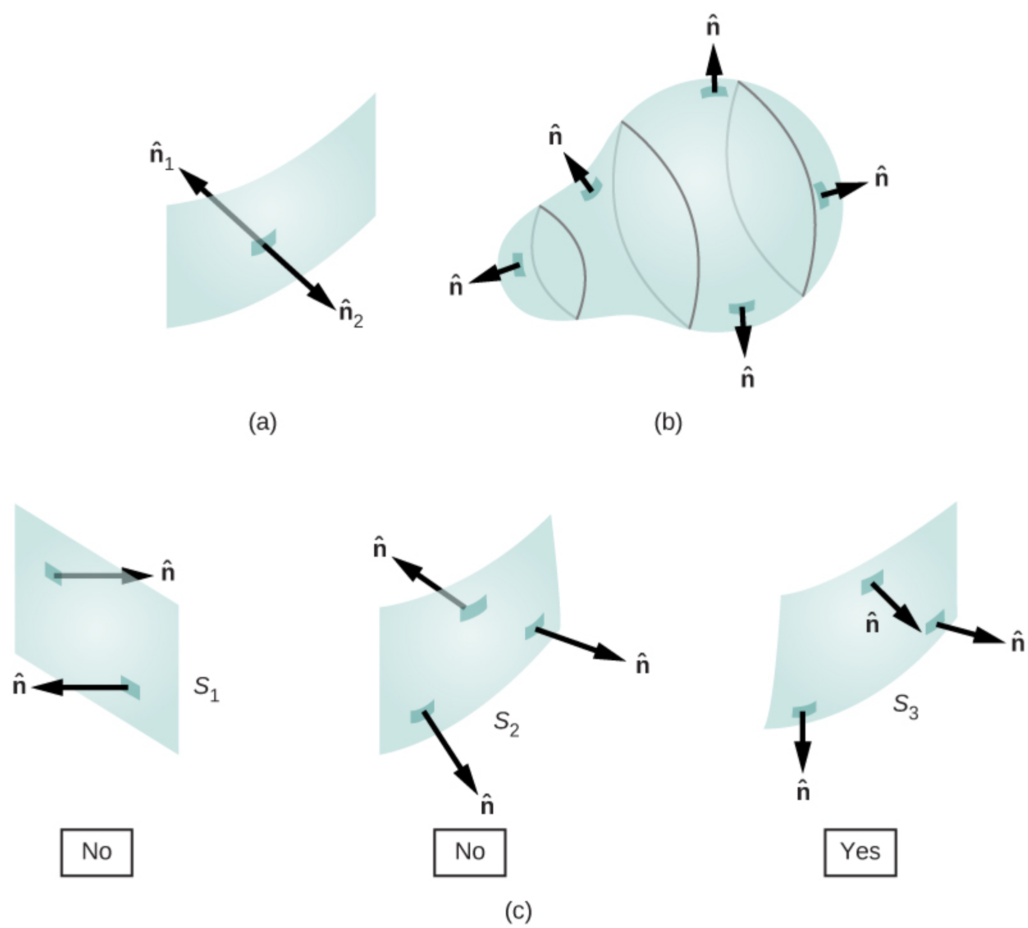

Figure 6.6 (a) Two potential normal vectors arise at every point on a surface. (b) The outward normal is used to calculate the flux through a closed surface. (c) Only S_3 has been given a consistent set of normal vectors that allows us to define the flux through the surface.

However, if a surface is closed, then the surface encloses a volume. In that case, the direction of the normal vector at any point on the surface points from the inside to the outside. On a *closed surface* such as that of Figure 6.6(b), $\hat{\mathbf{n}}$ is chosen to be the *outward normal* at every point, to be consistent with the sign convention for electric charge.

Electric Flux

Now that we have defined the area vector of a surface, we can define the electric flux of a uniform electric field through a flat area as the scalar product of the electric field and the area vector, as defined in Products of Vectors:

$$\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} \text{ (uniform } \vec{\mathbf{E}}\text{, flat surface).} \tag{6.1}$$

Figure 6.7 shows the electric field of an oppositely charged, parallel-plate system and an imaginary box between the plates. The electric field between the plates is uniform and points from the positive plate toward the negative plate. A calculation of the flux of this field through various faces of the box shows that the net flux through the box is zero. Why does the flux cancel out here?

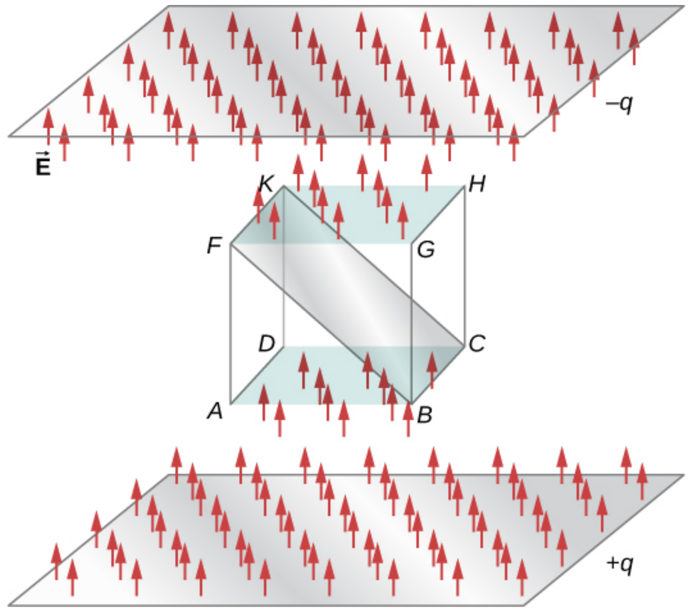

Figure 6.7 Electric flux through a cube, placed between two charged plates. Electric flux through the bottom face (*ABCD*) is negative, because $\vec{\mathbf{E}}$ is in the opposite direction to the normal to the surface. The electric flux through the top face (*FGHK*) is positive, because the electric field and the normal are in the same direction. The electric flux through the other faces is zero, since the electric field is perpendicular to the normal vectors of those faces. The net electric flux through the cube is the sum of fluxes through the six faces. Here, the net flux through the cube is equal to zero. The magnitude of the flux through rectangle *BCKF* is equal to the magnitudes of the flux through both the top and bottom faces.

The reason is that the sources of the electric field are outside the box. Therefore, if any electric field line enters the volume of the box, it must also exit somewhere on the surface because there is no charge inside for the lines to land on. Therefore, quite generally, electric flux through a closed surface is zero if there are no sources of electric field, whether positive or negative charges, inside the enclosed volume. In general, when field lines leave (or “flow out of”) a closed surface, Φ is positive; when they enter (or “flow into”) the surface, Φ is negative.

Any smooth, non-flat surface can be replaced by a collection of tiny, approximately flat surfaces, as shown in Figure 6.8. If we divide a surface S into small patches, then we notice that, as the patches become smaller, they can be approximated by flat surfaces. This is similar to the way we treat the surface of Earth as locally flat, even though we know that globally, it is approximately spherical.

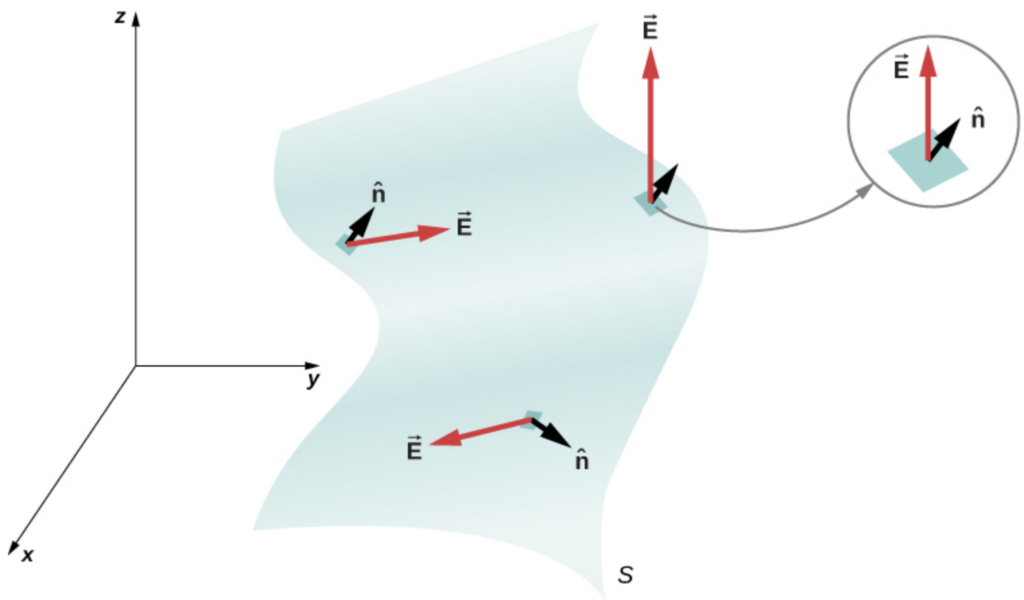

Figure 6.8 A surface is divided into patches to find the flux.

To keep track of the patches, we can number them from 1 through N. Now, we define the area vector for each patch as the area of the patch pointed in the direction of the normal. Let us denote the area vector for the ith patch by $\delta \vec{\mathbf{A}}_i$. (We have used the symbol δ to remind us that the area is of an arbitrarily small patch.) With sufficiently small patches, we may approximate the electric field over any given patch as uniform. Let us denote the average electric field at the location of the ith patch by $\vec{\mathbf{E}}_i$.

$$\vec{\mathbf{E}}_i = \text{average electric field over the } i\text{th patch.}$$

Therefore, we can write the electric flux Φ_i through the area of the ith patch as

$$\Phi_i = \vec{\mathbf{E}}_i \cdot \delta \vec{\mathbf{A}}_i \ (i\text{th patch}).$$

The flux through each of the individual patches can be constructed in this manner and then added to give us an estimate of the net flux through the entire surface S, which we denote simply as Φ.

$$\Phi = \sum_{i=1}^{N} \Phi_i = \sum_{i=1}^{N} \vec{\mathbf{E}}_i \cdot \delta \vec{\mathbf{A}}_i \ (N \text{ patch estimate}).$$

This estimate of the flux gets better as we decrease the size of the patches. However, when you use smaller patches, you need more of them to cover the same surface. In the limit of infinitesimally small patches, they may be considered to have area dA and unit normal $\hat{\mathbf{n}}$. Since the elements are infinitesimal, they may be assumed to be planar, and $\vec{\mathbf{E}}_i$ may be taken as constant over any element. Then the flux $d\Phi$ through an area dA is given by $d\Phi = \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA$. It is positive when the angle between $\vec{\mathbf{E}}_i$ and $\hat{\mathbf{n}}$ is less than $90°$ and negative when the angle is greater than $90°$. The net flux is the sum of the infinitesimal flux elements over the entire surface. With infinitesimally small patches, you need infinitely many patches, and the limit of the sum becomes a surface integral. With $\int_S$ representing the integral over S,

$$\Phi = \int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} \ (\text{open surface}). \tag{6.2}$$

In practical terms, surface integrals are computed by taking the antiderivatives of both dimensions defining the area, with the edges of the surface in question being the bounds of the integral.

To distinguish between the flux through an open surface like that of Figure 6.4 and the flux through a closed surface (one that completely bounds some volume), we represent flux through a closed surface by

$$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} \text{ (closed surface)} \tag{6.3}$$

where the circle through the integral symbol simply means that the surface is closed, and we are integrating over the entire thing. If you only integrate over a portion of a closed surface, that means you are treating a subset of it as an open surface.

EXAMPLE 6.1

Flux of a Uniform Electric Field

A constant electric field of magnitude E_0 points in the direction of the positive z-axis (Figure 6.9). What is the electric flux through a rectangle with sides a and b in the (a) xy-plane and in the (b) xz-plane?

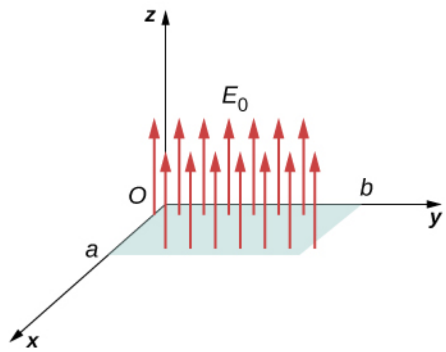

Figure 6.9 Calculating the flux of E_0 through a rectangular surface.

Strategy

Apply the definition of flux: $\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}$ (uniform $\vec{\mathbf{E}}$), where the definition of dot product is crucial.

Solution

a. In this case, $\Phi = \vec{\mathbf{E}}_0 \cdot \vec{\mathbf{A}} = E_0 A = E_0 ab$.
b. Here, the direction of the area vector is either along the positive y-axis or toward the negative y-axis. Therefore, the scalar product of the electric field with the area vector is zero, giving zero flux.

Significance

The relative directions of the electric field and area can cause the flux through the area to be zero.

EXAMPLE 6.2

Flux of a Uniform Electric Field through a Closed Surface

A constant electric field of magnitude E_0 points in the direction of the positive z-axis (Figure 6.10). What is the net electric flux through a cube?

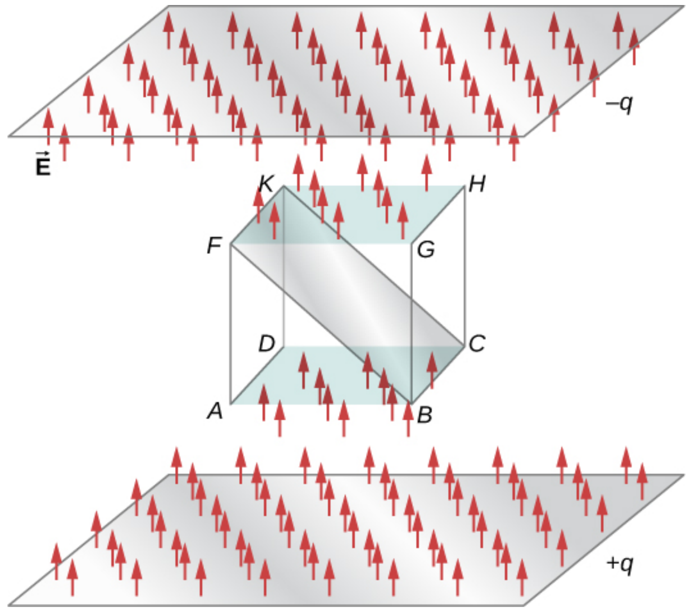

Figure 6.10 Calculating the flux of E_0 through a closed cubic surface.

Strategy

Apply the definition of flux: $\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}$ (uniform $\vec{\mathbf{E}}$), noting that a closed surface eliminates the ambiguity in the direction of the area vector.

Solution

Through the top face of the cube, $\Phi = \vec{\mathbf{E}}_0 \cdot \vec{\mathbf{A}} = E_0 A$.

Through the bottom face of the cube, $\Phi = \vec{\mathbf{E}}_0 \cdot \vec{\mathbf{A}} = -E_0 A$, because the area vector here points downward.

Along the other four sides, the direction of the area vector is perpendicular to the direction of the electric field. Therefore, the scalar product of the electric field with the area vector is zero, giving zero flux.

The net flux is $\Phi_{\text{net}} = E_0 A - E_0 A + 0 + 0 + 0 + 0 = 0$.

Significance

The net flux of a uniform electric field through a closed surface is zero.

EXAMPLE 6.3

Electric Flux through a Plane, Integral Method

A uniform electric field $\vec{\mathbf{E}}$ of magnitude 10 N/C is directed parallel to the yz-plane at 30° above the xy-plane, as shown in Figure 6.11. What is the electric flux through the plane surface of area 6.0 m^2 located in the xz-plane? Assume that $\hat{\mathbf{n}}$ points in the positive y-direction.

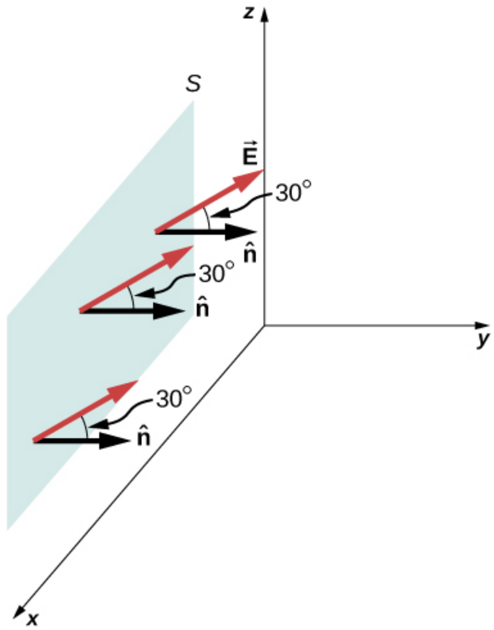

Figure 6.11 The electric field produces a net electric flux through the surface *S*.

Strategy

Apply $\Phi = \int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA$, where the direction and magnitude of the electric field are constant.

Solution

The angle between the uniform electric field $\vec{\mathbf{E}}$ and the unit normal $\hat{\mathbf{n}}$ to the planar surface is $30°$. Since both the direction and magnitude are constant, E comes outside the integral. All that is left is a surface integral over dA, which is A. Therefore, using the open-surface equation, we find that the electric flux through the surface is

$$\begin{aligned}\Phi &= \int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = EA\cos\theta \\ &= (10\text{ N/C})(6.0\text{ m}^2)(\cos 30°) = 52\text{ N}\cdot\text{m}^2/\text{C}.\end{aligned}$$

Significance

Again, the relative directions of the field and the area matter, and the general equation with the integral will simplify to the simple dot product of area and electric field.

CHECK YOUR UNDERSTANDING 6.1

What angle should there be between the electric field and the surface shown in Figure 6.11 in the previous example so that no electric flux passes through the surface?

EXAMPLE 6.4

Inhomogeneous Electric Field

What is the total flux of the electric field $\vec{\mathbf{E}} = cy^2\hat{\mathbf{k}}$ through the rectangular surface shown in Figure 6.12?

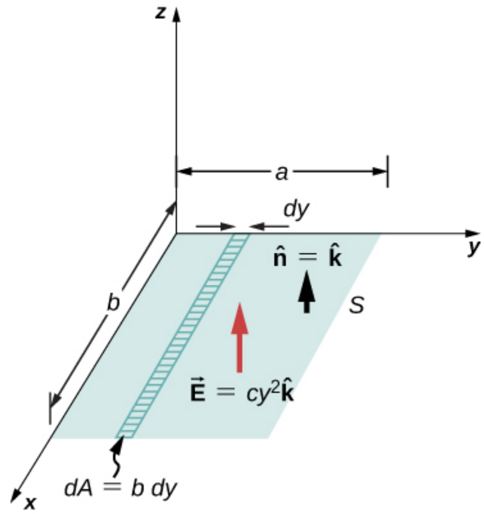

Figure 6.12 Since the electric field is not constant over the surface, an integration is necessary to determine the flux.

Strategy

Apply $\Phi = \int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA$. We assume that the unit normal $\hat{\mathbf{n}}$ to the given surface points in the positive z-direction, so $\hat{\mathbf{n}} = \hat{\mathbf{k}}$. Since the electric field is not uniform over the surface, it is necessary to divide the surface into infinitesimal strips along which $\vec{\mathbf{E}}$ is essentially constant. As shown in Figure 6.12, these strips are parallel to the x-axis, and each strip has an area $dA = b\, dy$.

Solution

From the open surface integral, we find that the net flux through the rectangular surface is

$$\begin{aligned}\Phi &= \int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \int_0^a (cy^2 \hat{\mathbf{k}}) \cdot \hat{\mathbf{k}}(b\, dy) \\ &= cb \int_0^a y^2\, dy = \frac{1}{3} a^3 bc.\end{aligned}$$

Significance

For a non-constant electric field, the integral method is required.

CHECK YOUR UNDERSTANDING 6.2

If the electric field in Example 6.4 is $\vec{\mathbf{E}} = mx\hat{\mathbf{k}}$, what is the flux through the rectangular area?

6.2 Explaining Gauss's Law

Learning Objectives

By the end of this section, you will be able to:

- State Gauss's law
- Explain the conditions under which Gauss's law may be used
- Apply Gauss's law in appropriate systems

We can now determine the electric flux through an arbitrary closed surface due to an arbitrary charge distribution. We found that if a closed surface does not have any charge inside where an electric field line can terminate, then any electric field line entering the surface at one point must necessarily exit at some other point of the surface. Therefore, if a closed surface does not have any charges inside the enclosed volume, then the electric flux through the surface is zero. Now, what happens to the electric flux if there are some charges inside the enclosed volume? Gauss's law gives a quantitative answer to this question.

To get a feel for what to expect, let's calculate the electric flux through a spherical surface around a positive point charge q, since we already know the electric field in such a situation. Recall that when we place the point charge at the origin of a coordinate system, the electric field at a point P that is at a distance r from the charge at the origin is given by

$$\vec{\mathbf{E}}_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}},$$

where $\hat{\mathbf{r}}$ is the radial vector from the charge at the origin to the point P. We can use this electric field to find the flux through the spherical surface of radius r, as shown in Figure 6.13.

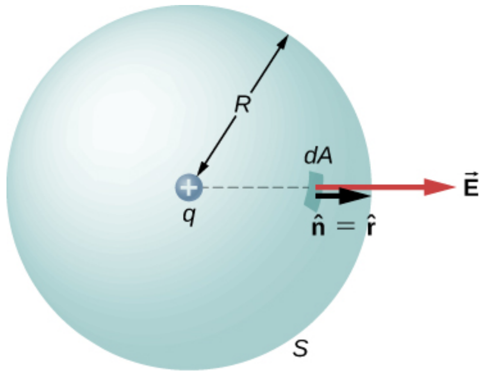

Figure 6.13 A closed spherical surface surrounding a point charge q.

Then we apply $\Phi = \int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA$ to this system and substitute known values. On the sphere, $\hat{\mathbf{n}} = \hat{\mathbf{r}}$ and $r = R$, so for an infinitesimal area dA,

$$d\Phi = \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}\, dA = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\, dA.$$

We now find the net flux by integrating this flux over the surface of the sphere:

$$\Phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\oint_S dA = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}(4\pi R^2) = \frac{q}{\varepsilon_0}.$$

where the total surface area of the spherical surface is $4\pi R^2$. This gives the flux through the closed spherical surface at radius r as

$$\Phi = \frac{q}{\varepsilon_0}. \quad \textbf{6.4}$$

A remarkable fact about this equation is that the flux is independent of the size of the spherical surface. This can be directly attributed to the fact that the electric field of a point charge decreases as $1/r^2$ with distance, which just cancels the r^2 rate of increase of the surface area.

Electric Field Lines Picture

An alternative way to see why the flux through a closed spherical surface is independent of the radius of the surface is to look at the electric field lines. Note that every field line from q that pierces the surface at radius R_1 also pierces the surface at R_2 (Figure 6.14).

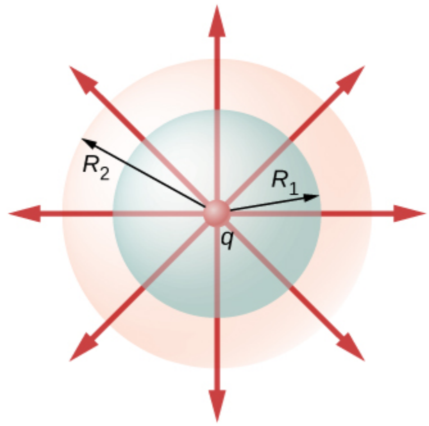

Figure 6.14 Flux through spherical surfaces of radii R_1 and R_2 enclosing a charge *q* are equal, independent of the size of the surface, since all *E*-field lines that pierce one surface from the inside to outside direction also pierce the other surface in the same direction.

Therefore, the net number of electric field lines passing through the two surfaces from the inside to outside direction is equal. This net number of electric field lines, which is obtained by subtracting the number of lines in the direction from outside to inside from the number of lines in the direction from inside to outside gives a visual measure of the electric flux through the surfaces.

You can see that if no charges are included within a closed surface, then the electric flux through it must be zero. A typical field line enters the surface at dA_1 and leaves at dA_2. Every line that enters the surface must also leave that surface. Hence the net "flow" of the field lines into or out of the surface is zero (Figure 6.15(a)). The same thing happens if charges of equal and opposite sign are included inside the closed surface, so that the total charge included is zero (part (b)). A surface that includes the same amount of charge has the same number of field lines crossing it, regardless of the shape or size of the surface, as long as the surface encloses the same amount of charge (part (c)).

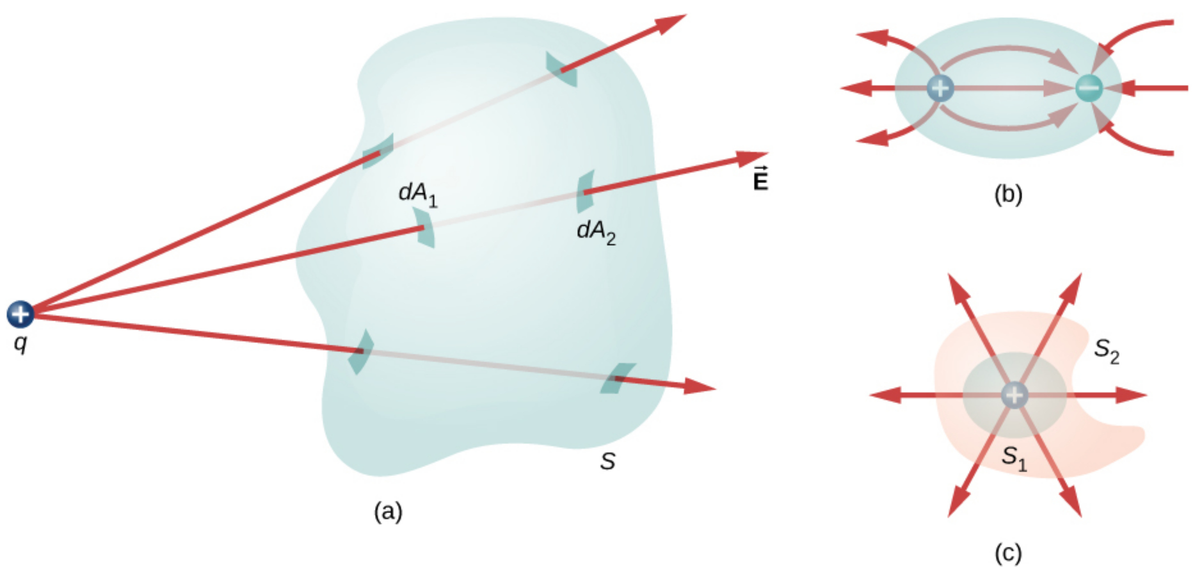

Figure 6.15 Understanding the flux in terms of field lines. (a) The electric flux through a closed surface due to a charge outside that surface is zero. (b) Charges are enclosed, but because the net charge included is zero, the net flux through the closed surface is also zero. (c) The shape and size of the surfaces that enclose a charge does not matter because all surfaces enclosing the same charge have the same flux.

Statement of Gauss's Law

Gauss's law generalizes this result to the case of any number of charges and any location of the charges in the space inside the closed surface. According to Gauss's law, the flux of the electric field $\vec{\mathbf{E}}$ through any closed surface, also called a **Gaussian surface**, is equal to the net charge enclosed (q_{enc}) divided by the permittivity of free space (ε_0):

$$\Phi_{\text{Closed Surface}} = \frac{q_{\text{enc}}}{\varepsilon_0}.$$

This equation holds for *charges of either sign*, because we define the area vector of a closed surface to point outward. If the enclosed charge is negative (see Figure 6.16(b)), then the flux through either S or S' is negative.

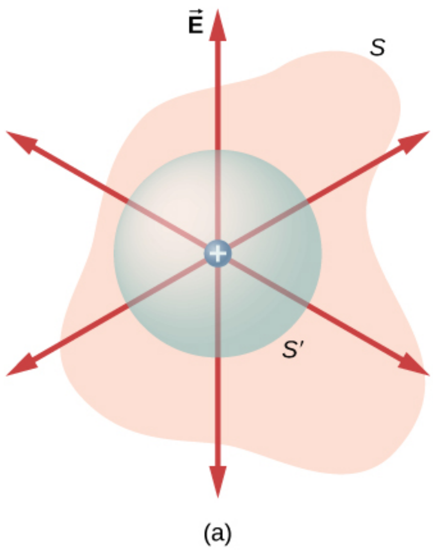

Figure 6.16 The electric flux through any closed surface surrounding a point charge q is given by Gauss's law. (a) Enclosed charge is positive. (b) Enclosed charge is negative.

The Gaussian surface does not need to correspond to a real, physical object; indeed, it rarely will. It is a mathematical construct that may be of any shape, provided that it is closed. However, since our goal is to integrate the flux over it, we tend to choose shapes that are highly symmetrical.

If the charges are discrete point charges, then we just add them. If the charge is described by a continuous distribution, then we need to integrate appropriately to find the total charge that resides inside the enclosed volume. For example, the flux through the Gaussian surface S of Figure 6.17 is $\Phi = (q_1 + q_2 + q_5)/\varepsilon_0$. Note that q_{enc} is simply the sum of the point charges. If the charge distribution were continuous, we would need to integrate appropriately to compute the total charge within the Gaussian surface.

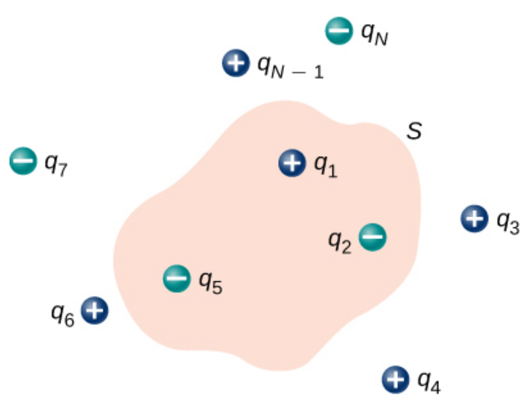

Figure 6.17 The flux through the Gaussian surface shown, due to the charge distribution, is $\Phi = |q_1| + |q_2| + |q_5|/\varepsilon_0$.

Recall that the principle of superposition holds for the electric field. Therefore, the total electric field at any point, including those on the chosen Gaussian surface, is the sum of all the electric fields present at this point. This allows us to write Gauss's law in terms of the total electric field.

Gauss's Law

The flux Φ of the electric field $\vec{\mathbf{E}}$ through any closed surface S (a Gaussian surface) is equal to the net charge enclosed (q_{enc}) divided by the permittivity of free space (ε_0) :

$$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \frac{q_{\text{enc}}}{\varepsilon_0}. \quad (6.5)$$

To use Gauss's law effectively, you must have a clear understanding of what each term in the equation represents. The field $\vec{\mathbf{E}}$ is the *total electric field* at every point on the Gaussian surface. This total field includes contributions from charges both inside and outside the Gaussian surface. However, q_{enc} is just the charge *inside* the Gaussian surface. Finally, the Gaussian surface is any closed surface in space. That surface can coincide with the actual surface of a conductor, or it can be an imaginary geometric surface. The only requirement imposed on a Gaussian surface is that it be closed (Figure 6.18).

Figure 6.18 A Klein bottle partially filled with a liquid. Could the Klein bottle be used as a Gaussian surface?

EXAMPLE 6.5

Electric Flux through Gaussian Surfaces

Calculate the electric flux through each Gaussian surface shown in Figure 6.19.

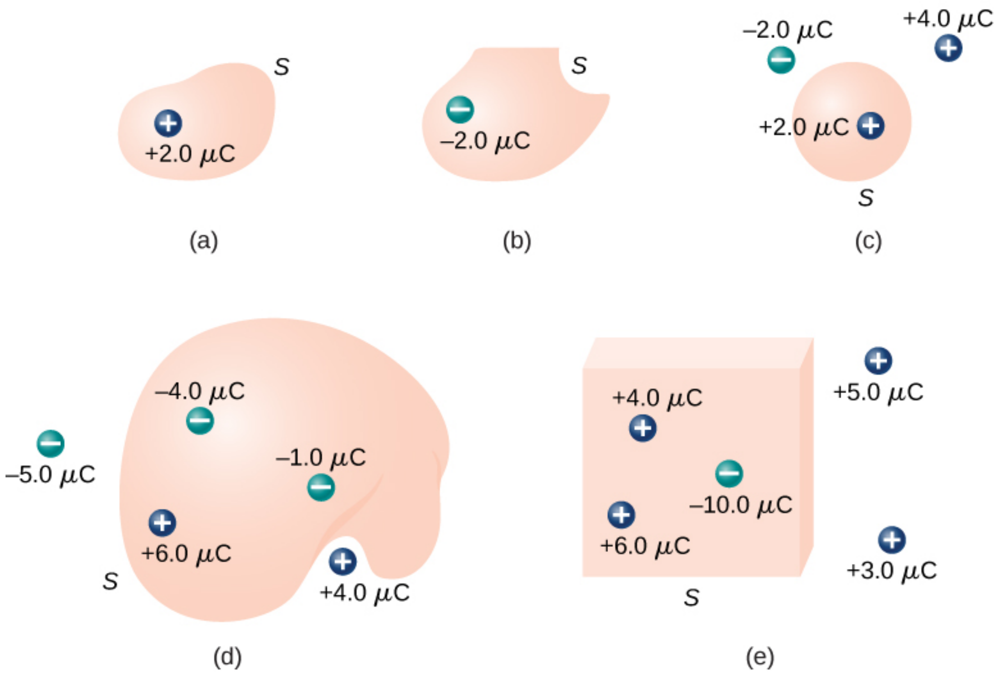

Figure 6.19 Various Gaussian surfaces and charges.

Strategy

From Gauss's law, the flux through each surface is given by $q_{\text{enc}}/\varepsilon_0$, where q_{enc} is the charge enclosed by that surface.

Solution

For the surfaces and charges shown, we find

a. $\Phi = \frac{2.0\ \mu\text{C}}{\varepsilon_0} = 2.3 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}.$

b. $\Phi = \frac{-2.0\ \mu\text{C}}{\varepsilon_0} = -2.3 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}.$

c. $\Phi = \frac{2.0\ \mu\text{C}}{\varepsilon_0} = 2.3 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}.$

d. $\Phi = \frac{-4.0\ \mu\text{C}+6.0\ \mu\text{C}-1.0\ \mu\text{C}}{\varepsilon_0} = 1.1 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}.$

e. $\Phi = \frac{4.0\ \mu\text{C}+6.0\ \mu\text{C}-10.0\ \mu\text{C}}{\varepsilon_0} = 0.$

Significance

In the special case of a closed surface, the flux calculations become a sum of charges. In the next section, this will allow us to work with more complex systems.

CHECK YOUR UNDERSTANDING 6.3

Calculate the electric flux through the closed cubical surface for each charge distribution shown in Figure 6.20.

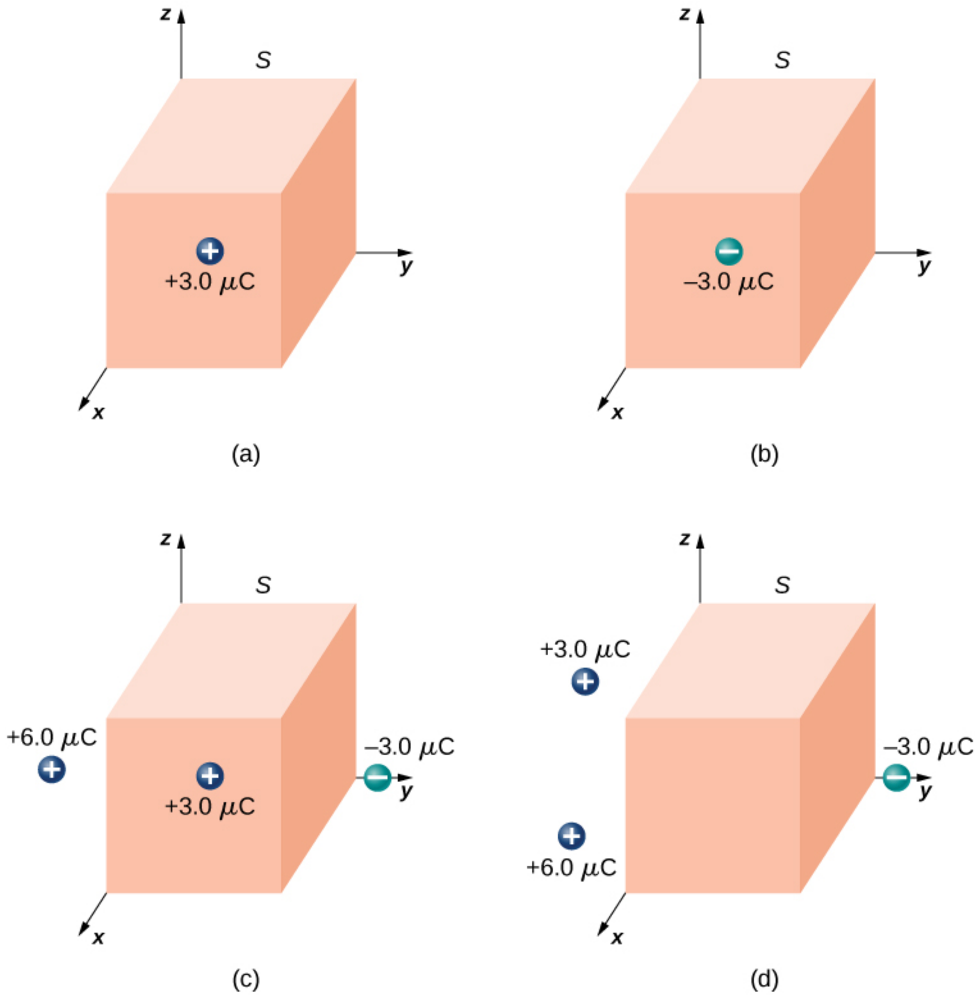

Figure 6.20 A cubical Gaussian surface with various charge distributions.

INTERACTIVE

Use this simulation (https://openstax.org/l/21gaussimulat) to adjust the magnitude of the charge and the radius of the Gaussian surface around it. See how this affects the total flux and the magnitude of the electric field at the Gaussian surface.

6.3 Applying Gauss's Law

Learning Objectives

By the end of this section, you will be able to:

- Explain what spherical, cylindrical, and planar symmetry are
- Recognize whether or not a given system possesses one of these symmetries
- Apply Gauss's law to determine the electric field of a system with one of these symmetries

Gauss's law is very helpful in determining expressions for the electric field, even though the law is not directly about the electric field; it is about the electric flux. It turns out that in situations that have certain symmetries (spherical, cylindrical, or planar) in the charge distribution, we can deduce the electric field based on knowledge of the electric flux. In these systems, we can find a Gaussian surface *S* over which the electric field has constant magnitude. Furthermore, if $\vec{\mathbf{E}}$ is parallel to $\hat{\mathbf{n}}$ everywhere on the surface, then $\vec{\mathbf{E}} \cdot \hat{\mathbf{n}} = E$. (If $\vec{\mathbf{E}}$ and $\hat{\mathbf{n}}$ are antiparallel everywhere on the surface, then $\vec{\mathbf{E}} \cdot \hat{\mathbf{n}} = -E$.) Gauss's law then simplifies to

$$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = E \oint_S dA = EA = \frac{q_{\text{enc}}}{\varepsilon_0}, \tag{6.6}$$

where A is the area of the surface. Note that these symmetries lead to the transformation of the flux integral into a product of the magnitude of the electric field and an appropriate area. When you use this flux in the expression for Gauss's law, you obtain an algebraic equation that you can solve for the magnitude of the electric field, which looks like

$$E \sim \frac{q_{\text{enc}}}{\varepsilon_0 \text{ area}}.$$

The direction of the electric field at point P is obtained from the symmetry of the charge distribution and the type of charge in the distribution. Therefore, Gauss's law can be used to determine $\vec{\mathbf{E}}$. Here is a summary of the steps we will follow:

PROBLEM-SOLVING STRATEGY

Gauss's Law

1. *Identify the spatial symmetry of the charge distribution*. This is an important first step that allows us to choose the appropriate Gaussian surface. As examples, an isolated point charge has spherical symmetry, and an infinite line of charge has cylindrical symmetry.
2. *Choose a Gaussian surface with the same symmetry as the charge distribution and identify its consequences*. With this choice, $\vec{\mathbf{E}} \cdot \hat{\mathbf{n}}$ is easily determined over the Gaussian surface.
3. *Evaluate the integral* $\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA$ *over the Gaussian surface, that is, calculate the flux through the surface*. The symmetry of the Gaussian surface allows us to factor $\vec{\mathbf{E}} \cdot \hat{\mathbf{n}}$ outside the integral.
4. *Determine the amount of charge enclosed by the Gaussian surface*. This is an evaluation of the right-hand side of the equation representing Gauss's law. It is often necessary to perform an integration to obtain the net enclosed charge.
5. *Evaluate the electric field of the charge distribution*. The field may now be found using the results of steps 3 and 4.

Basically, there are only three types of symmetry that allow Gauss's law to be used to deduce the electric field. They are

- A charge distribution with spherical symmetry
- A charge distribution with cylindrical symmetry
- A charge distribution with planar symmetry

To exploit the symmetry, we perform the calculations in appropriate coordinate systems and use the right kind of Gaussian surface for that symmetry, applying the remaining four steps.

Charge Distribution with Spherical Symmetry

A charge distribution has **spherical symmetry** if the density of charge depends only on the distance from a point in space and not on the direction. In other words, if you rotate the system, it doesn't look different. For instance, if a sphere of radius R is uniformly charged with charge density ρ_0 then the distribution has spherical symmetry (Figure 6.21(a)). On the other hand, if a sphere of radius R is charged so that the top half of the sphere has uniform charge density ρ_1 and the bottom half has a uniform charge density $\rho_2 \neq \rho_1$, then the sphere does not have spherical symmetry because the charge density depends on the direction (Figure 6.21(b)). Thus, it is not the shape of the object but rather the shape of the charge distribution that determines whether or not a system has spherical symmetry.

Figure 6.21(c) shows a sphere with four different shells, each with its own uniform charge density. Although this is a situation where charge density in the full sphere is not uniform, the charge density function depends only on the distance from the center and not on the direction. Therefore, this charge distribution does have spherical symmetry.

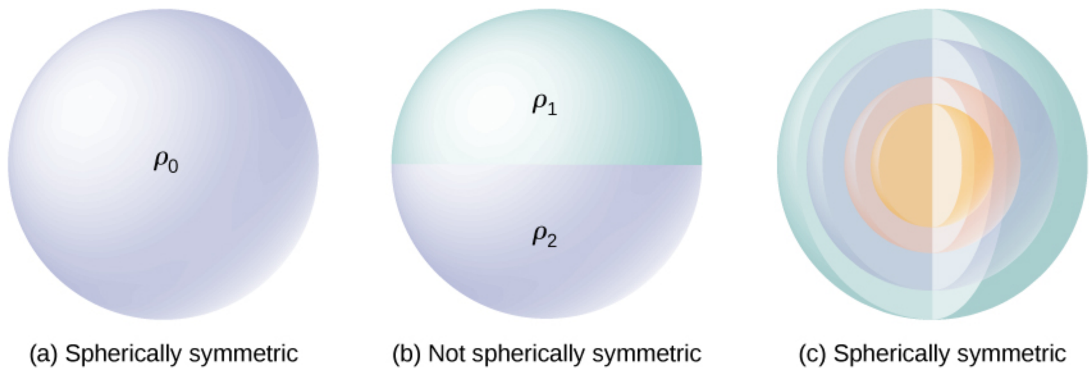

Figure 6.21 Illustrations of spherically symmetrical and nonsymmetrical systems. Different shadings indicate different charge densities. Charges on spherically shaped objects do not necessarily mean the charges are distributed with spherical symmetry. The spherical symmetry occurs only when the charge density does not depend on the direction. In (a), charges are distributed uniformly in a sphere. In (b), the upper half of the sphere has a different charge density from the lower half; therefore, (b) does not have spherical symmetry. In (c), the charges are in spherical shells of different charge densities, which means that charge density is only a function of the radial distance from the center; therefore, the system has spherical symmetry.

One good way to determine whether or not your problem has spherical symmetry is to look at the charge density function in spherical coordinates, $\rho(r, \theta, \phi)$. If the charge density is only a function of r, that is $\rho = \rho(r)$, then you have spherical symmetry. If the density depends on θ or ϕ, you could change it by rotation; hence, you would not have spherical symmetry.

Consequences of symmetry

In all spherically symmetrical cases, the electric field at any point must be radially directed, because the charge and, hence, the field must be invariant under rotation. Therefore, using spherical coordinates with their origins at the center of the spherical charge distribution, we can write down the expected form of the electric field at a point P located at a distance r from the center:

$$\text{Spherical symmetry: } \vec{\mathbf{E}}_P = E_P(r)\hat{\mathbf{r}}, \tag{6.7}$$

where $\hat{\mathbf{r}}$ is the unit vector pointed in the direction from the origin to the field point P. The radial component E_P of the electric field can be positive or negative. When $E_P > 0$, the electric field at P points away from the origin, and when $E_P < 0$, the electric field at P points toward the origin.

Gaussian surface and flux calculations

We can now use this form of the electric field to obtain the flux of the electric field through the Gaussian surface. For spherical symmetry, the Gaussian surface is a closed spherical surface that has the same center as the center of the charge distribution. Thus, the direction of the area vector of an area element on the Gaussian surface at any point is parallel to the direction of the electric field at that point, since they are both radially directed outward (Figure 6.22).

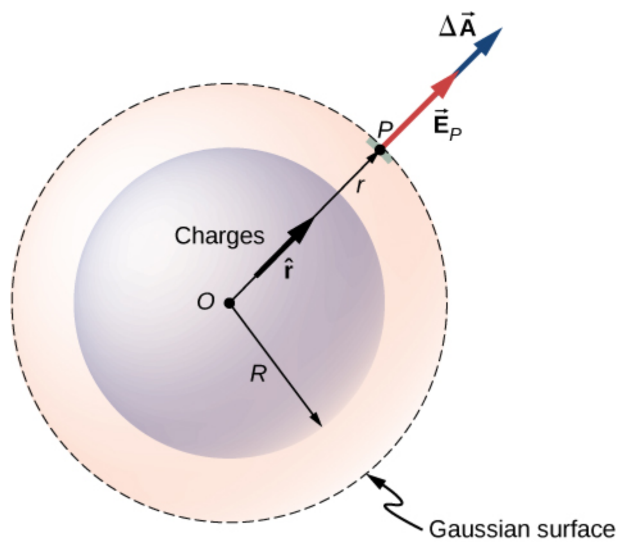

Figure 6.22 The electric field at any point of the spherical Gaussian surface for a spherically symmetrical charge distribution is parallel to the area element vector at that point, giving flux as the product of the magnitude of electric field and the value of the area. Note that the radius *R* of the charge distribution and the radius *r* of the Gaussian surface are different quantities.

The magnitude of the electric field $\vec{\mathbf{E}}$ must be the same everywhere on a spherical Gaussian surface concentric with the distribution. For a spherical surface of radius *r*,

$$\Phi = \oint_S \vec{\mathbf{E}}_P \cdot \hat{\mathbf{n}}\, dA = E_P \oint_S dA = E_P\, 4\pi r^2.$$

Using Gauss's law

According to Gauss's law, the flux through a closed surface is equal to the total charge enclosed within the closed surface divided by the permittivity of vacuum ε_0. Let q_{enc} be the total charge enclosed inside the distance *r* from the origin, which is the space inside the Gaussian spherical surface of radius *r*. This gives the following relation for Gauss's law:

$$4\pi r^2 E = \frac{q_{\text{enc}}}{\varepsilon_0}.$$

Hence, the electric field at point *P* that is a distance *r* from the center of a spherically symmetrical charge distribution has the following magnitude and direction:

$$\text{Magnitude: } E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2} \tag{6.8}$$

Direction: radial from *O* to *P* or from *P* to *O*.

The direction of the field at point *P* depends on whether the charge in the sphere is positive or negative. For a net positive charge enclosed within the Gaussian surface, the direction is from *O* to *P*, and for a net negative charge, the direction is from *P* to *O*. This is all we need for a point charge, and you will notice that the result above is identical to that for a point charge. However, Gauss's law becomes truly useful in cases where the charge occupies a finite volume.

Computing enclosed charge

The more interesting case is when a spherical charge distribution occupies a volume, and asking what the electric field inside the charge distribution is thus becomes relevant. In this case, the charge enclosed depends on the distance *r* of the field point relative to the radius of the charge distribution *R*, such as that shown in Figure 6.23.

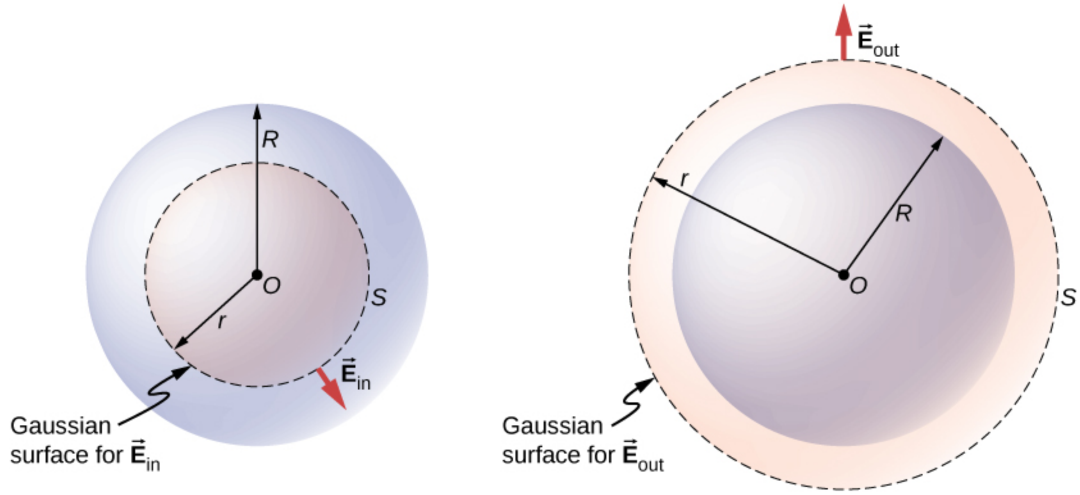

Figure 6.23 A spherically symmetrical charge distribution and the Gaussian surface used for finding the field (a) inside and (b) outside the distribution.

If point P is located outside the charge distribution—that is, if $r \geq R$—then the Gaussian surface containing P encloses all charges in the sphere. In this case, q_{enc} equals the total charge in the sphere. On the other hand, if point P is within the spherical charge distribution, that is, if $r < R$, then the Gaussian surface encloses a smaller sphere than the sphere of charge distribution. In this case, q_{enc} is less than the total charge present in the sphere. Referring to Figure 6.23, we can write q_{enc} as

$$q_{enc} = \begin{cases} q_{tot} \text{(total charge) if } r \geq R \\ q_{\text{within } r < R} \text{(only charge within } r < R\text{) if } r < R \end{cases}.$$

The field at a point outside the charge distribution is also called $\vec{E}_{out}$, and the field at a point inside the charge distribution is called $\vec{E}_{in}$. Focusing on the two types of field points, either inside or outside the charge distribution, we can now write the magnitude of the electric field as

$$P \text{ outside sphere } E_{out} = \frac{1}{4\pi\varepsilon_0} \frac{q_{tot}}{r^2} \tag{6.9}$$

$$P \text{ inside sphere } E_{in} = \frac{1}{4\pi\varepsilon_0} \frac{q_{\text{within } r < R}}{r^2}. \tag{6.10}$$

Note that the electric field outside a spherically symmetrical charge distribution is identical to that of a point charge at the center that has a charge equal to the total charge of the spherical charge distribution. This is remarkable since the charges are not located at the center only. We now work out specific examples of spherical charge distributions, starting with the case of a uniformly charged sphere.

EXAMPLE 6.6

Uniformly Charged Sphere

A sphere of radius R, such as that shown in Figure 6.23, has a uniform volume charge density ρ_0. Find the electric field at a point outside the sphere and at a point inside the sphere.

Strategy

Apply the Gauss's law problem-solving strategy, where we have already worked out the flux calculation.

Solution

The charge enclosed by the Gaussian surface is given by

$$q_{enc} = \int \rho_0 \, dV = \int_0^r \rho_0 4\pi r'^2 \, dr' = \rho_0 \left(\frac{4}{3}\pi r^3\right).$$

The answer for electric field amplitude can then be written down immediately for a point outside the sphere, labeled E_{out}, and a point inside the sphere, labeled E_{in}.

$$\begin{aligned} E_{\text{out}} &= \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{tot}}}{r^2},\ q_{\text{tot}} = \frac{4}{3}\pi R^3\ \rho_0, \\ E_{\text{in}} &= \frac{q_{\text{enc}}}{4\pi\varepsilon_0 r^2} = \frac{\rho_0 r}{3\varepsilon_0},\ \text{since } q_{\text{enc}} = \frac{4}{3}\pi r^3\ \rho_0. \end{aligned}$$

It is interesting to note that the magnitude of the electric field increases inside the material as you go out, since the amount of charge enclosed by the Gaussian surface increases with the volume. Specifically, the charge enclosed grows $\propto r^3$, whereas the field from each infinitesimal element of charge drops off $\propto 1/r^2$ with the net result that the electric field within the distribution increases in strength linearly with the radius. The magnitude of the electric field outside the sphere decreases as you go away from the charges, because the included charge remains the same but the distance increases. Figure 6.24 displays the variation of the magnitude of the electric field with distance from the center of a uniformly charged sphere.

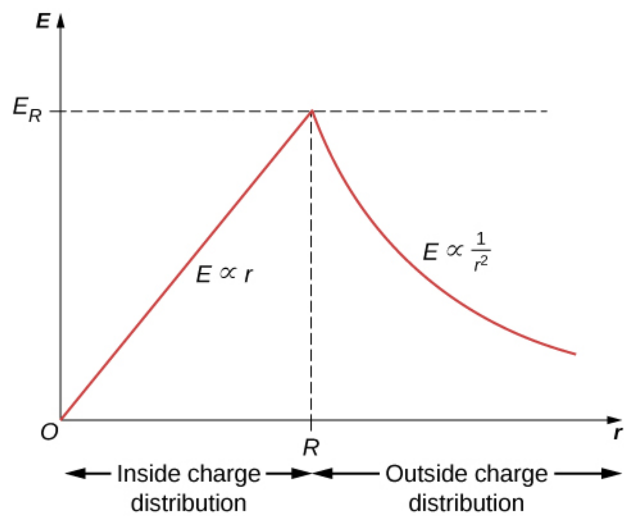

Figure 6.24 Electric field of a uniformly charged, non-conducting sphere increases inside the sphere to a maximum at the surface and then decreases as $1/r^2$. Here, $E_R = \frac{\rho_0 R}{3\varepsilon_0}$. The electric field is due to a spherical charge distribution of uniform charge density and total charge Q as a function of distance from the center of the distribution.

The direction of the electric field at any point P is radially outward from the origin if ρ_0 is positive, and inward (i.e., toward the center) if ρ_0 is negative. The electric field at some representative space points are displayed in Figure 6.25 whose radial coordinates r are $r = R/2$, $r = R$, and $r = 2R$.

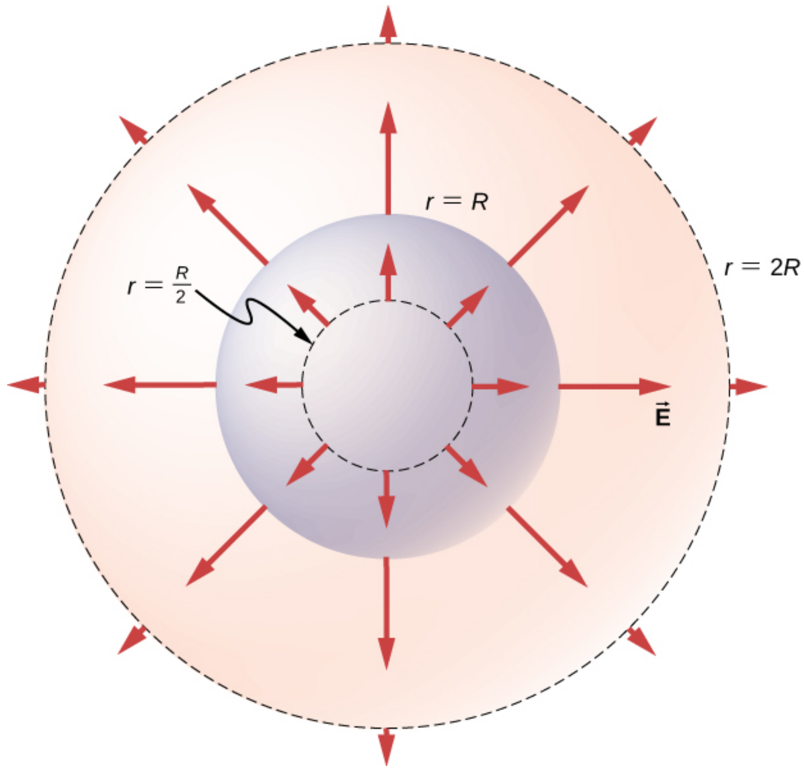

Figure 6.25 Electric field vectors inside and outside a uniformly charged sphere.

Significance

Notice that E_{out} has the same form as the equation of the electric field of an isolated point charge. In determining the electric field of a uniform spherical charge distribution, we can therefore assume that all of the charge inside the appropriate spherical Gaussian surface is located at the center of the distribution.

EXAMPLE 6.7

Non-Uniformly Charged Sphere

A non-conducting sphere of radius R has a non-uniform charge density that varies with the distance from its center as given by

$$\rho(r) = ar^n \ (r \leq R; n \geq 0),$$

where a is a constant. We require $n \geq 0$ so that the charge density is not undefined at $r = 0$. Find the electric field at a point outside the sphere and at a point inside the sphere.

Strategy

Apply the Gauss's law strategy given above, where we work out the enclosed charge integrals separately for cases inside and outside the sphere.

Solution

Since the given charge density function has only a radial dependence and no dependence on direction, we have a spherically symmetrical situation. Therefore, the magnitude of the electric field at any point is given above and the direction is radial. We just need to find the enclosed charge q_{enc}, which depends on the location of the field point.

A note about symbols: We use r' for locating charges in the charge distribution and r for locating the field point(s) at the Gaussian surface(s). The letter R is used for the radius of the charge distribution.

As charge density is not constant here, we need to integrate the charge density function over the volume enclosed by the Gaussian surface. Therefore, we set up the problem for charges in one spherical shell, say between r' and $r' + dr'$, as shown in Figure 6.26. The volume of charges in the shell of infinitesimal width is equal to the product of the area of surface $4\pi r'^2$ and the thickness dr'. Multiplying the volume with the density at this location, which is ar'^n, gives the charge in the shell:

$$dq = ar'^n \, 4\pi r'^2 dr'.$$

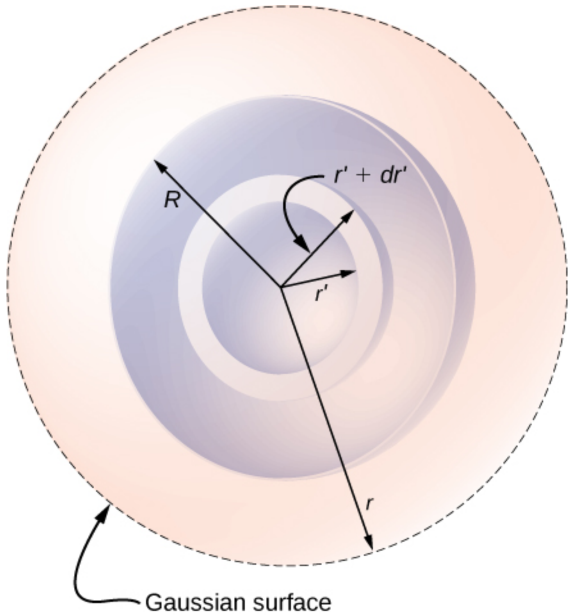

Figure 6.26 Spherical symmetry with non-uniform charge distribution. In this type of problem, we need four radii: R is the radius of the charge distribution, r is the radius of the Gaussian surface, r' is the inner radius of the spherical shell, and $r' + dr'$ is the outer radius of the spherical shell. The spherical shell is used to calculate the charge enclosed within the Gaussian surface. The range for r' is from 0 to r for the field at a point inside the charge distribution and from 0 to R for the field at a point outside the charge distribution. If $r > R$, then the Gaussian surface encloses more volume than the charge distribution, but the additional volume does not contribute to q_{enc}.

(a) **Field at a point outside the charge distribution.** In this case, the Gaussian surface, which contains the field point P, has a radius r that is greater than the radius R of the charge distribution, $r > R$. Therefore, all charges of the charge distribution are enclosed within the Gaussian surface. Note that the space between $r' = R$ and $r' = r$ is empty of charges and therefore does not contribute to the integral over the volume enclosed by the Gaussian surface:

$$q_{enc} = \int dq = \int_0^R ar'^n \, 4\pi r'^2 dr' = \frac{4\pi a}{n+3} R^{n+3}.$$

This is used in the general result for $\vec{\mathbf{E}}_{out}$ above to obtain the electric field at a point outside the charge distribution as

$$\vec{\mathbf{E}}_{out} = \left[\frac{aR^{n+3}}{\varepsilon_0(n+3)}\right] \frac{1}{r^2} \hat{\mathbf{r}},$$

where $\hat{\mathbf{r}}$ is a unit vector in the direction from the origin to the field point at the Gaussian surface.

(b) **Field at a point inside the charge distribution.** The Gaussian surface is now buried inside the charge distribution, with $r < R$. Therefore, only those charges in the distribution that are within a distance r of the center of the spherical charge distribution count in r_{enc}:

$$q_{enc} = \int_0^r ar'^n 4\pi r'^2 \, dr' = \frac{4\pi a}{n+3} r^{n+3}.$$

Now, using the general result above for $\vec{\mathbf{E}}_{\text{in}}$, we find the electric field at a point that is a distance r from the center and lies within the charge distribution as

$$\vec{\mathbf{E}}_{\text{in}} = \left[\frac{a}{\varepsilon_0(n+3)}\right] r^{n+1}\hat{\mathbf{r}},$$

where the direction information is included by using the unit radial vector.

CHECK YOUR UNDERSTANDING 6.4

Check that the electric fields for the sphere reduce to the correct values for a point charge.

Charge Distribution with Cylindrical Symmetry

A charge distribution has **cylindrical symmetry** if the charge density depends only upon the distance r from the axis of a cylinder and must not vary along the axis or with direction about the axis. In other words, if your system varies if you rotate it around the axis, or shift it along the axis, you do not have cylindrical symmetry.

Figure 6.27 shows four situations in which charges are distributed in a cylinder. A uniform charge density ρ_0. in an infinite straight wire has a cylindrical symmetry, and so does an infinitely long cylinder with constant charge density ρ_0. An infinitely long cylinder that has different charge densities along its length, such as a charge density ρ_1 for $z > 0$ and $\rho_2 \neq \rho_1$ for $z < 0$, does not have a usable cylindrical symmetry for this course. Neither does a cylinder in which charge density varies with the direction, such as a charge density ρ_1 for $0 \leq \theta < \pi$ and $\rho_2 \neq \rho_1$ for $\pi \leq \theta < 2\pi$. A system with concentric cylindrical shells, each with uniform charge densities, albeit different in different shells, as in Figure 6.27(d), does have cylindrical symmetry if they are infinitely long. The infinite length requirement is due to the charge density changing along the axis of a finite cylinder. In real systems, we don't have infinite cylinders; however, if the cylindrical object is considerably longer than the radius from it that we are interested in, then the approximation of an infinite cylinder becomes useful.

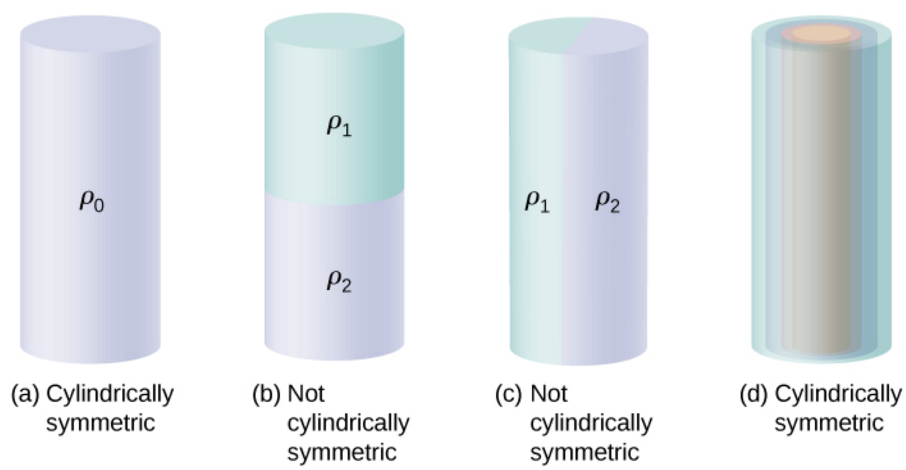

Figure 6.27 To determine whether a given charge distribution has cylindrical symmetry, look at the cross-section of an "infinitely long" cylinder. If the charge density does not depend on the polar angle of the cross-section or along the axis, then you have cylindrical symmetry. (a) Charge density is constant in the cylinder; (b) upper half of the cylinder has a different charge density from the lower half; (c) left half of the cylinder has a different charge density from the right half; (d) charges are constant in different cylindrical rings, but the density does not depend on the polar angle. Cases (a) and (d) have cylindrical symmetry, whereas (b) and (c) do not.

Consequences of symmetry

In all cylindrically symmetrical cases, the electric field $\vec{\mathbf{E}}_P$ at any point P must also display cylindrical symmetry.

Cylindrical symmetry: $\vec{\mathbf{E}}_P = E_P(r)\hat{\mathbf{r}},$

where r is the distance from the axis and $\hat{\mathbf{r}}$ is a unit vector directed perpendicularly away from the axis (Figure 6.28).

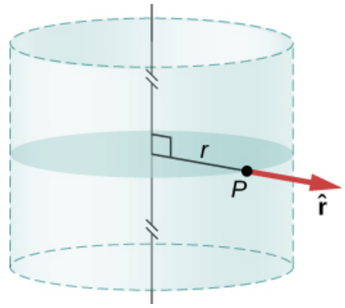

Figure 6.28 The electric field in a cylindrically symmetrical situation depends only on the distance from the axis. The direction of the electric field is pointed away from the axis for positive charges and toward the axis for negative charges.

Gaussian surface and flux calculation

To make use of the direction and functional dependence of the electric field, we choose a closed Gaussian surface in the shape of a cylinder with the same axis as the axis of the charge distribution. The flux through this surface of radius s and height L is easy to compute if we divide our task into two parts: (a) a flux through the flat ends and (b) a flux through the curved surface (Figure 6.29).

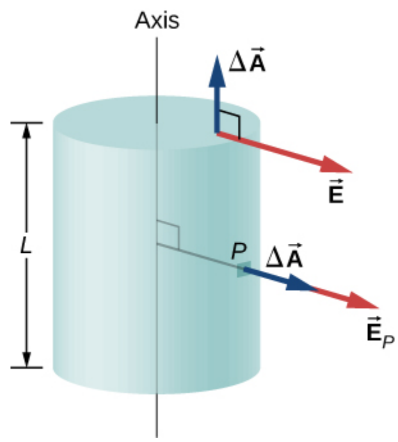

Figure 6.29 The Gaussian surface in the case of cylindrical symmetry. The electric field at a patch is either parallel or perpendicular to the normal to the patch of the Gaussian surface.

The electric field is perpendicular to the cylindrical side and parallel to the planar end caps of the surface. The flux through the cylindrical part is

$$\int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = E \int_S dA = E(2\pi r L),$$

whereas the flux through the end caps is zero because $\vec{\mathbf{E}} \cdot \hat{\mathbf{n}} = 0$ there. Thus, the flux is

$$\int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = E(2\pi r L) + 0 + 0 = 2\pi r L E.$$

Using Gauss’s law

According to Gauss’s law, the flux must equal the amount of charge within the volume enclosed by this surface, divided by the permittivity of free space. When you do the calculation for a cylinder of length L, you find that q_{enc} of Gauss’s law is directly proportional to L. Let us write it as charge per unit length (λ_{enc}) times length L:

$$q_{\text{enc}} = \lambda_{\text{enc}} L.$$

Hence, Gauss’s law for any cylindrically symmetrical charge distribution yields the following magnitude of the electric field a distance s away from the axis:

$$\text{Magnitude: } E(r) = \frac{\lambda_{enc}}{2\pi\varepsilon_0}\frac{1}{r}.$$

The charge per unit length λ_{enc} depends on whether the field point is inside or outside the cylinder of charge distribution, just as we have seen for the spherical distribution.

Computing enclosed charge

Let R be the radius of the cylinder within which charges are distributed in a cylindrically symmetrical way. Let the field point P be at a distance s from the axis. (The side of the Gaussian surface includes the field point P.) When $r > R$ (that is, when P is outside the charge distribution), the Gaussian surface includes all the charge in the cylinder of radius R and length L. When $r < R$ (P is located inside the charge distribution), then only the charge within a cylinder of radius s and length L is enclosed by the Gaussian surface:

$$\lambda_{enc} L = \begin{cases} \text{(total charge) if } r \geq R \\ \text{(only charge within } r < R\text{) if } r < R \end{cases}.$$

EXAMPLE 6.8

Uniformly Charged Cylindrical Shell

A very long non-conducting cylindrical shell of radius R has a uniform surface charge densityσ_0. Find the electric field (a) at a point outside the shell and (b) at a point inside the shell.

Strategy

Apply the Gauss's law strategy given earlier, where we treat the cases inside and outside the shell separately.

Solution

a. **Electric field at a point outside the shell.** For a point outside the cylindrical shell, the Gaussian surface is the surface of a cylinder of radius $r > R$ and length L, as shown in Figure 6.30. The charge enclosed by the Gaussian cylinder is equal to the charge on the cylindrical shell of length L. Therefore, λ_{enc} is given by

$$\lambda_{enc} = \frac{\sigma_0 2\pi RL}{L} = 2\pi R\sigma_0.$$

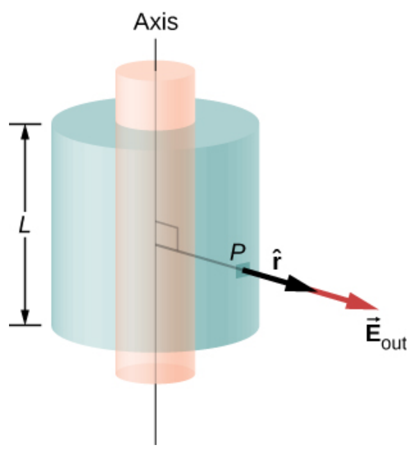

Figure 6.30 A Gaussian surface surrounding a cylindrical shell.

Hence, the electric field at a point P outside the shell at a distance r away from the axis is

$$\vec{\mathbf{E}} = \frac{2\pi R\sigma_0}{2\pi\varepsilon_o}\frac{1}{r}\hat{\mathbf{r}} = \frac{R\sigma_0}{\varepsilon_o}\frac{1}{r}\hat{\mathbf{r}}\ (r > R)$$

where $\hat{\mathbf{r}}$ is a unit vector, perpendicular to the axis and pointing away from it, as shown in the figure. The electric field at P points in the direction of $\hat{\mathbf{r}}$ given in Figure 6.30 if $\sigma_0 > 0$ and in the opposite direction to $\hat{\mathbf{r}}$ if $\sigma_0 < 0$.

b. **Electric field at a point inside the shell.** For a point inside the cylindrical shell, the Gaussian surface is a cylinder whose radius r is less than R (Figure 6.31). This means no charges are included inside the Gaussian surface:

$$\lambda_{\text{enc}} = 0.$$

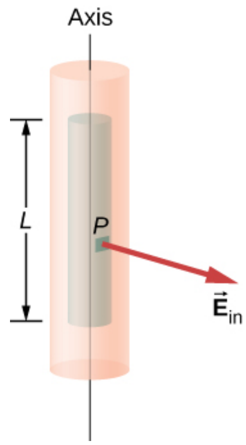

Figure 6.31 A Gaussian surface within a cylindrical shell.

This gives the following equation for the magnitude of the electric field E_{in} at a point whose r is less than R of the shell of charges.

$$E_{\text{in}}\ 2\pi r L = 0\ (r < R),$$

This gives us

$$E_{\text{in}} = 0\ (r < R).$$

Significance

Notice that the result inside the shell is exactly what we should expect: No enclosed charge means zero electric field. Outside the shell, the result becomes identical to a wire with uniform charge $R\sigma_0$.

CHECK YOUR UNDERSTANDING 6.5

A thin straight wire has a uniform linear charge density λ_0. Find the electric field at a distance d from the wire, where d is much less than the length of the wire.

Charge Distribution with Planar Symmetry

A **planar symmetry** of charge density is obtained when charges are uniformly spread over a large flat surface. In planar symmetry, all points in a plane parallel to the plane of charge are identical with respect to the charges.

Consequences of symmetry

We take the plane of the charge distribution to be the xy-plane and we find the electric field at a space point P with coordinates (x, y, z). Since the charge density is the same at all (x, y)-coordinates in the $z = 0$ plane, by symmetry, the electric field at P cannot depend on the x- or y-coordinates of point P, as shown in Figure 6.32. Therefore, the electric field at P can only depend on the distance from the plane and has a direction either toward the plane or away from the plane. That is, the electric field at P has only a nonzero z-component.

Uniform charges in xy plane: $\vec{\mathbf{E}} = E(z)\hat{\mathbf{z}}$

where z is the distance from the plane and $\hat{\mathbf{z}}$ is the unit vector normal to the plane. Note that in this system, $E\,(z) = E\,(-z)$, although of course they point in opposite directions.

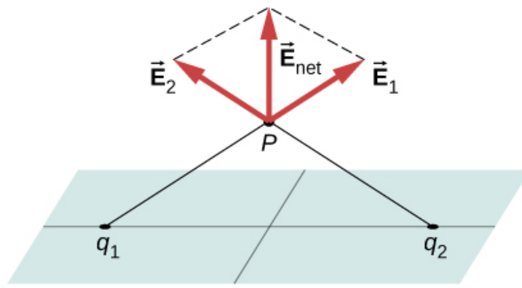

Figure 6.32 The components of the electric field parallel to a plane of charges cancel out the two charges located symmetrically from the field point *P*. Therefore, the field at any point is pointed vertically from the plane of charges. For any point *P* and charge q_1, we can always find a q_2 with this effect.

Gaussian surface and flux calculation

In the present case, a convenient Gaussian surface is a box, since the expected electric field points in one direction only. To keep the Gaussian box symmetrical about the plane of charges, we take it to straddle the plane of the charges, such that one face containing the field point *P* is taken parallel to the plane of the charges. In Figure 6.33, sides I and II of the Gaussian surface (the box) that are parallel to the infinite plane have been shaded. They are the only surfaces that give rise to nonzero flux because the electric field and the area vectors of the other faces are perpendicular to each other.

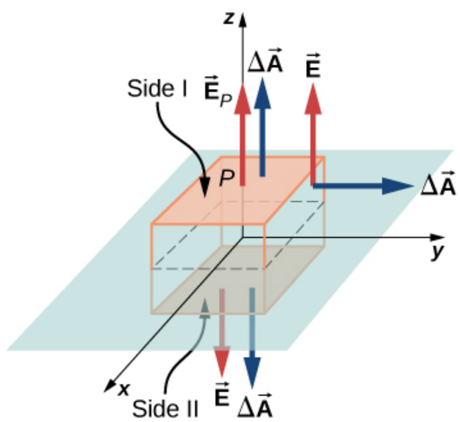

Figure 6.33 A thin charged sheet and the Gaussian box for finding the electric field at the field point *P*. The normal to each face of the box is from inside the box to outside. On two faces of the box, the electric fields are parallel to the area vectors, and on the other four faces, the electric fields are perpendicular to the area vectors.

Let A be the area of the shaded surface on each side of the plane and E_P be the magnitude of the electric field at point *P*. Since sides I and II are at the same distance from the plane, the electric field has the same magnitude at points in these planes, although the directions of the electric field at these points in the two planes are opposite to each other.

Magnitude at I or II: $E(z) = E_P$.

If the charge on the plane is positive, then the direction of the electric field and the area vectors are as shown in Figure 6.33. Therefore, we find for the flux of electric field through the box

$$\Phi = \oint_S \vec{\mathbf{E}}_P \cdot \hat{\mathbf{n}}\, dA = E_P A + E_P A + 0 + 0 + 0 + 0 = 2E_P A \qquad 6.11$$

where the zeros are for the flux through the other sides of the box. Note that if the charge on the plane is negative, the directions of electric field and area vectors for planes I and II are opposite to each other, and we get a negative sign for the flux. According to Gauss's law, the flux must equal $q_{\text{enc}}/\varepsilon_0$. From Figure 6.33, we see that the charges inside the volume enclosed by the Gaussian box reside on an area A of the xy-plane. Hence,

$$q_{\text{enc}} = \sigma_0 A. \qquad 6.12$$

Using the equations for the flux and enclosed charge in Gauss's law, we can immediately determine the electric field at a point at height z from a uniformly charged plane in the xy-plane:

$$\vec{\mathbf{E}}_P = \frac{\sigma_0}{2\varepsilon_0}\hat{\mathbf{n}}.$$

The direction of the field depends on the sign of the charge on the plane and the side of the plane where the field point P is located. Note that above the plane, $\hat{\mathbf{n}} = +\hat{\mathbf{z}}$, while below the plane, $\hat{\mathbf{n}} = -\hat{\mathbf{z}}$.

You may be surprised to note that the electric field does not actually depend on the distance from the plane; this is an effect of the assumption that the plane is infinite. In practical terms, the result given above is still a useful approximation for finite planes near the center.

6.4 Conductors in Electrostatic Equilibrium

Learning Objectives

By the end of this section, you will be able to:

- Describe the electric field within a conductor at equilibrium
- Describe the electric field immediately outside the surface of a charged conductor at equilibrium
- Explain why if the field is not as described in the first two objectives, the conductor is not at equilibrium

So far, we have generally been working with charges occupying a volume within an insulator. We now study what happens when free charges are placed on a conductor. Generally, in the presence of a (generally external) electric field, the free charge in a conductor redistributes and very quickly reaches electrostatic equilibrium. The resulting charge distribution and its electric field have many interesting properties, which we can investigate with the help of Gauss's law and the concept of electric potential.

The Electric Field inside a Conductor Vanishes

If an electric field is present inside a conductor, it exerts forces on the **free electrons** (also called conduction electrons), which are electrons in the material that are not bound to an atom. These free electrons then accelerate. However, moving charges by definition means nonstatic conditions, contrary to our assumption. Therefore, when electrostatic equilibrium is reached, the charge is distributed in such a way that the electric field inside the conductor vanishes.

If you place a piece of a metal near a positive charge, the free electrons in the metal are attracted to the external positive charge and migrate freely toward that region. The region the electrons move to then has an excess of electrons over the protons in the atoms and the region from where the electrons have migrated has more protons than electrons. Consequently, the metal develops a negative region near the charge and a positive region at the far end (Figure 6.34). As we saw in the preceding chapter, this separation of equal magnitude and opposite type of electric charge is called polarization. If you remove the external charge, the electrons migrate back and neutralize the positive region.

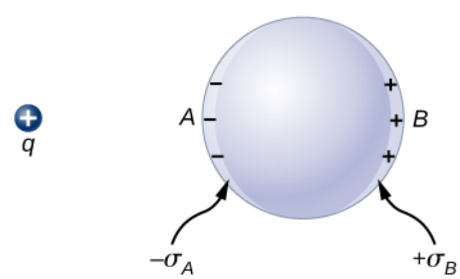

Figure 6.34 Polarization of a metallic sphere by an external point charge $+q$. The near side of the metal has an opposite surface charge compared to the far side of the metal. The sphere is said to be polarized. When you remove the external charge, the polarization of the metal also disappears.

The polarization of the metal happens only in the presence of external charges. You can think of this in terms of electric fields. The external charge creates an external electric field. When the metal is placed in the region of this electric field, the electrons and protons of the metal experience electric forces due to this external electric field, but only the conduction electrons are free to move in the metal over macroscopic distances. The movement of the conduction electrons leads to the polarization, which creates an induced electric field in addition to the external electric field (Figure 6.35). The net electric field is a vector sum of the fields of $+q$ and the surface charge densities $-\sigma_A$ and $+\sigma_B$. This means that the net field inside the conductor is different from the field outside the conductor.

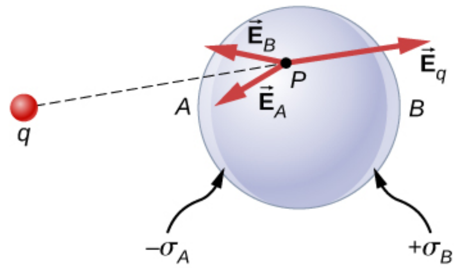

Figure 6.35 In the presence of an external charge q, the charges in a metal redistribute. The electric field at any point has three contributions, from $+q$ and the induced charges $-\sigma_A$ and $+\sigma_B$. Note that the surface charge distribution will not be uniform in this case.

The redistribution of charges is such that the sum of the three contributions at any point P inside the conductor is

$$\vec{\mathbf{E}}_P = \vec{\mathbf{E}}_q + \vec{\mathbf{E}}_B + \vec{\mathbf{E}}_A = \vec{\mathbf{0}}.$$

Now, thanks to Gauss's law, we know that there is no net charge enclosed by a Gaussian surface that is solely within the volume of the conductor at equilibrium. That is, $q_{\text{enc}} = 0$ and hence

$$\vec{\mathbf{E}}_{\text{net}} = \vec{\mathbf{0}} \text{ (at points inside a conductor).} \quad (6.13)$$

Charge on a Conductor

An interesting property of a conductor in static equilibrium is that extra charges on the conductor end up on the outer surface of the conductor, regardless of where they originate. Figure 6.36 illustrates a system in which we bring an external positive charge inside the cavity of a metal and then touch it to the inside surface. Initially, the inside surface of the cavity is negatively charged and the outside surface of the conductor is positively charged. When we touch the inside surface of the cavity, the induced charge is neutralized, leaving the outside surface and the whole metal charged with a net positive charge.

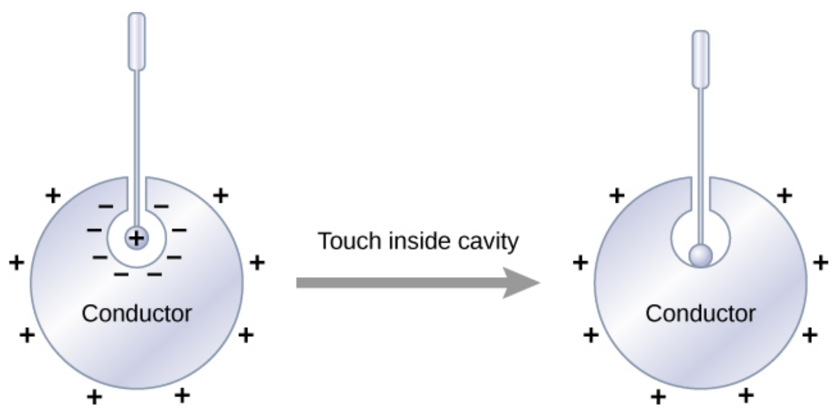

Figure 6.36 Electric charges on a conductor migrate to the outside surface no matter where you put them initially.

To see why this happens, note that the Gaussian surface in Figure 6.37 (the dashed line) follows the contour of the actual surface of the conductor and is located an infinitesimal distance *within* it. Since $E = 0$ everywhere inside a conductor,

$$\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = 0.$$

Thus, from Gauss's law, there is no net charge inside the Gaussian surface. But the Gaussian surface lies just below the actual surface of the conductor; consequently, there is no net charge inside the conductor. Any excess charge must lie on its surface.

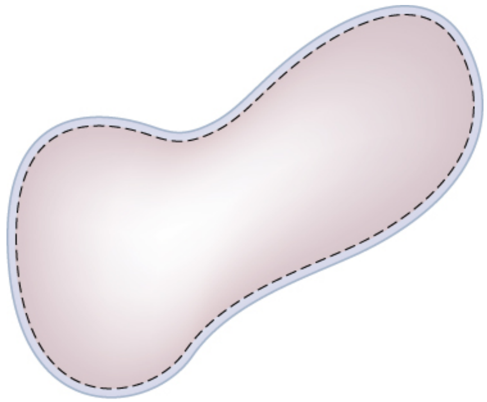

Figure 6.37 The dashed line represents a Gaussian surface that is just beneath the actual surface of the conductor.

This particular property of conductors is the basis for an extremely accurate method developed by Plimpton and Lawton in 1936 to verify Gauss's law and, correspondingly, Coulomb's law. A sketch of their apparatus is shown in Figure 6.38. Two spherical shells are connected to one another through an electrometer E, a device that can detect a very slight amount of charge flowing from one shell to the other. When switch S is thrown to the left, charge is placed on the outer shell by the battery B. Will charge flow through the electrometer to the inner shell?

No. Doing so would mean a violation of Gauss's law. Plimpton and Lawton did not detect any flow and, knowing the sensitivity of their electrometer, concluded that if the radial dependence in Coulomb's law were $1/r^{(2+\delta)}$, δ would be less than 2×10^{-9}[1]. More recent measurements place δ at less than 3×10^{-16}[2], a number so small that the validity of Coulomb's law seems indisputable.

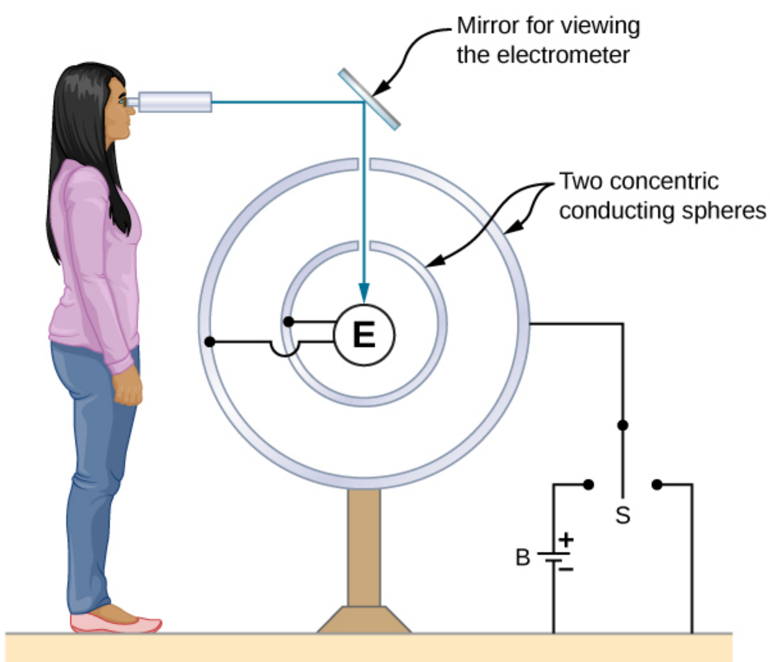

Figure 6.38 A representation of the apparatus used by Plimpton and Lawton. Any transfer of charge between the spheres is detected by

1 S. Plimpton and W. Lawton. 1936. "A Very Accurate Test of Coulomb's Law of Force between Charges." *Physical Review* 50, No. 11: 1066, doi:10.1103/PhysRev.50.1066

2 E. Williams, J. Faller, and H. Hill. 1971. "New Experimental Test of Coulomb's Law: A Laboratory Upper Limit on the Photon Rest Mass." *Physical Review Letters* 26 , No. 12: 721, doi:10.1103/PhysRevLett.26.721

the electrometer E.

The Electric Field at the Surface of a Conductor

If the electric field had a component parallel to the surface of a conductor, free charges on the surface would move, a situation contrary to the assumption of electrostatic equilibrium. Therefore, the electric field is always perpendicular to the surface of a conductor.

At any point just above the surface of a conductor, the surface charge density σ and the magnitude of the electric field E are related by

$$E = \frac{\sigma}{\varepsilon_0}. \tag{6.14}$$

To see this, consider an infinitesimally small Gaussian cylinder that surrounds a point on the surface of the conductor, as in Figure 6.39. The cylinder has one end face inside and one end face outside the surface. The height and cross-sectional area of the cylinder are δ and ΔA, respectively. The cylinder's sides are perpendicular to the surface of the conductor, and its end faces are parallel to the surface. Because the cylinder is infinitesimally small, the charge density σ is essentially constant over the surface enclosed, so the total charge inside the Gaussian cylinder is $\sigma\Delta A$. Now E is perpendicular to the surface of the conductor outside the conductor and vanishes within it, because otherwise, the charges would accelerate, and we would not be in equilibrium. Electric flux therefore crosses only the outer end face of the Gaussian surface and may be written as $E\Delta A$, since the cylinder is assumed to be small enough that E is approximately constant over that area. From Gauss' law,

$$E\Delta A = \frac{\sigma\Delta A}{\varepsilon_0}.$$

Thus,

$$E = \frac{\sigma}{\varepsilon_0}.$$

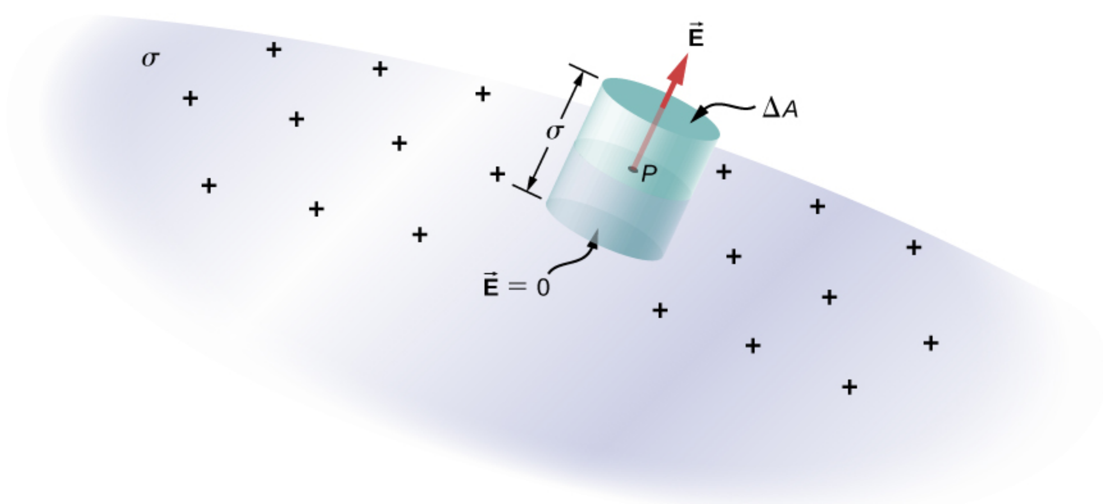

Figure 6.39 An infinitesimally small cylindrical Gaussian surface surrounds point *P*, which is on the surface of the conductor. The field $\vec{E}$ is perpendicular to the surface of the conductor outside the conductor and vanishes within it.

EXAMPLE 6.9

Electric Field of a Conducting Plate

The infinite conducting plate in Figure 6.40 has a uniform surface charge density σ. Use Gauss' law to find the electric field outside the plate. Compare this result with that previously calculated directly.

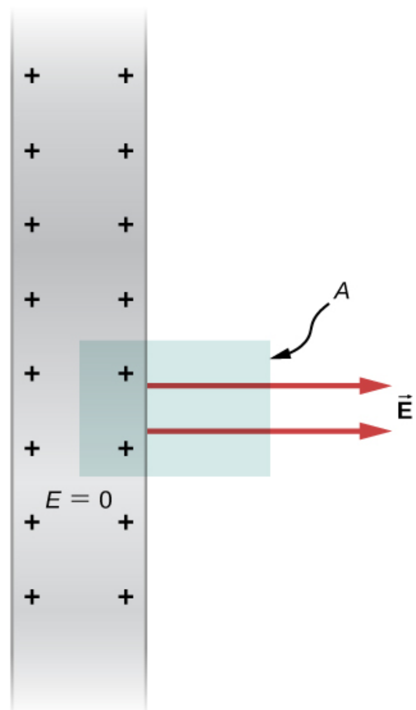

Figure 6.40 A side view of an infinite conducting plate and Gaussian cylinder with cross-sectional area *A*.

Strategy

For this case, we use a cylindrical Gaussian surface, a side view of which is shown.

Solution

The flux calculation is similar to that for an infinite sheet of charge from the previous chapter with one major exception: The left face of the Gaussian surface is inside the conductor where $\vec{\mathbf{E}} = \vec{\mathbf{0}}$, so the total flux through the Gaussian surface is EA rather than $2EA$. Then from Gauss' law,

$$EA = \frac{\sigma A}{\varepsilon_0}$$

and the electric field outside the plate is

$$E = \frac{\sigma}{\varepsilon_0}.$$

Significance

This result is in agreement with the result from the previous section, and consistent with the rule stated above.

EXAMPLE 6.10

Electric Field between Oppositely Charged Parallel Plates

Two large conducting plates carry equal and opposite charges, with a surface charge density σ of magnitude $6.81 \times 10^{-7}\ \text{C/m}^2$, as shown in Figure 6.41. The separation between the plates is $l = 6.50\ \text{mm}$. What is the electric field between the plates?

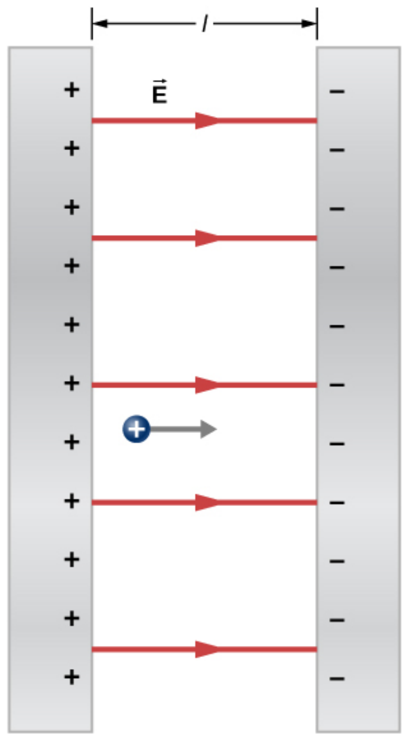

Figure 6.41 The electric field between oppositely charged parallel plates. A test charge is released at the positive plate.

Strategy

Note that the electric field at the surface of one plate only depends on the charge on that plate. Thus, apply $E = \sigma/\varepsilon_0$ with the given values.

Solution

The electric field is directed from the positive to the negative plate, as shown in the figure, and its magnitude is given by

$$E = \frac{\sigma}{\varepsilon_0} = \frac{6.81 \times 10^{-7}\ \text{C/m}^2}{8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2} = 7.69 \times 10^4\ \text{N/C}.$$

Significance

This formula is applicable to more than just a plate. Furthermore, two-plate systems will be important later.

EXAMPLE 6.11

A Conducting Sphere

The isolated conducting sphere (Figure 6.42) has a radius R and an excess charge q. What is the electric field both inside and outside the sphere?

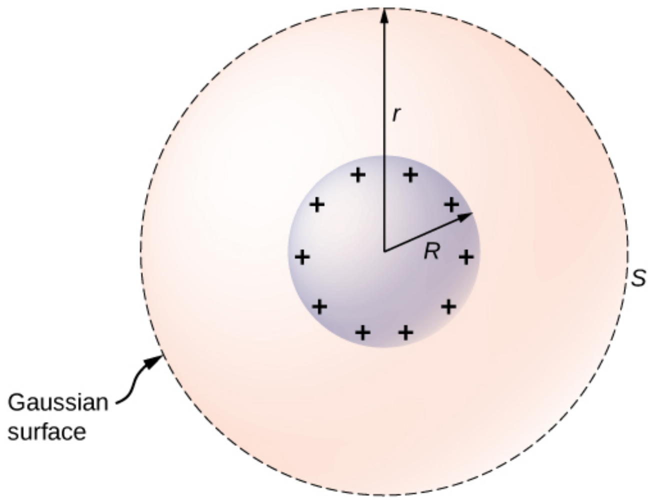

Figure 6.42 An isolated conducting sphere.

Strategy

The sphere is isolated, so its surface change distribution and the electric field of that distribution are spherically symmetrical. We can therefore represent the field as $\vec{\mathbf{E}} = E(r)\hat{\mathbf{r}}$. To calculate $E(r)$, we apply Gauss's law over a closed spherical surface S of radius r that is concentric with the conducting sphere.

Solution

Since r is constant and $\hat{\mathbf{n}} = \hat{\mathbf{r}}$ on the sphere,

$$\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = E(r) \oint_S dA = E(r)\, 4\pi r^2.$$

For $r < R$, S is within the conductor, so $q_{\text{enc}} = 0$, and Gauss's law gives

$$E(r) = 0,$$

as expected inside a conductor. If $r > R$, S encloses the conductor so $q_{\text{enc}} = q$. From Gauss's law,

$$E(r)\, 4\pi r^2 = \frac{q}{\varepsilon_0}.$$

The electric field of the sphere may therefore be written as

$$\begin{aligned} \vec{\mathbf{E}} &= \vec{\mathbf{0}} && (r < R), \\ \vec{\mathbf{E}} &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} && (r \geq R). \end{aligned}$$

Significance

Notice that in the region $r \geq R$, the electric field due to a charge q placed on an isolated conducting sphere of radius R is identical to the electric field of a point charge q located at the center of the sphere. The difference between the charged metal and a point charge occurs only at the space points inside the conductor. For a point charge placed at the center of the sphere, the electric field is not zero at points of space occupied by the sphere, but a conductor with the same amount of charge has a zero electric field at those points (Figure 6.43). However, there is no distinction at the outside points in space where $r > R$, and we can replace the isolated charged spherical conductor by a point charge at its center with impunity.

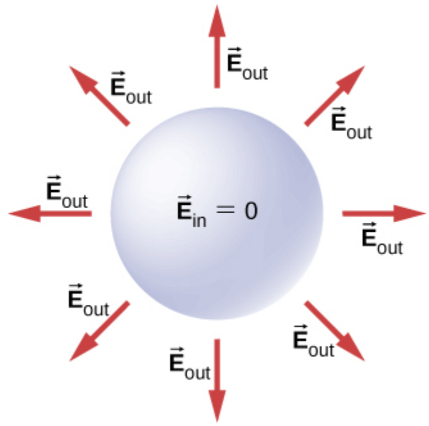

Figure 6.43 Electric field of a positively charged metal sphere. The electric field inside is zero, and the electric field outside is same as the electric field of a point charge at the center, although the charge on the metal sphere is at the surface.

CHECK YOUR UNDERSTANDING 6.6

How will the system above change if there are charged objects external to the sphere?

For a conductor with a cavity, if we put a charge $+q$ inside the cavity, then the charge separation takes place in the conductor, with $-q$ amount of charge on the inside surface and a $+q$ amount of charge at the outside surface (Figure 6.44(a)). For the same conductor with a charge $+q$ outside it, there is no excess charge on the inside surface; both the positive and negative induced charges reside on the outside surface (Figure 6.44(b)).

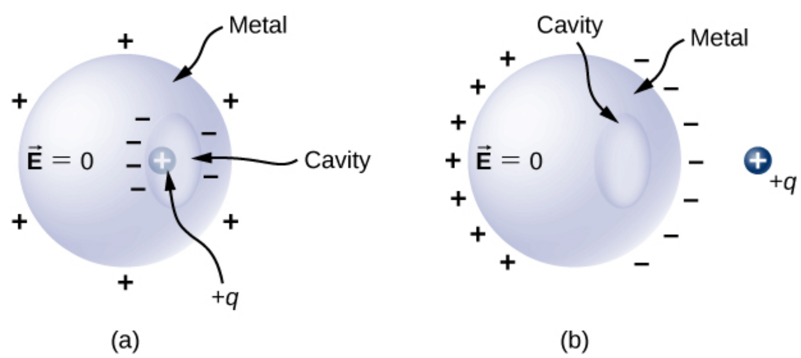

Figure 6.44 (a) A charge inside a cavity in a metal. The distribution of charges at the outer surface does not depend on how the charges are distributed at the inner surface, since the *E*-field inside the body of the metal is zero. That magnitude of the charge on the outer surface does depend on the magnitude of the charge inside, however. (b) A charge outside a conductor containing an inner cavity. The cavity remains free of charge. The polarization of charges on the conductor happens at the surface.

If a conductor has two cavities, one of them having a charge $+q_a$ inside it and the other a charge $-q_b$, the polarization of the conductor results in $-q_a$ on the inside surface of the cavity *a*, $+q_b$ on the inside surface of the cavity *b*, and $q_a - q_b$ on the outside surface (Figure 6.45). The charges on the surfaces may not be uniformly spread out; their spread depends upon the geometry. The only rule obeyed is that when the equilibrium has been reached, the charge distribution in a conductor is such that the electric field by the charge distribution in the conductor cancels the electric field of the external charges at all space points inside the body of the conductor.

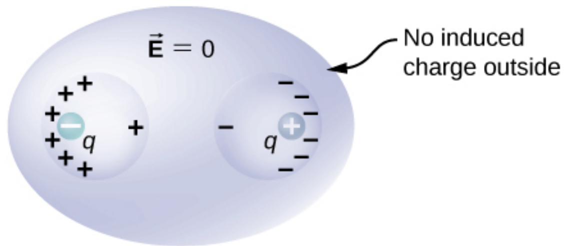

Figure 6.45 The charges induced by two equal and opposite charges in two separate cavities of a conductor. If the net charge on the cavity is nonzero, the external surface becomes charged to the amount of the net charge.

CHAPTER REVIEW

Key Terms

area vector vector with magnitude equal to the area of a surface and direction perpendicular to the surface

cylindrical symmetry system only varies with distance from the axis, not direction

electric flux dot product of the electric field and the area through which it is passing

flux quantity of something passing through a given area

free electrons also called conduction electrons, these are the electrons in a conductor that are not bound to any particular atom, and hence are free to move around

Gaussian surface any enclosed (usually imaginary) surface

planar symmetry system only varies with distance from a plane

spherical symmetry system only varies with the distance from the origin, not in direction

Key Equations

Definition of electric flux, for uniform electric field	$\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} \rightarrow EA \cos\theta$
Electric flux through an open surface	$\Phi = \int_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$
Electric flux through a closed surface	$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$
Gauss's law	$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \frac{q_{enc}}{\varepsilon_0}$
Gauss's Law for systems with symmetry	$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = E \oint_S dA = EA = \frac{q_{enc}}{\varepsilon_0}$
The magnitude of the electric field just outside the surface of a conductor	$E = \frac{\sigma}{\varepsilon_0}$

Summary

6.1 Electric Flux

- The electric flux through a surface is proportional to the number of field lines crossing that surface. Note that this means the magnitude is proportional to the portion of the field perpendicular to the area.
- The electric flux is obtained by evaluating the surface integral

$$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}},$$

  where the notation used here is for a closed surface S.

6.2 Explaining Gauss's Law

- Gauss's law relates the electric flux through a closed surface to the net charge within that surface,

$$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \frac{q_{enc}}{\varepsilon_0},$$

  where q_{enc} is the total charge inside the Gaussian surface S.
- All surfaces that include the same amount of charge have the same number of field lines crossing it, regardless of the shape or size of the surface, as long as the surfaces enclose the same amount of charge.

6.3 Applying Gauss's Law

- For a charge distribution with certain spatial symmetries (spherical, cylindrical, and planar), we can find a Gaussian surface over which $\vec{\mathbf{E}} \cdot \hat{\mathbf{n}} = E$, where E is constant over the surface.

The electric field is then determined with Gauss's law.

- For spherical symmetry, the Gaussian surface is also a sphere, and Gauss's law simplifies to $4\pi r^2 E = \frac{q_{\text{enc}}}{\varepsilon_0}$.
- For cylindrical symmetry, we use a cylindrical Gaussian surface, and find that Gauss's law simplifies to $2\pi r L E = \frac{q_{\text{enc}}}{\varepsilon_0}$.
- For planar symmetry, a convenient Gaussian surface is a box penetrating the plane, with two faces parallel to the plane and the remainder perpendicular, resulting in Gauss's law being $2AE = \frac{q_{\text{enc}}}{\varepsilon_0}$.

6.4 Conductors in Electrostatic Equilibrium

- The electric field inside a conductor vanishes.
- Any excess charge placed on a conductor resides entirely on the surface of the conductor.
- The electric field is perpendicular to the surface of a conductor everywhere on that surface.
- The magnitude of the electric field just above the surface of a conductor is given by $E = \frac{\sigma}{\varepsilon_0}$.

Conceptual Questions

6.1 Electric Flux

1. Discuss how to orient a planar surface of area A in a uniform electric field of magnitude E_0 to obtain (a) the maximum flux and (b) the minimum flux through the area.

2. What are the maximum and minimum values of the flux in the preceding question?

3. The net electric flux crossing a closed surface is always zero. True or false?

4. The net electric flux crossing an open surface is never zero. True or false?

6.2 Explaining Gauss's Law

5. Two concentric spherical surfaces enclose a point charge q. The radius of the outer sphere is twice that of the inner one. Compare the electric fluxes crossing the two surfaces.

6. Compare the electric flux through the surface of a cube of side length a that has a charge q at its center to the flux through a spherical surface of radius a with a charge q at its center.

7. (a) If the electric flux through a closed surface is zero, is the electric field necessarily zero at all points on the surface? (b) What is the net charge inside the surface?

8. Discuss how Gauss's law would be affected if the electric field of a point charge did not vary as $1/r^2$.

9. Discuss the similarities and differences between the gravitational field of a point mass m and the electric field of a point charge q.

10. Discuss whether Gauss's law can be applied to other forces, and if so, which ones.

11. Is the term $\vec{\mathbf{E}}$ in Gauss's law the electric field produced by just the charge inside the Gaussian surface?

12. Reformulate Gauss's law by choosing the unit normal of the Gaussian surface to be the one directed inward.

6.3 Applying Gauss's Law

13. Would Gauss's law be helpful for determining the electric field of two equal but opposite charges a fixed distance apart?

14. Discuss the role that symmetry plays in the application of Gauss's law. Give examples of continuous charge distributions in which Gauss's law is useful and not useful in determining the electric field.

15. Discuss the restrictions on the Gaussian surface used to discuss planar symmetry. For example, is its length important? Does the cross-section have to be square? Must the end faces be on opposite sides of the sheet?

6.4 Conductors in Electrostatic Equilibrium

16. Is the electric field inside a metal always zero?

17. Under electrostatic conditions, the excess charge on a conductor resides on its surface. Does this mean that all the conduction electrons in a conductor are on the surface?

18. A charge q is placed in the cavity of a conductor as shown below. Will a charge outside the conductor experience an electric field due to the presence of q?