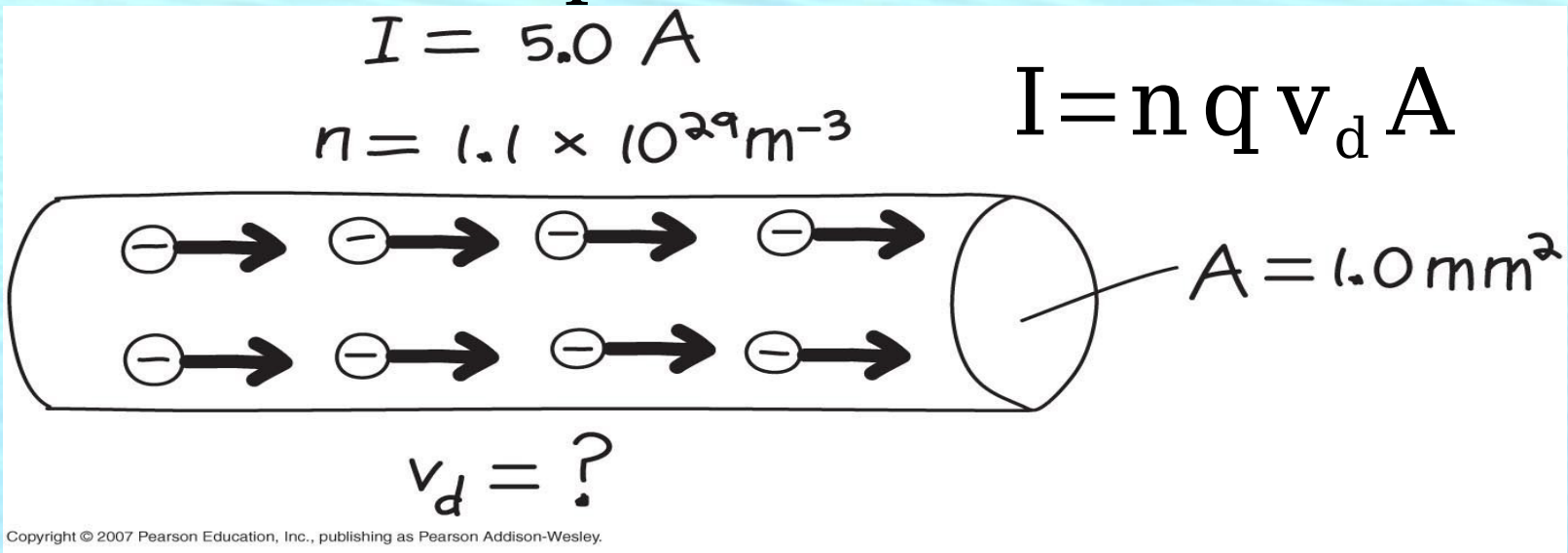
# Physics 122 – Class #24 – (4/9/15)

- Drude Model of resistivity
- Kirchoff's Laws
- Analyzing Series and Parallel Circuits
  - Equivalent resistance
- RC Circuits

# Drude Model of Conductance

You can derive Ohm's law by assuming a metal is a box full of loose electrons that bump into "scattering centers" every $\tau$ Seconds. (Tau is < 1 picosecond for most solids at room temp.)




$$I = n q v_d A$$

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$$\vec{J} = n q \vec{v}_d = n q a \tau = \frac{n q^2 \tau}{m} \vec{E} = \sigma \vec{E}$$

**Derived electrical resistance from Classical Mechanics!**

$$\sigma = \frac{n q^2 \tau}{m}$$

In semiconductor, n is small, so rho is larger than a conductor. In insulator, n is nearly zero. $\tau$

$$\rho = \frac{m}{n q^2 \tau}$$

In ionic conductor, m is larger (ions not electrons) so conductivity is Lower.

A semiconductor may be “doped”

# DC Circuits I – Basic ideas

Current leaves + terminal of battery, passes through circuit and returns to – terminal.

Current does NOT get "used up"

Voltage drops as you move through the circuit.

Energy gets "used up" (the energy put out by the battery gets used in each of the "load" resistors.

Light bulbs get brighter if you put more power through them.

# DC Circuits II – Kirchoff's Laws

**Voltage drops sum to zero around any loop in a circuit**

Sum of voltages across all series elements = battery voltage.

Voltages are same across all parallel elements.

**Current into a node is same as current out of a node**

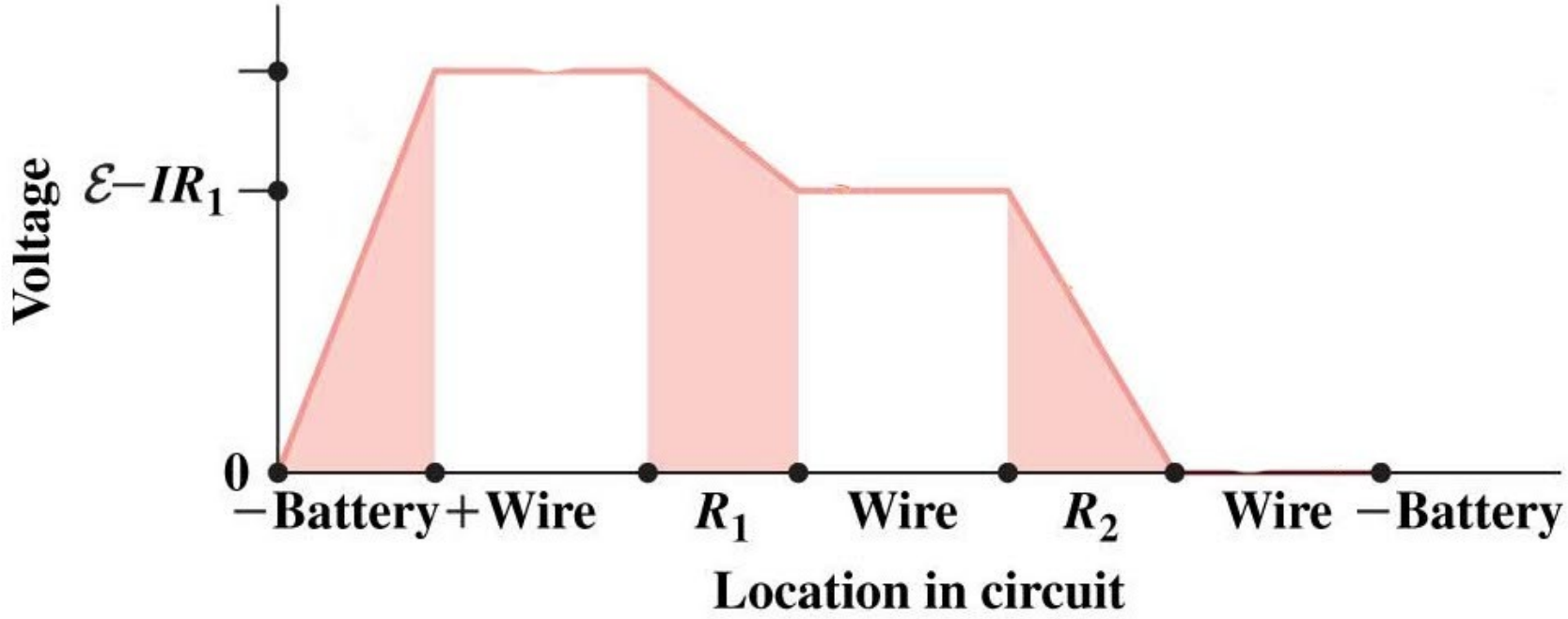
Current is the same in all series elements.

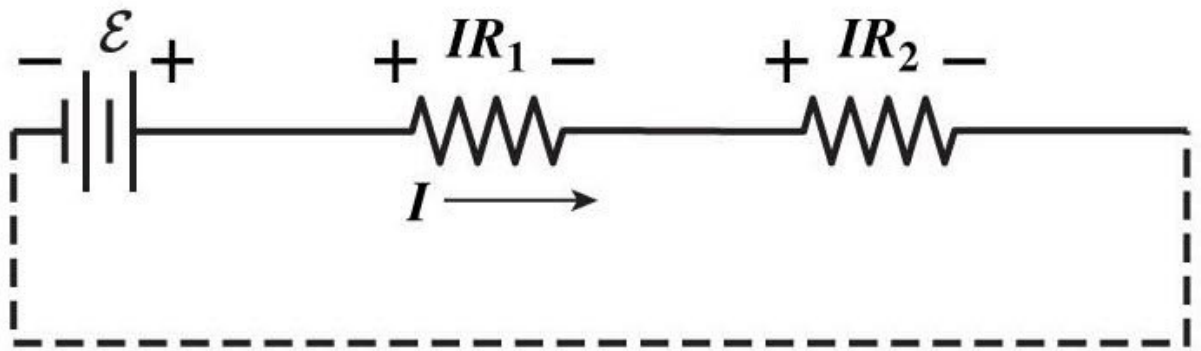
Current splits across parallel elements.

**Series circuits:**

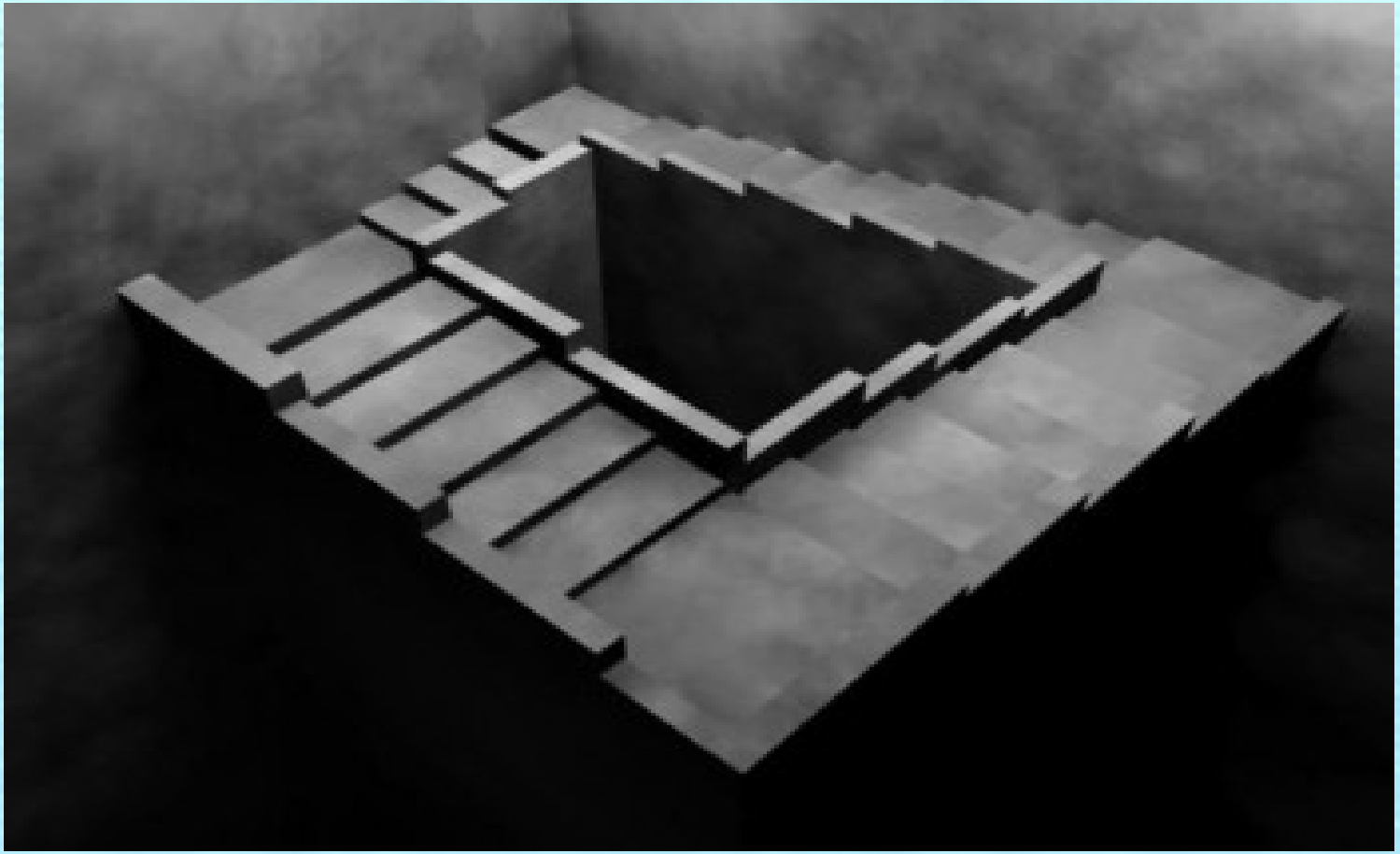
**Same current in every part of circuit**

**Voltage drops at every resistor**



**You can't keep going downhill (in voltage)**
**in a full loop!  (Unless you live inside an M. C. Escher drawing.)**

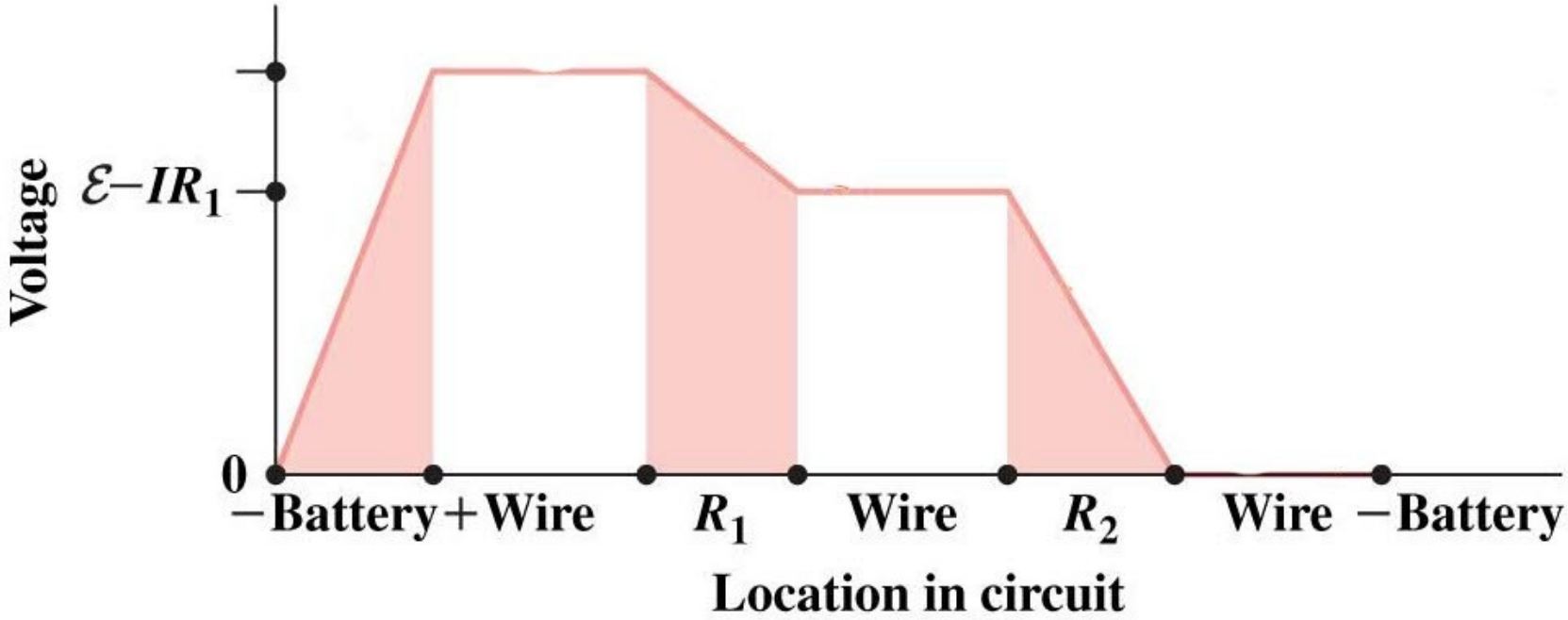


**Kirchoff's Voltage Rule (Loop rule):**

Sum of voltage drops around circuit (including battery) is zero.

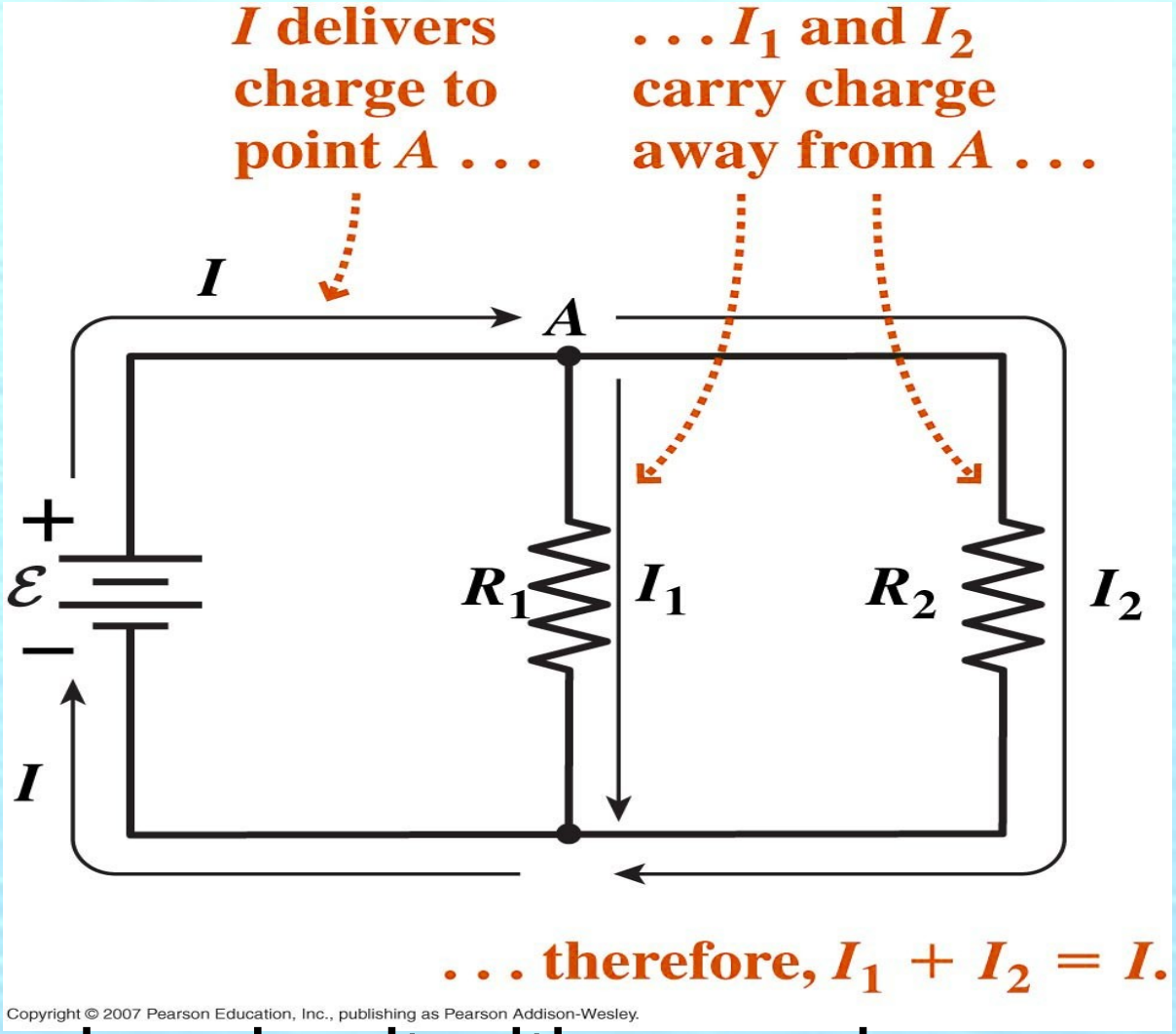
OR

Sum of voltage drops across circuit element = battery voltage.



# Kirchoff's Current Rule (Node rule):

Sum of currents entering any node is sum of currents leaving the node.

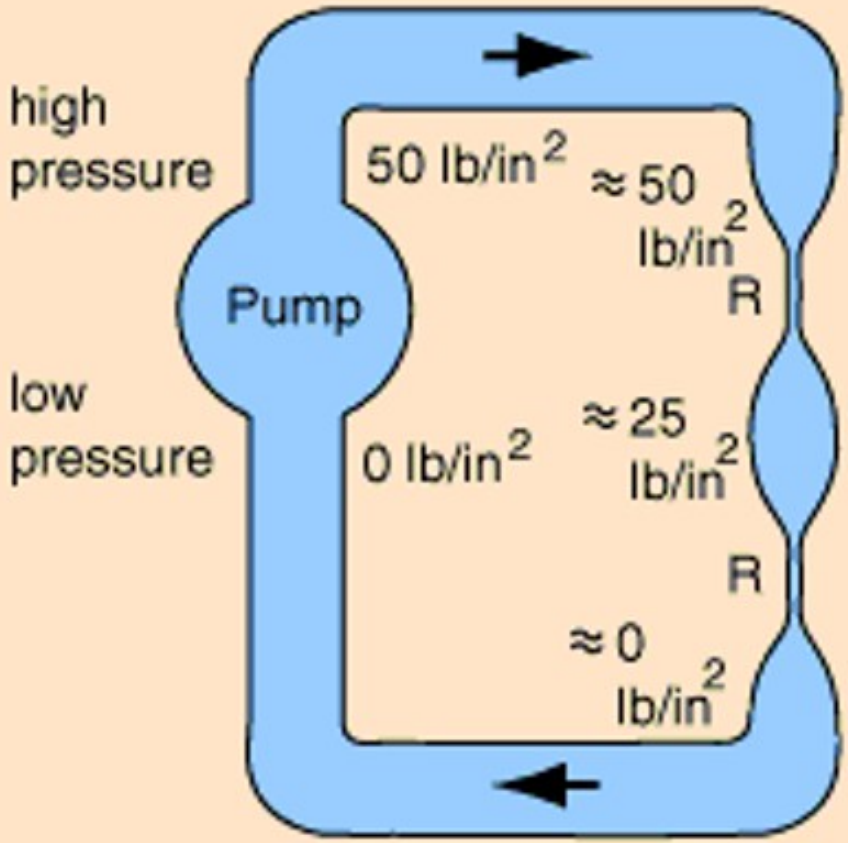


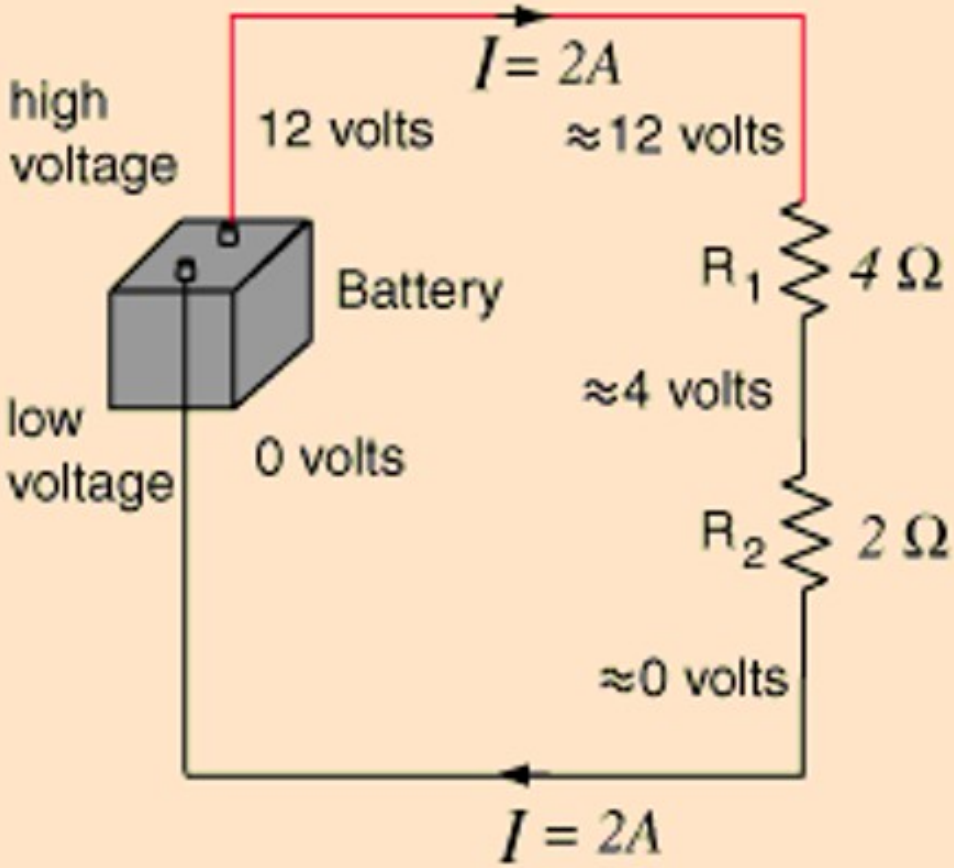

So in a series circuit with no nodes, current is same everywhere.

**Series circuits:**

Same current in every part of circuit.

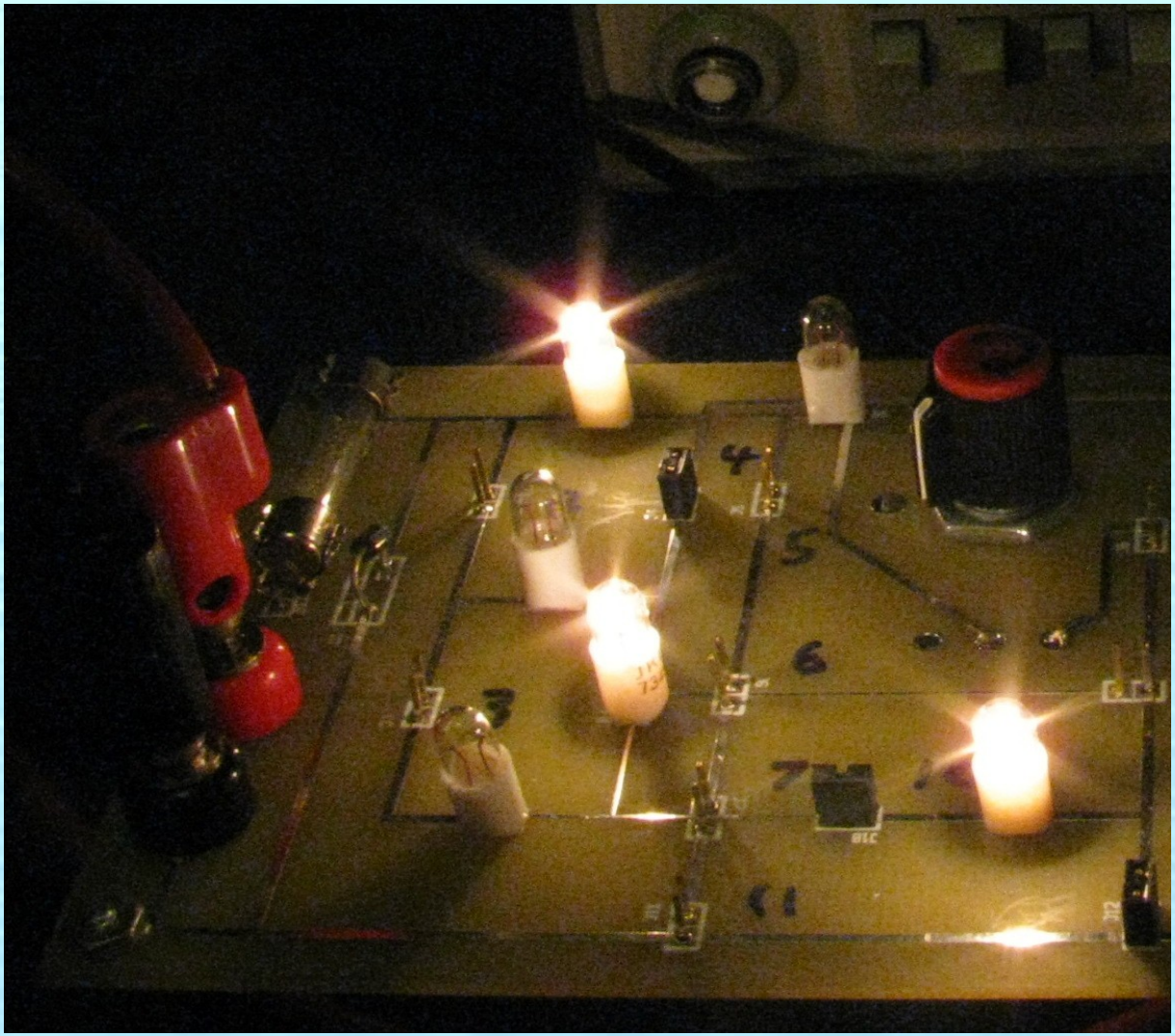
Voltage drops at every resistor.



# Series circuits:

## All light bulbs equally bright …

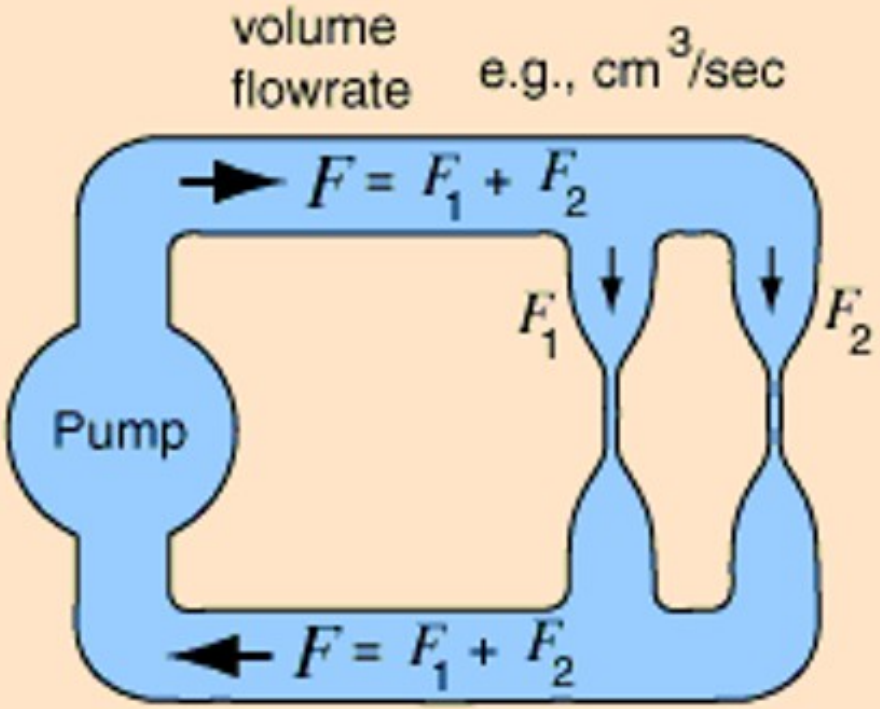


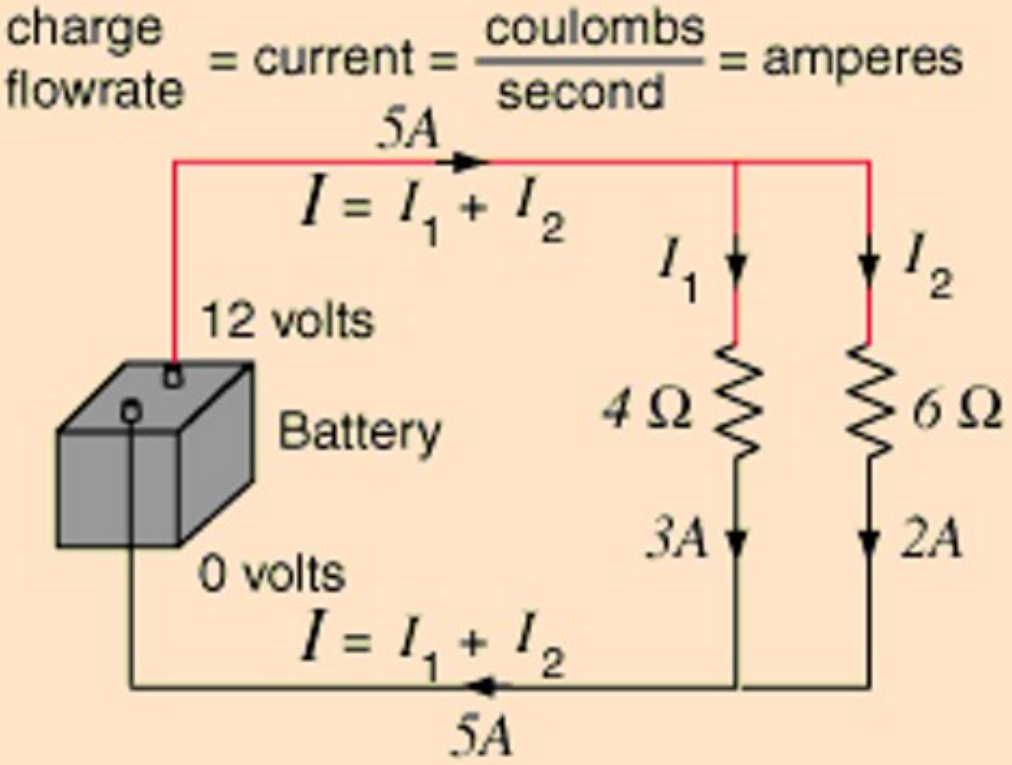
**Parallel circuits:**

**Same voltage drop across resistors.**

**Current splits between resistors.**

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$



**Parallel circuits:**

**Same voltage drop across resistors.**

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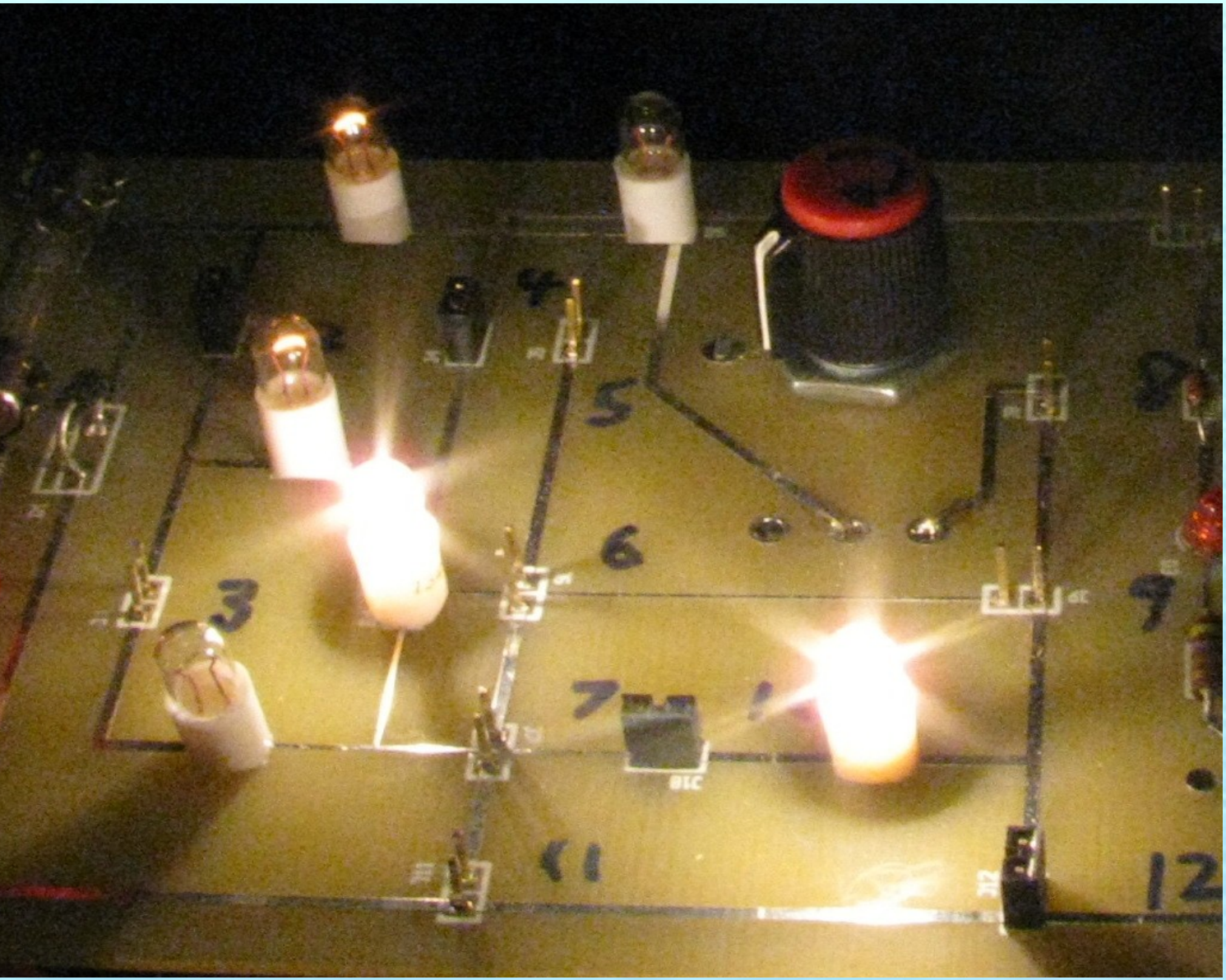
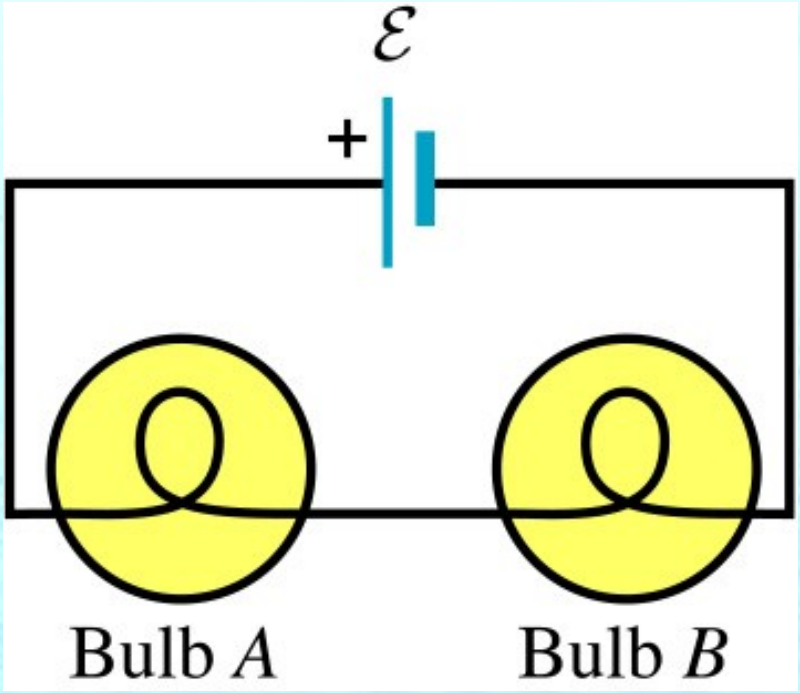


**Clickers:** A 2.0 Ω resistor is in series with a 6.0 Ω resistor. If the 2.0 Ω resistor has a current of 1.0 Amp, what is the current through the 6.0 Ω resistor?

[A] also 1.0 Amp
[B] 0.333 Amp
[C] 6.0 Amps
[D] 3.0 Amps
[E] Need more information to decide.

In the circuit shown, the two bulbs *A* and *B* are identical. Compared to bulb *A*,



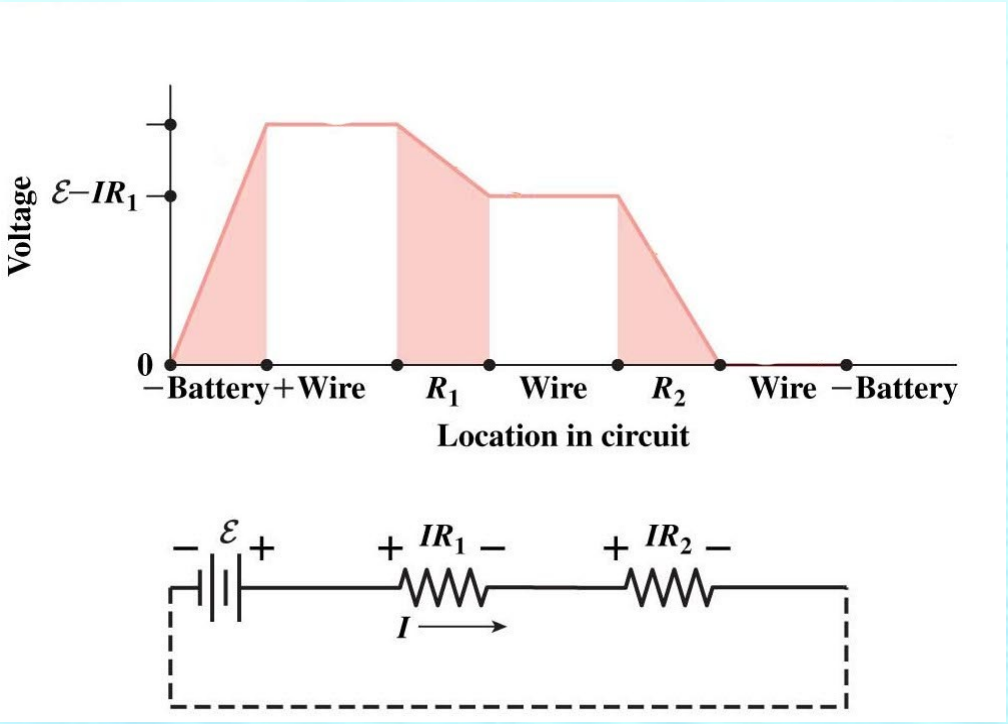

A. bulb *B* glows more brightly

B. bulb *B* glows less brightly

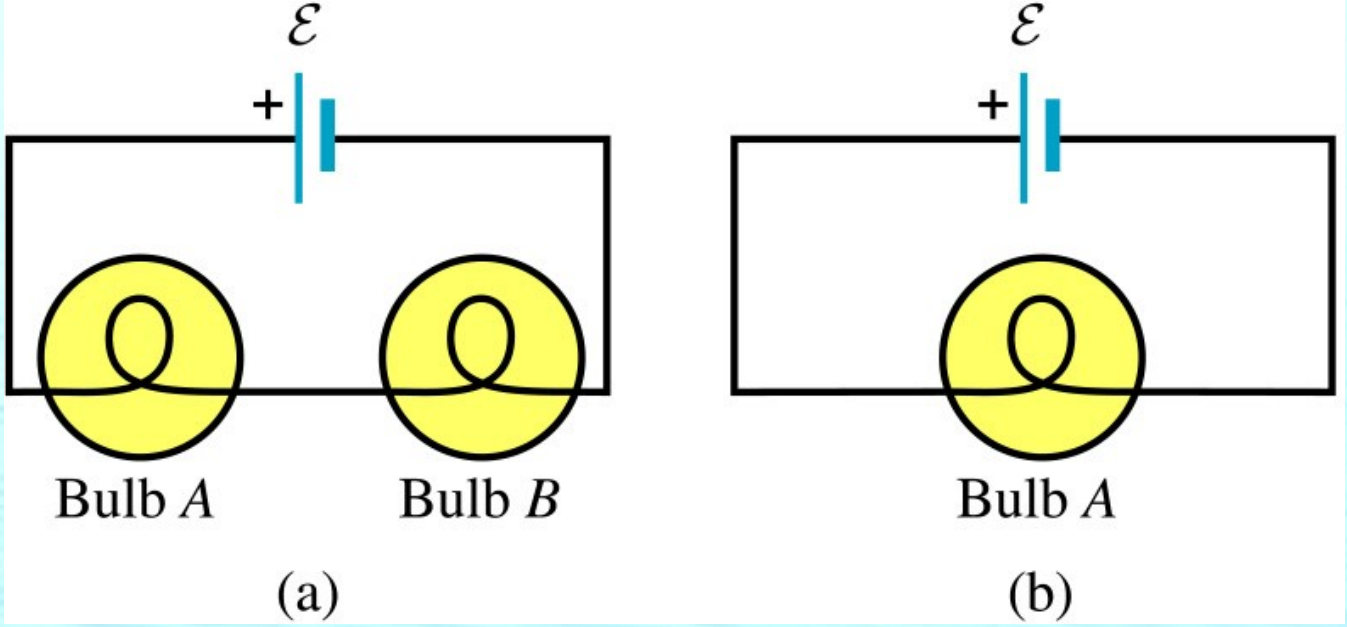
C. bulb *B* glows just as brightly

D. answer depends on whether the mobile charges in the wires are positively or negatively charged

# pHeT




http://phet.colorado.edu/en/simulation/circuit-construction-kit-dc




In the circuit shown in (a), the two bulbs *A* and *B* are identical. Bulb *B* is removed and the circuit is completed as shown in (b). Compared to the brightness of bulb *A* in (a), bulb *A* in (b) is

A. brighter

B. less bright

C. just as bright

D. any of the above

# Resistors and Capacitors in Series & Parallel

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

$$R_{series} = R_1 + R_2 + R_3 + \ldots$$

$$C_{parallel.} = C_1 + C_2 + C_3 + \ldots$$

$$\frac{1}{R_{parallel.}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Resistors(and capacitors) in parallel have equal voltages.

Resistors (and capacitors) in series have equal currents.

# PheT – Circuit Lab

**To observe.**

Voltage sums to zero around a circuit.

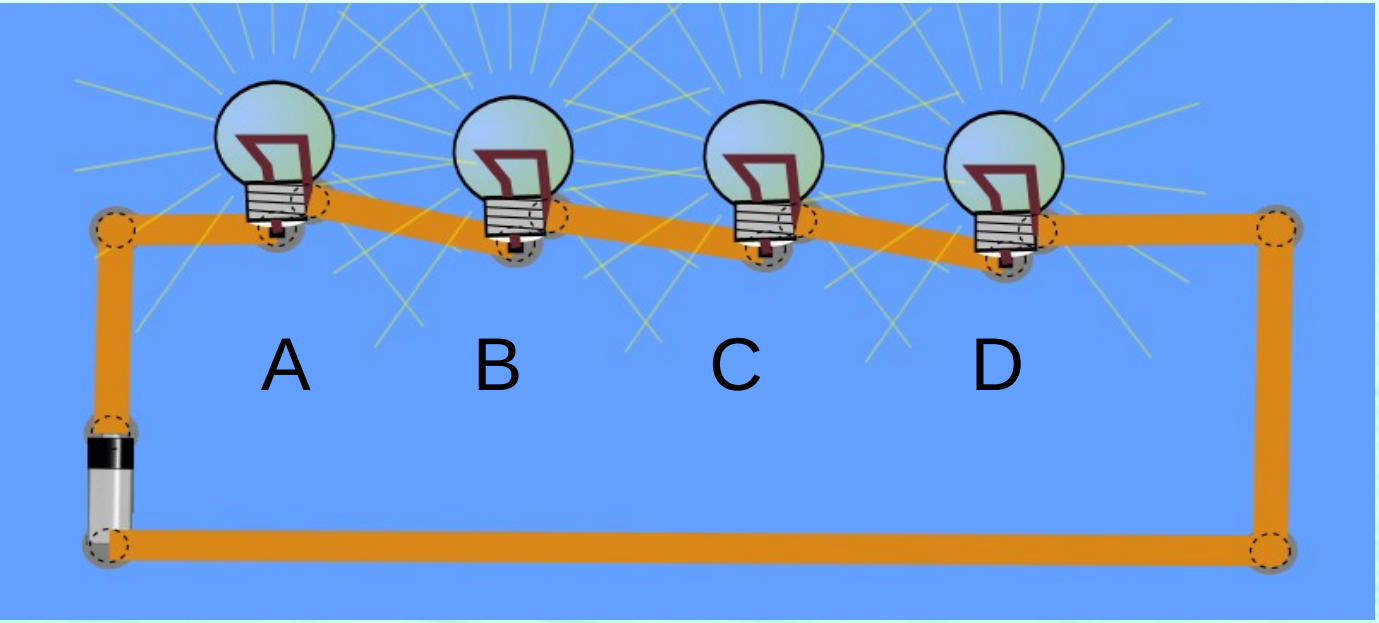
Voltage drops across every resistor, and is constant along wires.

Batteries are voltage sources … current is set by equivalent resistance.

Current is same everywhere in series.

Current splits across parallel legs.

Lightbulb brightness is proportional to power dissipated in each bulb




Four light bulbs are in series as shown. Light bulb “C” Is removed. What happens?

A. Bulbs “A” and “B” and “D” remain lit the same brightness

B. Bulbs “A”, “B”, and “D” remain lit and get brighter

C. Bulbs “A” and “B” get brighter, and bulb “D” goes out

D. Bulbs “A” and “B” remain lit thes same brightness, and bulb “D” goes out

E. Bulbs “A”, “B” and “D” go out.

# Properties of Equivalent Resistors

If you replace a circuit with its equivalent resistor then:

1) The current through the equivalent resistor is the same as the TOTAL current through the original circuit.

2) The power used by the equivalent resistor is the same as the TOTAL power used by the original circuit.

3) The sum of the power used at each original resistor equals the power used by the equivalent resistor.

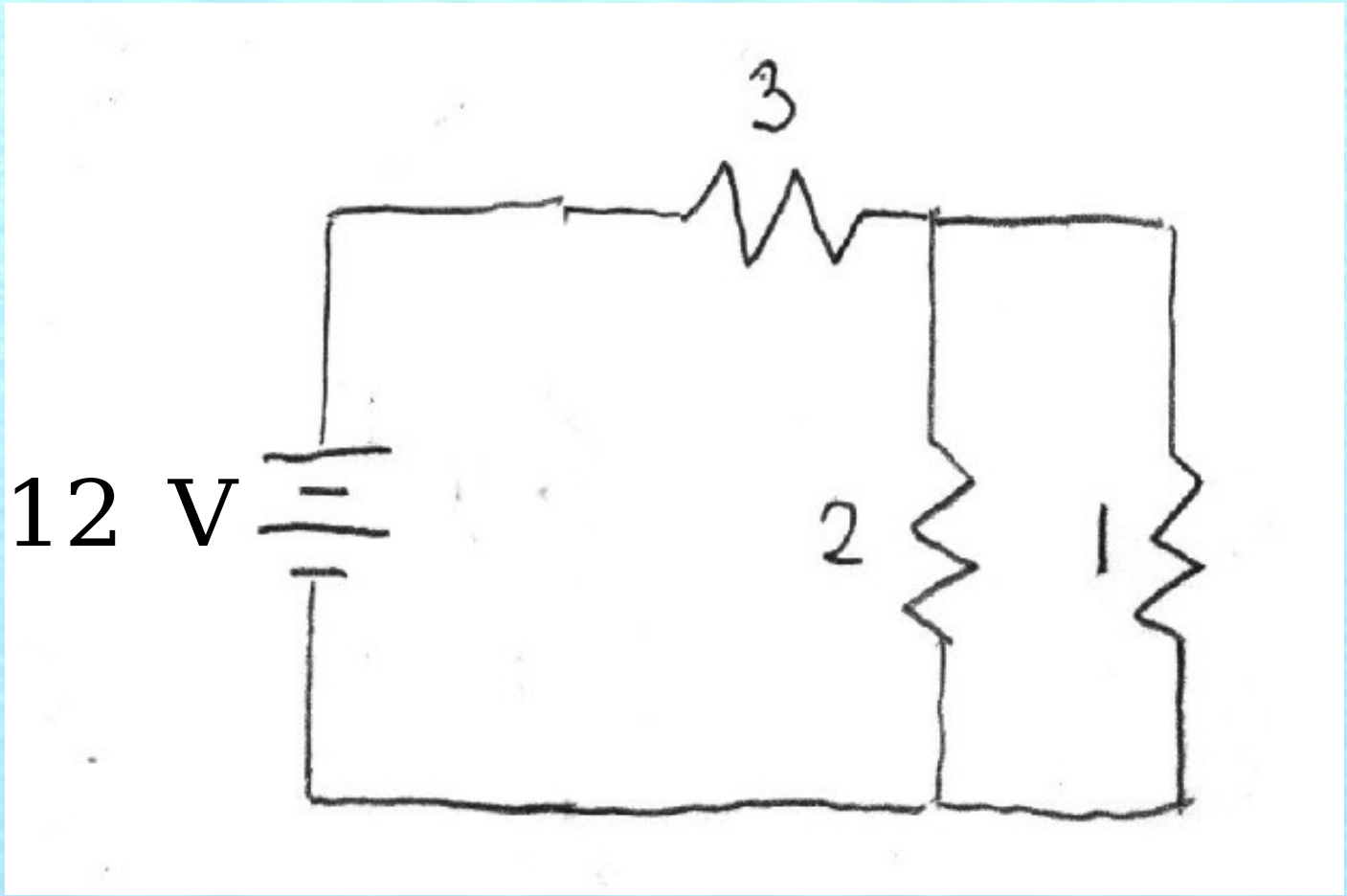
4) The sum of the currents thru all parallel legs is the same as the current through the equivalent resistor.

What is the equivalent resistance of this circuit?

What is the voltage across the 2-Ohm resistor?

What is the voltage across the 3-Ohm resistor?

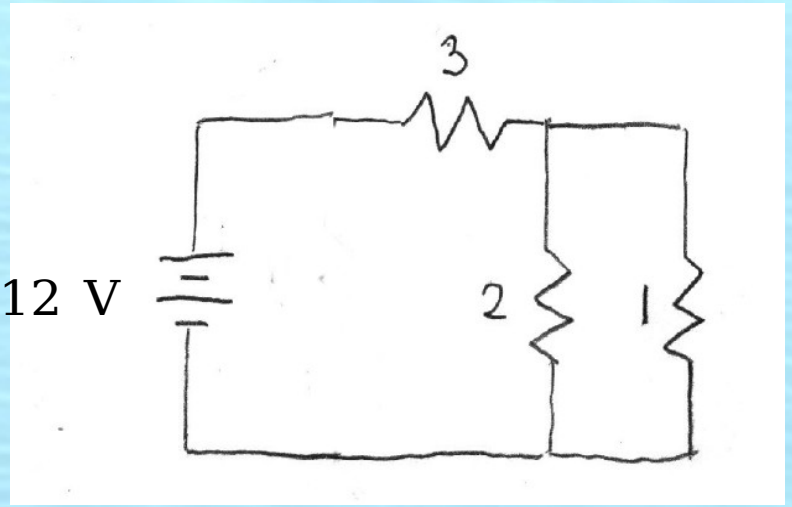
$V_B = 12$ V



$R_{eq}=?...$

$\Delta V_2=?...$

$\Delta V_3=?...$




$V_B=12\ V$

# Solving resistor network problems

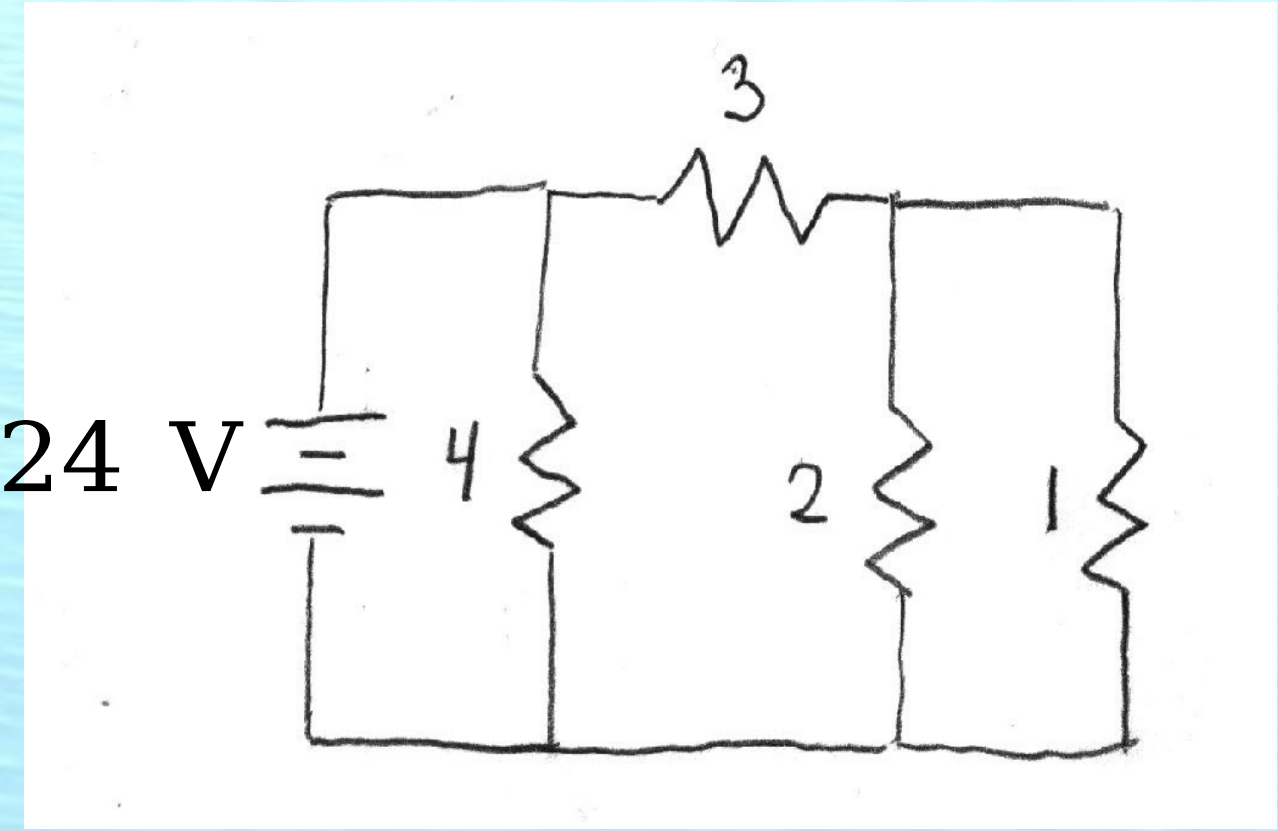
1) Redraw the circuit as needed to see what parts are in series and what parts are in parallel.
2) Replace each section with an equivalent resistor for that section.
3) Combine equivalent resistors to get down to a single resistor.  Get total current.
4) Apply current to all series sections to get voltage drop.
5) Apply voltage drop to all parallel sections to get current.

What is the equivalent resistance of this circuit?

What is the voltage across the 2-Ohm resistor?

What is the voltage across the 3-Ohm resistor?

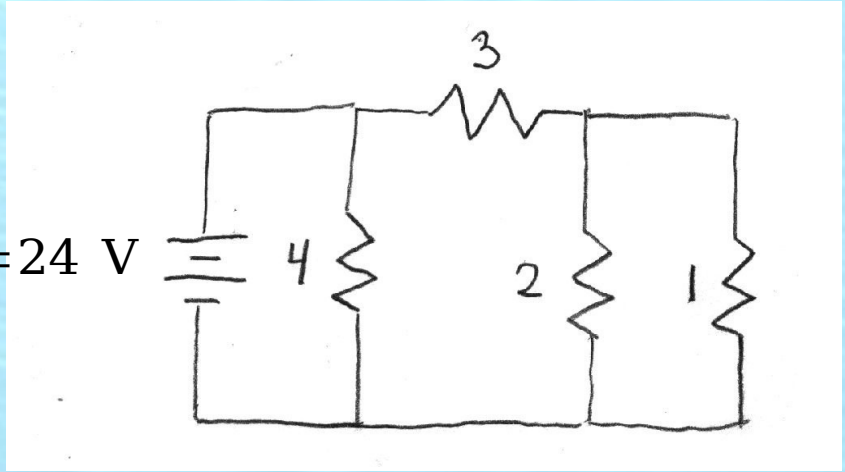
$V_B = 24$ V



$R_{eq}$=?...

$\Delta V_2$=?...

$\Delta V_3$=?...



Redraw this circuit to show parallel/series combos better.
Given V = 11 Volts. Example of Kirchhoff's Voltage law.

$I_4 = \frac{11}{4}$ A $\quad R_{eqv_{12}} = \frac{2}{3}$ Ω $\quad R_{eqv_{123}} = \frac{11}{3}$ Ω

$I_3 = 3$ A

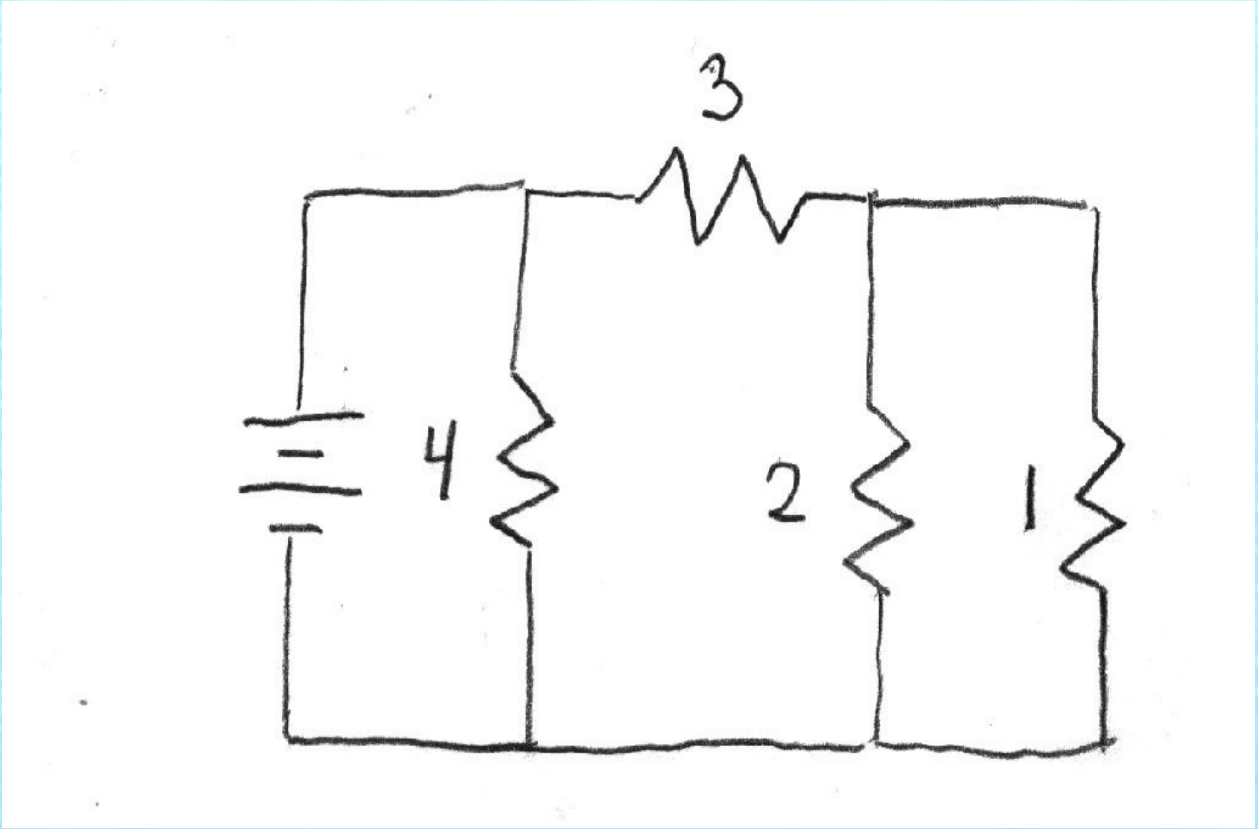
$I_1 = 2$ A

$I_2 = 1$ A

$V_3 = 9$ V

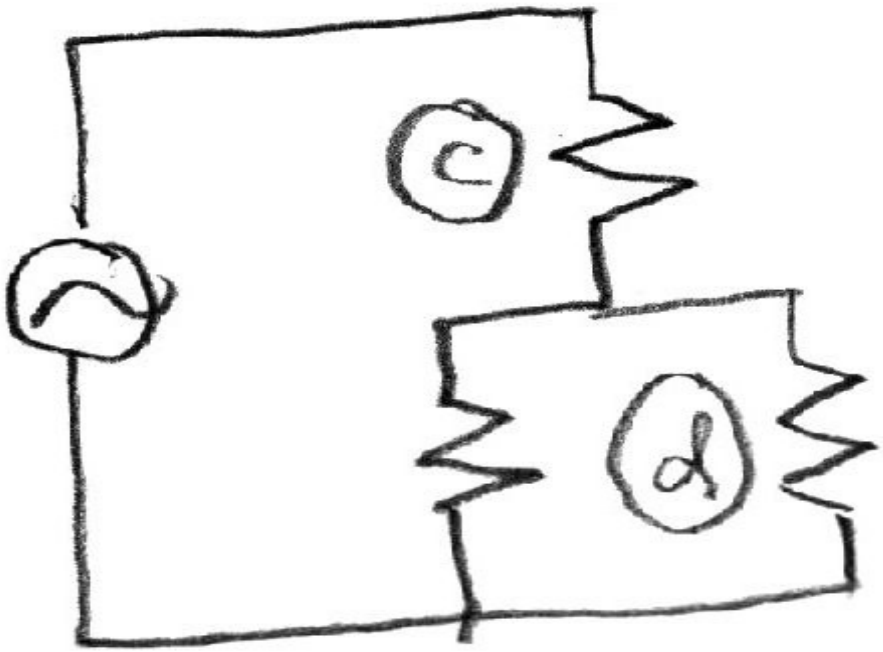
$I_{total} = 5.75$ A

$R_{eqv_{total}} = \frac{44}{23}$ Ω



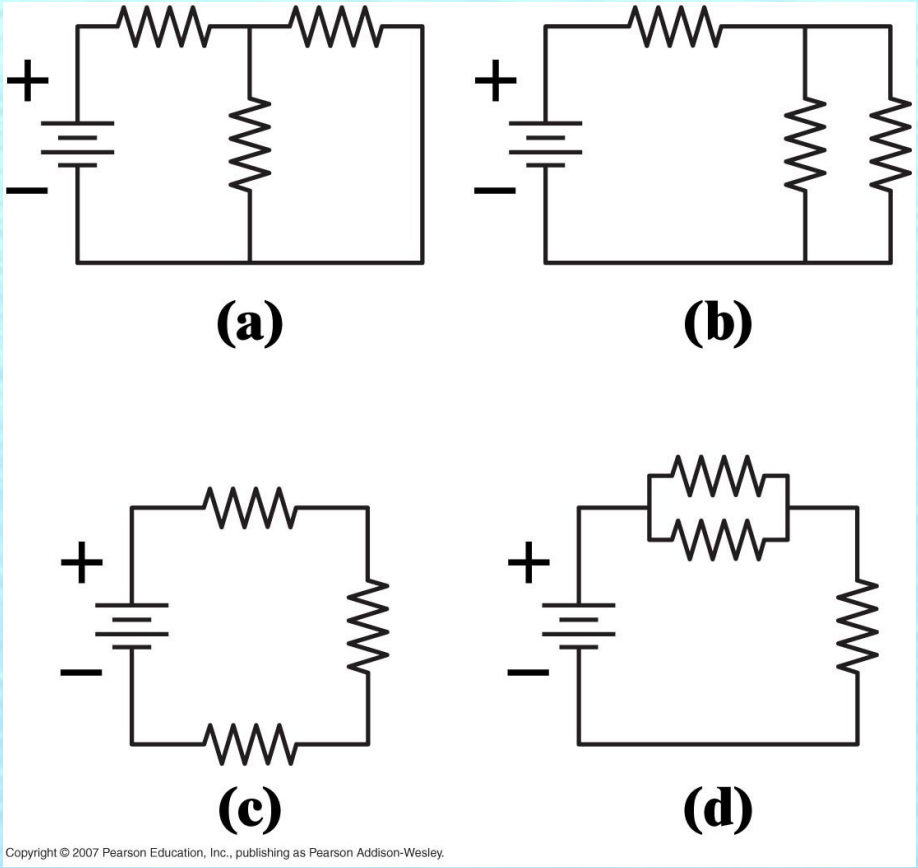
Each resistor is 1 Ohm. The equivalent resistance of this circuit is?

(A) 1 Ohm
(B) 3 Ohms
(C) 1/3 Ohm
(D) 2.5 Ohms
(E) 1.5 Ohm



All of the resistors shown below are equal. All of the batteries have the same EMF (or voltage).

Which circuit below draws the smallest current from its battery?



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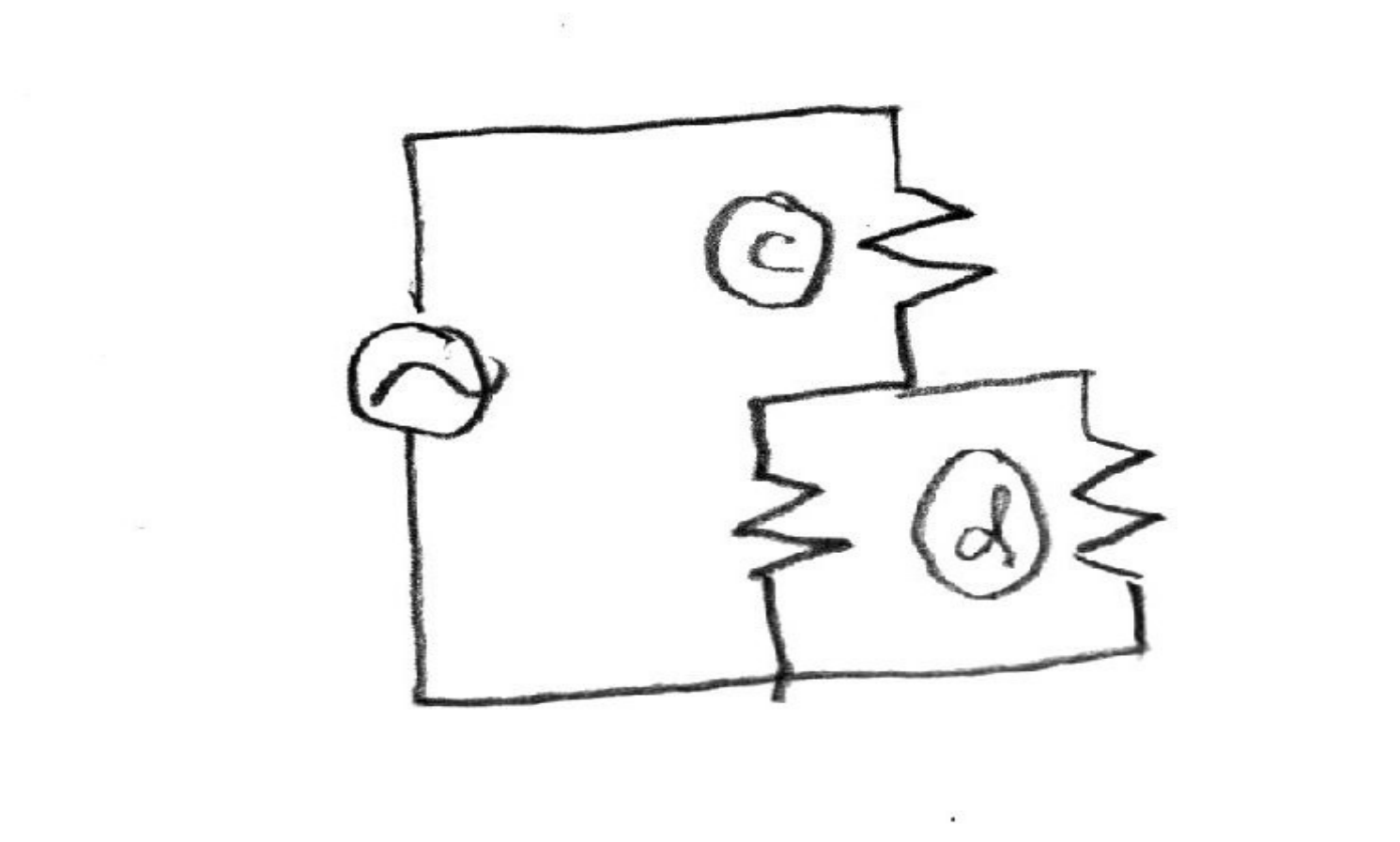
# Do Problem 31.60

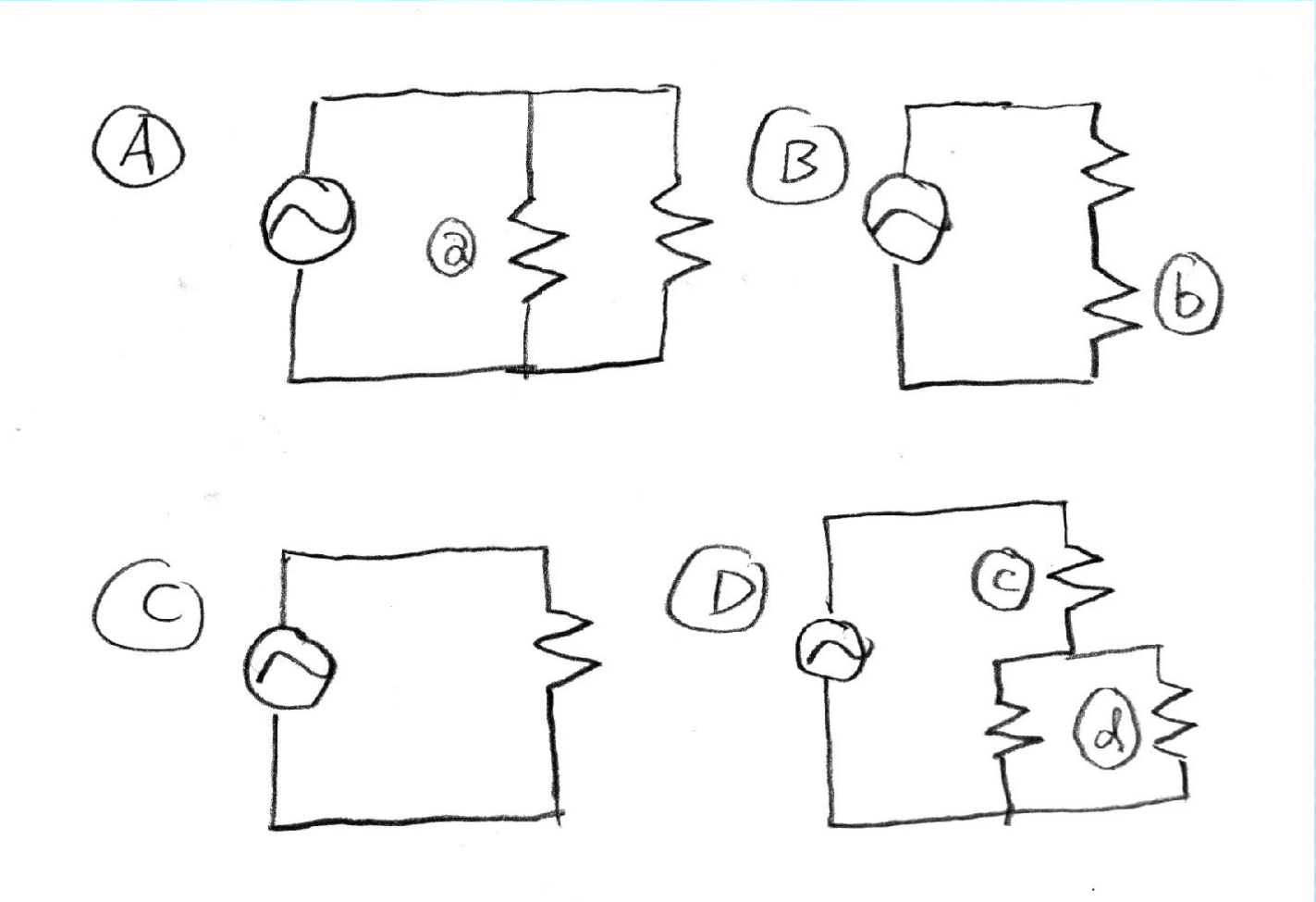
If all three resistors have the same value, and the power-supply voltage is 9-V, the voltage across resistor “d” is:

(A) 3 V
(B) 4.5 V
(C) 6 V
(D) 9 V



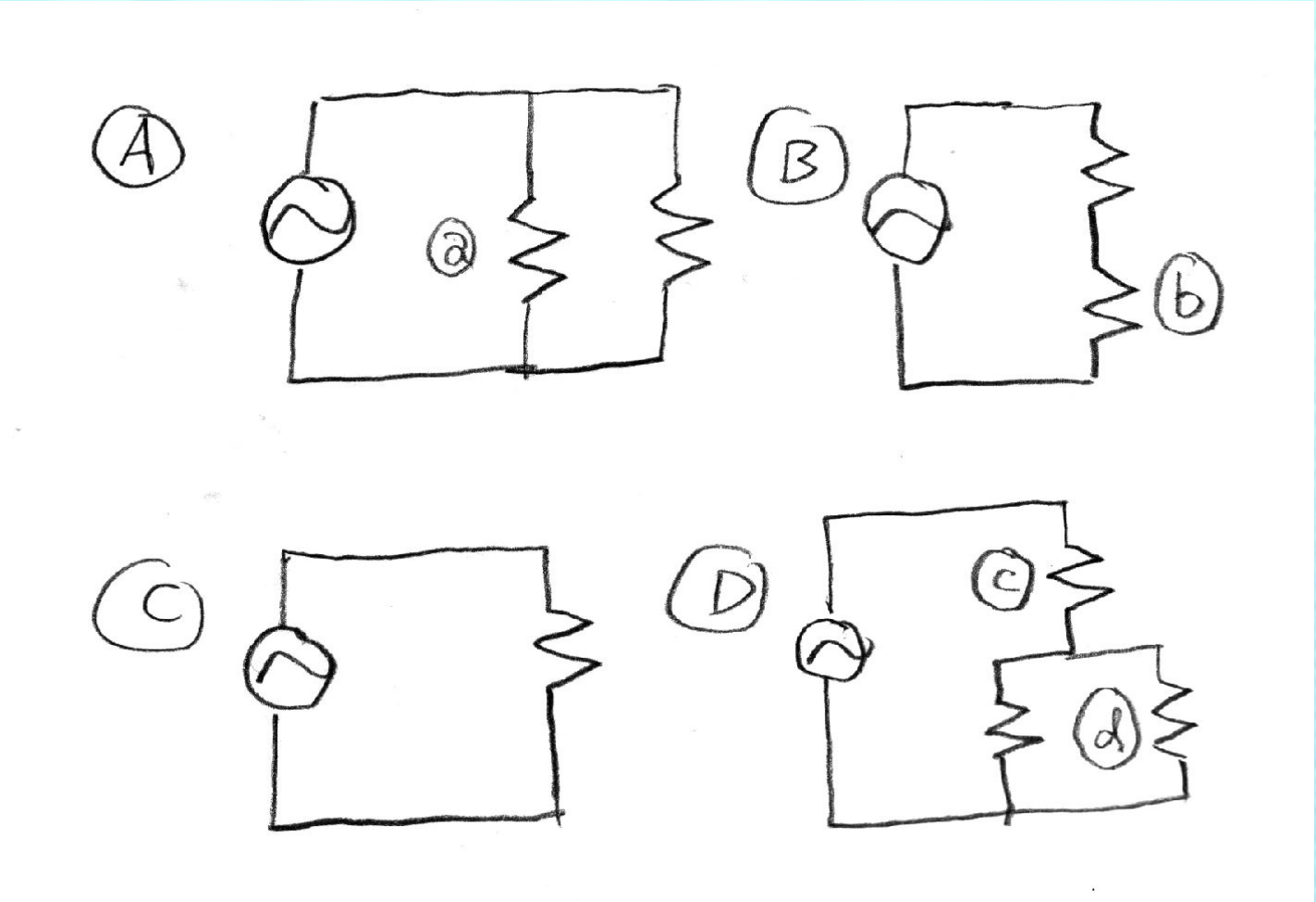
All of the resistors have the same value.
Each of the power supplies is a constant voltage supply (and all are equal).
Which circuit uses the most power?

(A) A
(B) B
(C) C
(D) D



All of the resistors have the same value. Each of the power supplies is a constant voltage supply (and all are equal). Which circuit uses the *least* power?

(A) A
(B) B
(C) C
(D) D



**Clickers:** You have a single 60-W light-bulb attached to an “ideal” battery.
You want to add
another 60-W bulb to make the room brighter. How should it be added?

[A] Put it in series with first bulb.

[B] Put in in parallel with first bulb.

[C] Both “A” and “B” will make the room brighter. (Brightness set by Wattage of lightbulb)

[D] Neither “A” or “B” will help – the battery puts out constant power.

# RC Circuits

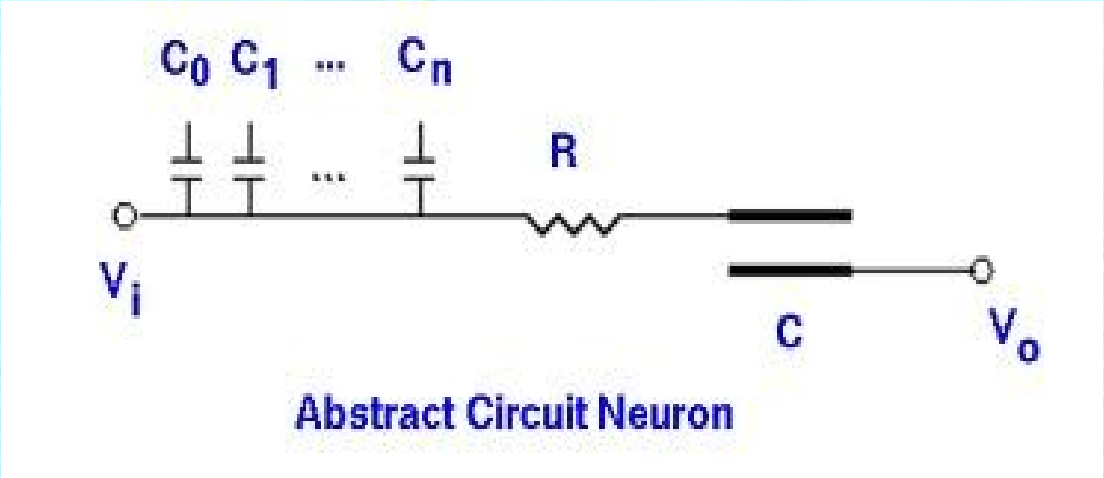
A circuit with both a resistor and a capacitor in it is called an “RC-circuit”.

A circuit of pure resistors comes to equilibrium in less than a picosecond.

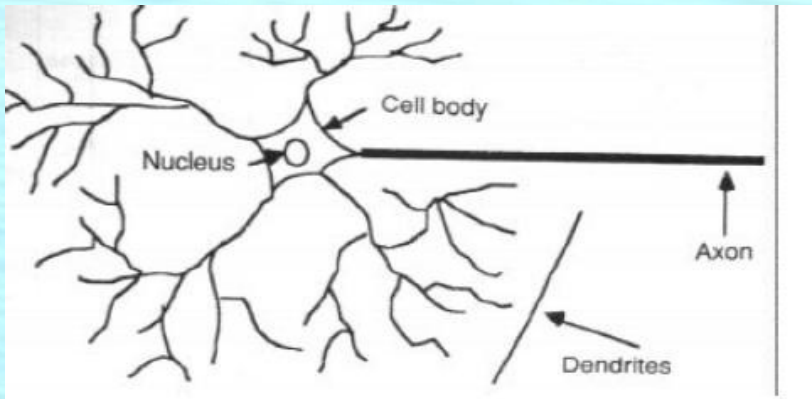
RC circuits have a characteristic time $(\tau = RC)$ Before they reach equilibrium.

# For the Biology majors …
# (or anyone with neurons)

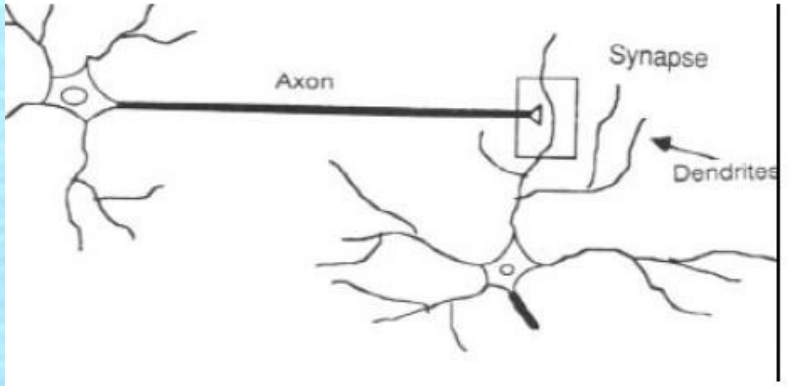
Try Googling “RC model of a neuron”.



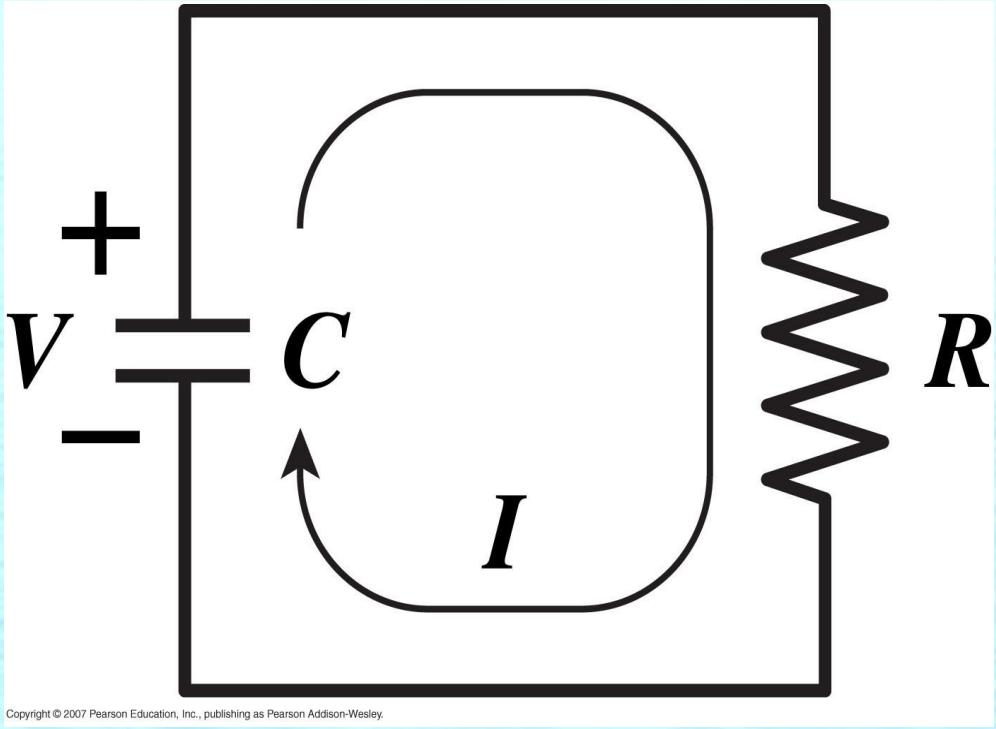
Abstract Circuit Neuron



Components of a Biological Neuron



The Synapse



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If capacitor begins charged ---> $Q_C = Q_0 e^{-t/RC}$

$V_C = V_0 e^{-t/RC}$

$I = I_0 e^{-t/RC}$

# Is an Ohm Farad really a second?

$$Q = C\,V \rightarrow C = \frac{Q}{V} \rightarrow [\text{Farad}] = \frac{[\text{Coul}]}{[\text{Volt}]}$$

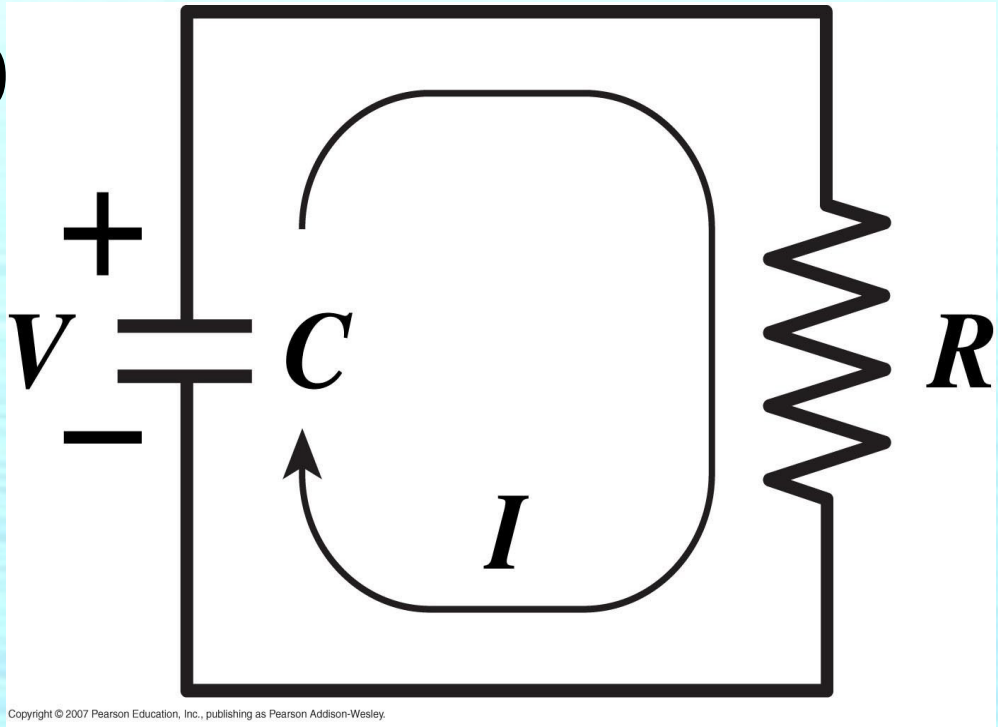
$$V = I\,R \rightarrow R = \frac{V}{I} \rightarrow [\text{Ohm}] = \frac{[\text{Volt}]}{[\text{Amp}]}$$

$$RC = [\text{Ohm}][\text{Farad}] = \frac{[\text{Volt}]}{[\text{Amp}]} \times \frac{[\text{Coul}]}{[\text{Volt}]}$$

$$RC = \frac{[\text{Coulomb}]}{[\text{Amp}]} = \frac{[\text{Coul}]}{[\text{Coul/second}]} = \text{seconds}!$$

If capacitor begins charged …

$$V + IR = 0 \rightarrow \frac{Q}{C} + \frac{dQ}{dt} R = 0$$



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If capacitor begins charged …

$$V + IR = 0 \rightarrow \frac{Q}{C} + \frac{dQ}{dt} R = 0$$

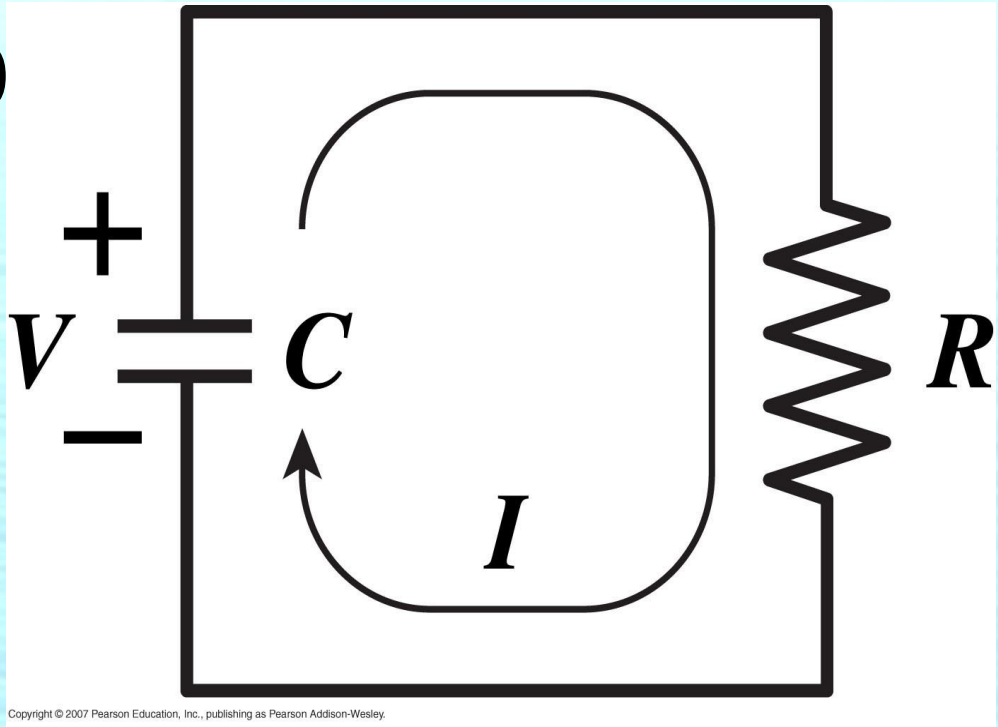
$$\frac{dQ}{dt} = -\frac{Q}{RC}$$

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

$$\int \frac{dQ}{Q} = -\frac{1}{RC} \int dt$$

$$\ln Q = -\frac{t}{RC}$$

$$Q = Q_0 e^{-t/RC}$$



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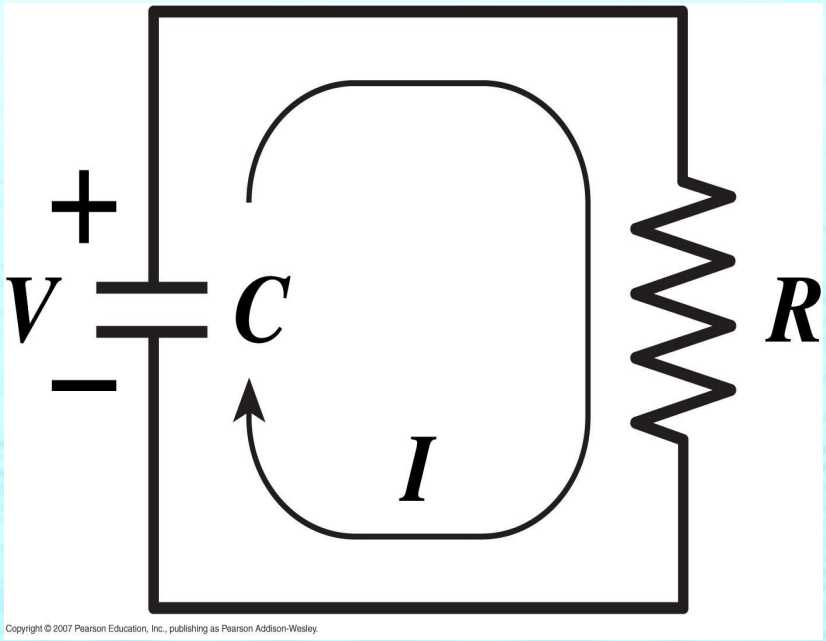
$$V_C = \frac{Q_0}{C} e^{-t/RC}$$

What is I(t)?

Given $Q(t) = Q_0 e^{-t/RC}$

And $I(t) = \dfrac{-dQ}{dt}$

What is $I(t)$ ?



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[A] $Q_0 e^{-t/RC}$

[B] $Q_0 RC e^{-t/RC}$

[C] $-Q_0 RC e^{-t/RC}$

[D] $\dfrac{1}{RC} e^{-t/RC}$

[E] $\dfrac{Q_0}{RC} e^{-t/RC}$

# RC Circuits

RC circuits have a characteristic time $(\tau = RC)$ Before they reach equilibrium. Note that R and C never appear alone

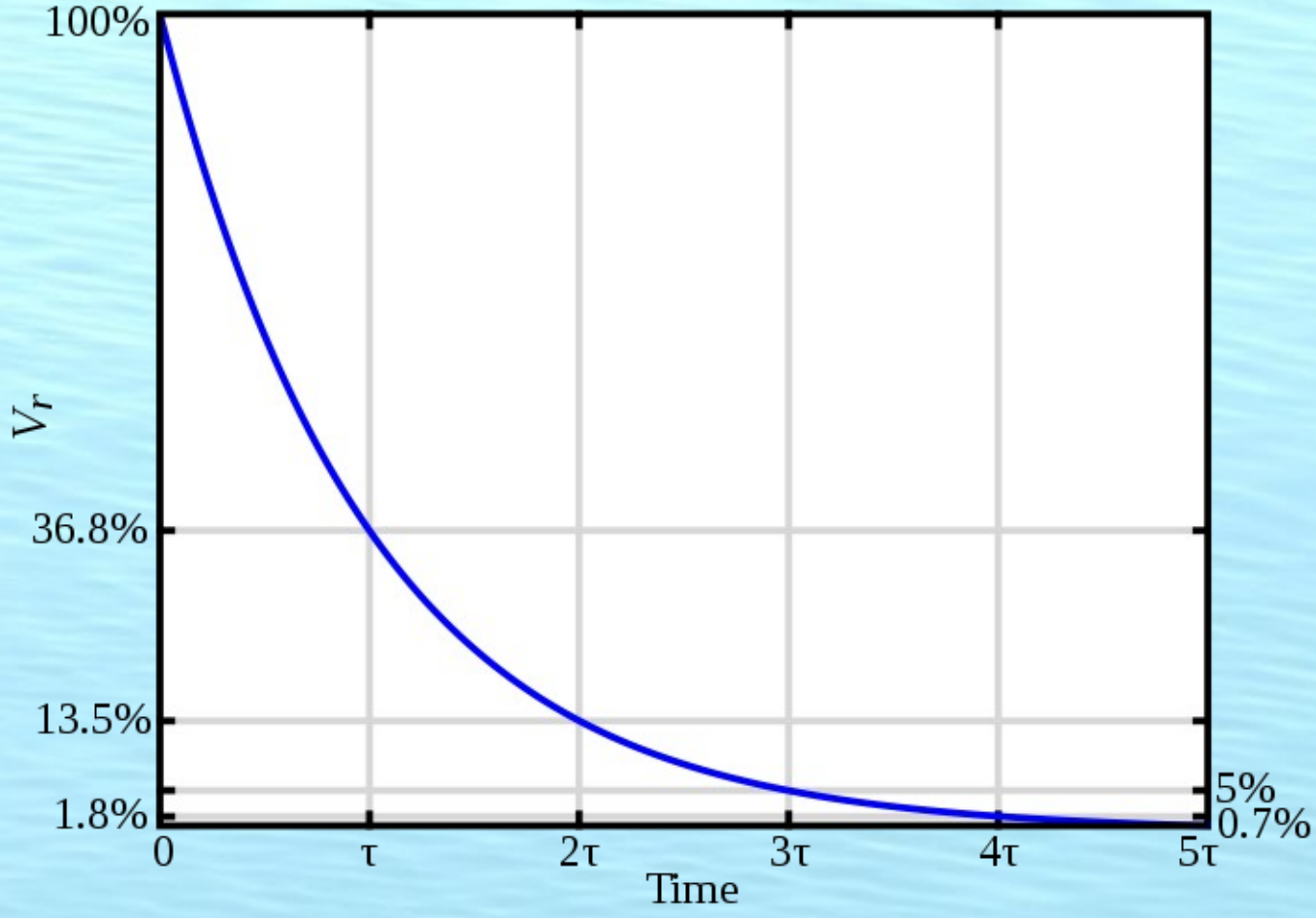
$$Q = Q_0 e^{-t/RC}$$

$$I = I_0 e^{-t/RC}$$

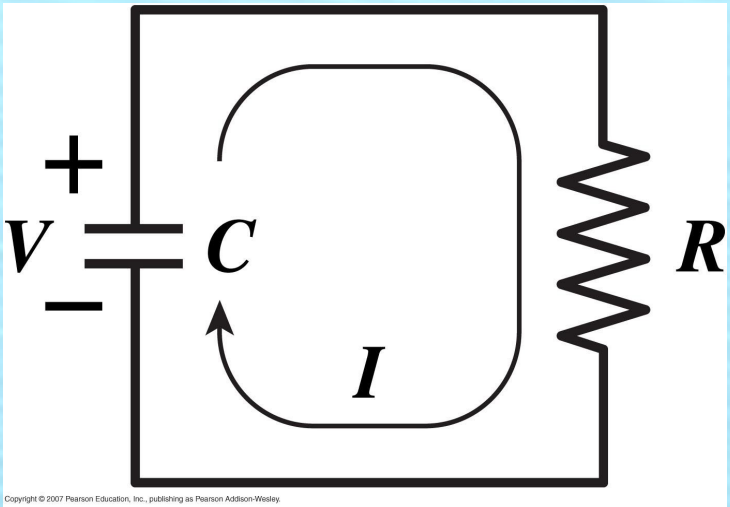
$$I = -\frac{Q_0}{RC} e^{-t/RC}$$

$$V_C = V_0 e^{-t/RC}$$

$$V_C = \frac{Q_0}{C} e^{-t/RC}$$



A 200 $\mathrm{k}\Omega$ resistor is in series with $\mathrm{C}=10\ \mu\mathrm{F}$. A t=0, the voltage on the capacitor is 8 V. What is the voltage on the capacitor and the current through the resistor after 1 sec, 2 sec and 4 seconds?



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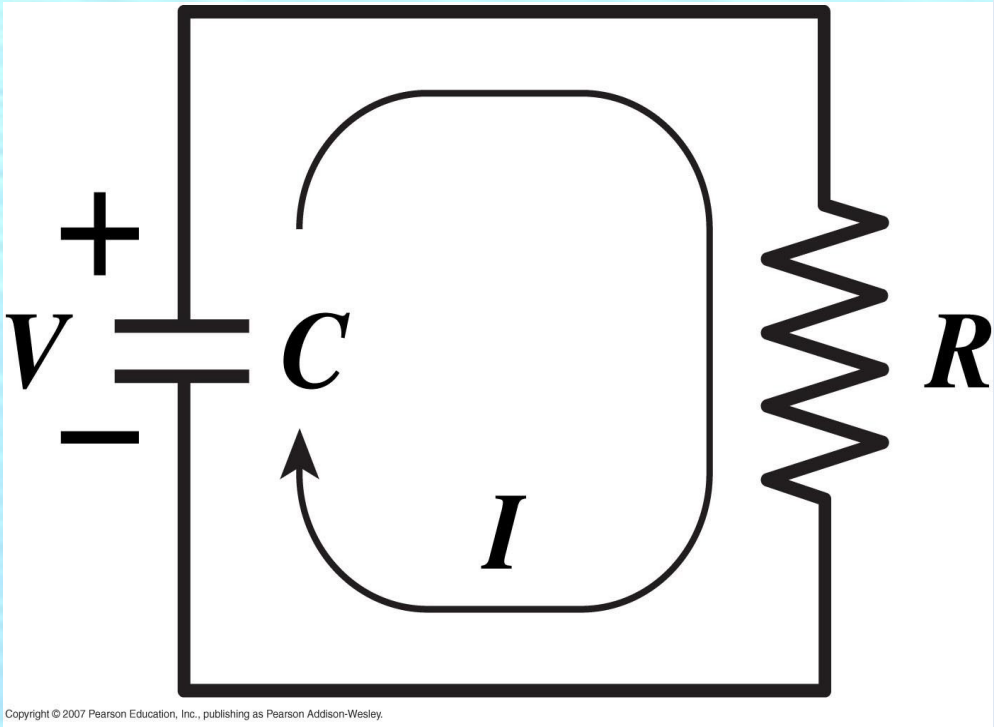
$$I=I_0 e^{-t/RC} \qquad I=\frac{V_0}{R}e^{-t/RC}$$

$$V=V_0 e^{-t/RC}$$

$$RC=(2.0\times10^5\ \Omega)(1.0\times10^{-5}\ \mathrm{F})$$

$$V(4)=8e^{-4.00\mathrm{s}/2.00\mathrm{s}}\ \text{Volts}$$

$$V(4)=8\times0.135=1.08\ \text{Volts}$$



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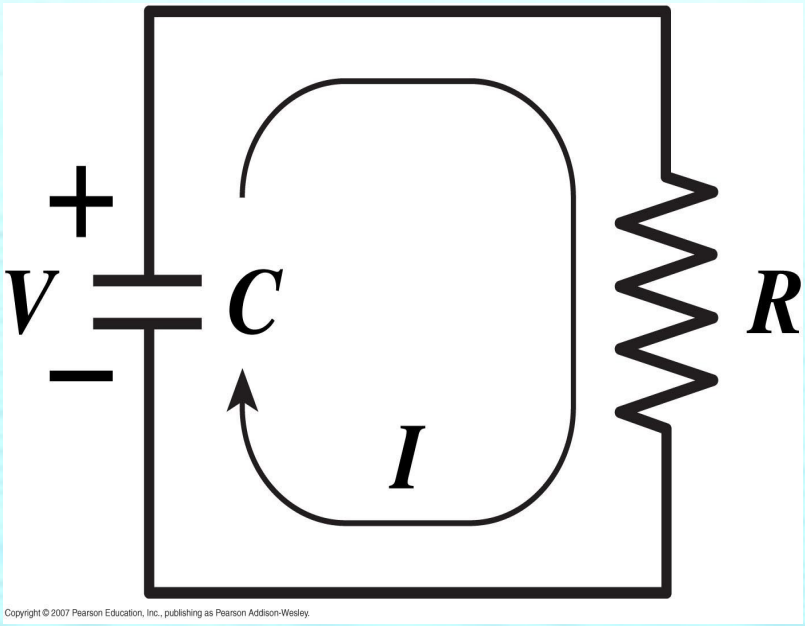
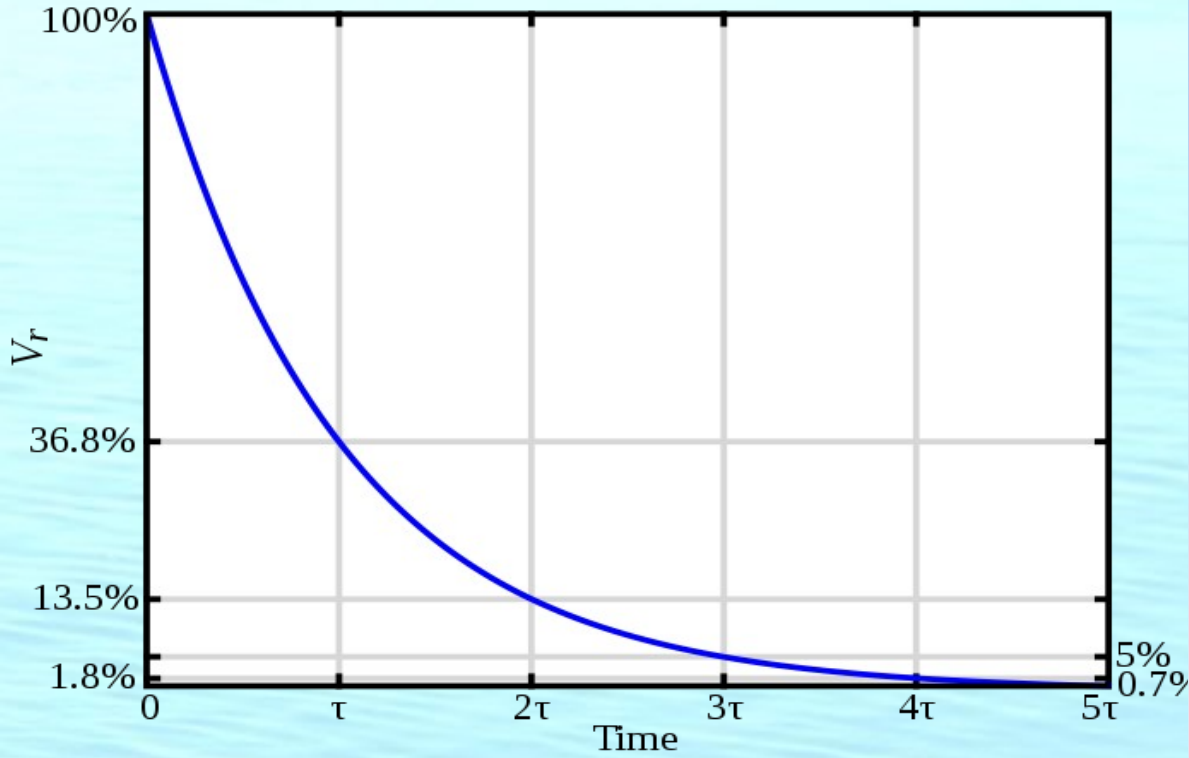
# Time for pHeT  (AC Circuits Lab)




In this circuit, where capacitor begins charged --->

$$V_C = V_0 e^{-t/RC}$$

$$I = I_0 e^{-t/RC}$$

Find the resistance needed in an RC circuit to discharge a 20 $\mu$F capacitor
to 55% of full charge in 140 ms.

Find the resistance needed in an RC circuit to discharge a $20\ \mu F$ capacitor to 55% of full charge in 140 ms.

$$Q_C = Q_0 e^{-t/RC}$$

$$0.55 Q_0 = Q_0 e^{-t/RC}$$

$$0.55 = e^{-t/RC}$$

$$\ln(0.55) = \ln\left(e^{-t/RC}\right)$$

$$-0.597 = \frac{-t}{RC}$$

$$R = \frac{t}{0.597 C}$$

$$R = \frac{1.40 \times 10^{-1}\ s}{0.597 \cdot 2.0 \times 10^{-5}\ F}$$

$$R = 11.74\ k\Omega$$

# RC Circuits Demo