Physics 122 – Class #22 – (4/2/15)

- Energy in a capacitor
- Supercaps and regenerative braking
- Energy in an electric field
- Current
- Resistance, Power and Ohm's Law
  - Examples
- Water Analogy
- Conductors
- Resistance and Resistivity
- Drude Model of Conduction

Deriving Capacitor Energy Formula

$$[1a]\ Q = C\,V$$

$$[1b]\ dQ = C dV$$

$$[2a]\ U = Q\,V$$

$$[2b]\ dU = V\,dQ$$

$$[3]\ dU = V\,C dV$$

$$[3a]\ U = \int_0^V V'\,C\,dV'$$

$$U = \frac{1}{2} C\,V^2$$

Q is charge on capacitor
C is capacitance
V is voltage on capacitor
U is potential energy of capacitor

Deriving Capacitor Energy Formula

$$[1a]\ Q = C\,V$$ Definition of Capacitance

$$[1b]\ dQ = C\,dV$$ Calculus – for C and constant

$$[2a]\ U = Q\,V$$ Definition of Potential

$$[2b]\ dU = V\,dQ$$ Calculus – for V “constant-ish”

$$[3\ ]\ dU = V\,C\,dV$$ Plug 1b into 2b

$$[3a]\ U = \int_0^V V'\,C\,dV'$$

$$U = \frac{1}{2}C\,V^2$$

Q is charge on capacitor
C is capacitance
V is voltage on capacitor
U is potential energy of capacitor

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Energy Storage:
A 5000 kg
bus traveling at 10 m/s
dumps its kinetic energy
into ultra-capacitors
For a quick restart.
How much capacitance
Is needed, assuming the
Ultra-caps can tolerate 16 Volts.

Energy Storage:

m=5000 kg

$$v_{initial} = 10\ m/s$$

$$V_{max} = 16\ V$$

$$K_{initial} = \frac{1}{2} m v_i^2 = 0.5 \times 5000 \times 10^2 = 2.5 \times 10^5\,J$$

$$U_{final} = \frac{1}{2} C V_{max}^2 \rightarrow C = \frac{2U_{final}}{V_{max}^2}$$

$$C = \frac{(2)(2.5 \times 10^5)}{16^2} = 1950\,\text{Farads} = \ \$2000$$

http://www.ebay.com/bhp/100-farad-capacitor

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Deriving Energy in an Electric Field … part 1

$$[1]\ U=\frac{1}{2}CV^2$$

Energy in a Capacitor

$$[2]\ C=\epsilon_0\frac{A}{d}$$

Parallel plate capacitor formula

$$[3]\ V=Ed$$

$$[4]\ U=\frac{1}{2}\left[\epsilon_0\frac{A}{d}\right][Ed]^2$$

$$[5]\ U=\frac{1}{2}\epsilon_0E^2(Ad)$$

Deriving Energy in an Electric Field … part 1

$$[1]\ U=\frac{1}{2}CV^2$$

Energy in a Capacitor

$$[2]\ C=\epsilon_0\frac{A}{d}$$

Parallel plate capacitor formula

$$[3]\ V=Ed$$

Relation between voltage and field
In a parallel plate capacitor

$$[4]\ U=\frac{1}{2}\left[\epsilon_0\frac{A}{d}\right][Ed]^2$$

Plug 2 and 3 into 1

$$[5]\ U=\frac{1}{2}\epsilon_0 E^2(Ad)$$

Rearrange and cancel

Deriving Energy in an Electric Field … part 2

$$[5]\; U = \frac{1}{2}\epsilon_0 E^2 (A\,d)$$

From last page

$$[6]\; U = \frac{1}{2}\epsilon_0 E^2 (\text{Volume})$$

Def. Of Volume!

$$[7]\; \frac{U}{\text{Volume}} = \frac{1}{2}\epsilon_0 E^2$$

$$[8]\; u_E = \frac{1}{2}\epsilon_0 E^2$$

Energy Density is Energy per Unit Volume

Total energy vs. energy density

Similar to 29.29

A parallel plate capacitor with area of ½ a square meter and a spacing of 0.8 mm is charged to 1000 V. What is the electric field?

[A] 1250 kVolt/meter

[B] 1000 kWatts

[C] 1 kJoule

[D] 1 kVolt/meter

[E] Not enough information given

Total energy vs. energy density

Similar to 29.29

A parallel plate capacitor with area of ½ a square meter and a spacing of 0.8 mm is charged to 1000 V. What is the electric energy density? What is the total energy stored in the electric field?

$$u_E = \frac{1}{2} \epsilon_0 E^2 \qquad U = \frac{1}{2} C V^2$$

Total energy vs. energy density

Similar to 29.29

A parallel plate capacitor with area of ½ a square meter and a spacing of 0.8 mm is charged to 1000 V. What is the electric energy density? What is the total energy stored in the electric field?

$$\mathrm{u_E} = \frac{1}{2}\epsilon_0 \mathrm{E}^2 = \frac{1}{2}\left(8.86\times10^{-12}\right)\left(1.25\times10^{6}\right)^2 = 6.9\ \mathrm{J/m^3}$$

$$C = \epsilon_0 \frac{A}{d} = 8.86\times10^{-12}\, F/m \frac{\left(0.5\ m^2\right)}{\left(8\times10^{-4}\ m\right)} = 5.5\, nF$$

$$\mathrm{U} = \frac{1}{2}\mathrm{C}\,\mathrm{V}^2 = \frac{1}{2}\left(5.5\times10^{-9}\right)\left(1000\right)^2 = 2.75\,\mathrm{mJ}$$

$$\mathrm{U} = \mathrm{u_E}\times\mathrm{Volume} = 6.9\times\left(0.5\right)\left(8\times10^{-4}\right) = 2.75\,\mathrm{mJ}$$

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Water current is total mass that passes an observer per second.

Electrical current is charge flow rate past a fixed point.
Units (C/s)

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Also

$$V_{cap} = \frac{Q}{C} = \frac{\int I dt}{C}$$

Chapter 30 Formulae (some)

Power is the product of voltage and current (True for ALL devices)

$$P = I V$$

Ohm's law (resistors only)

$$V = I R$$

Resistor power dissipation

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

Resistance in terms of resistivity

$$R = \rho \frac{L}{A} = \frac{L}{\sigma A}$$

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Find the resistance of a heating coil that draws 4.8 A when the voltage across it is 120 V. What power is dissipated in this coil?

$$P = IV$$

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

What current flows when a 45 V potential difference is imposed across a 1.8 $\mathrm{k}\Omega$ resistor? What power is dissipated in this resistor?

$$P = IV$$

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

An electric burner with 35 Ω resistance consumes 1.5 kiloWatts. At what voltage does it operate?

$$P = IV$$

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

(A) 120 V
(B) 230 V
(C) 52,500 V
(D) 14,400 V
(E) 42.8 V

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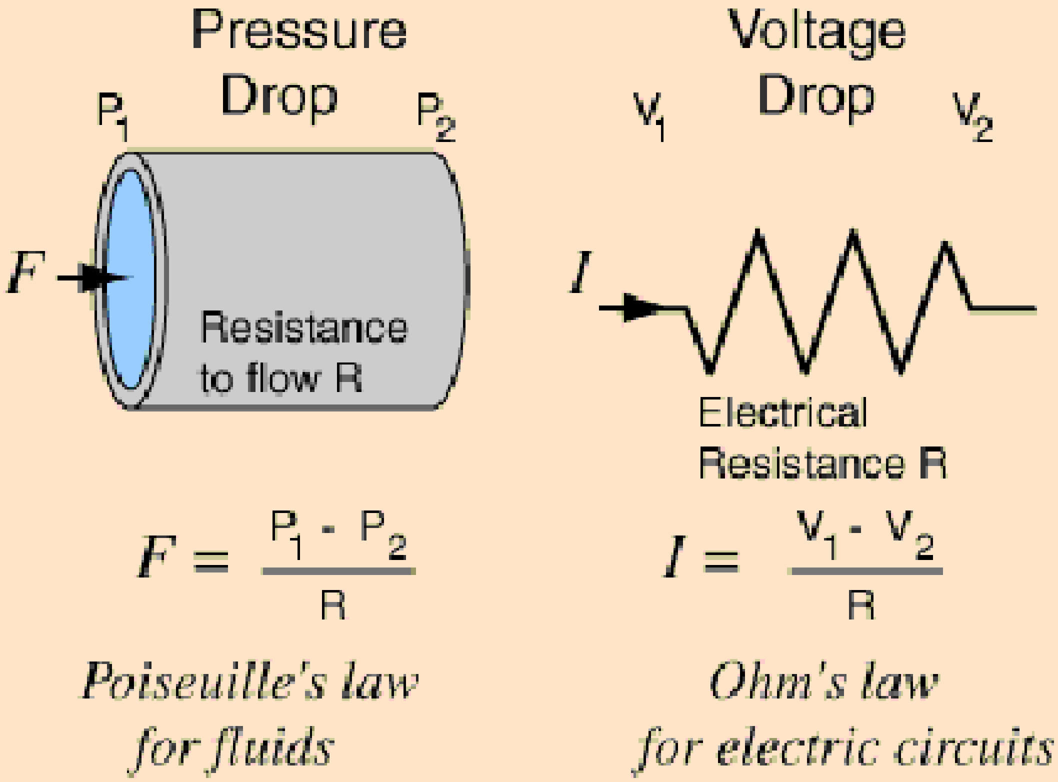
Ohm's Law-Poiseuille's Law

Ohm's law for electric current flow and Poiseuille's law for the smooth flow of fluids are of the same form.



Water analogy to DC circuits

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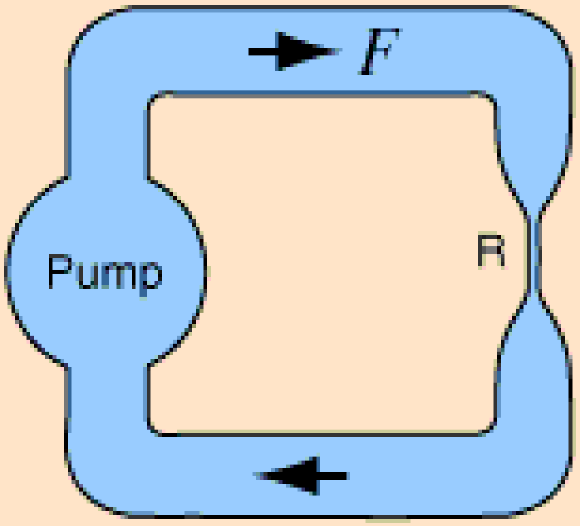
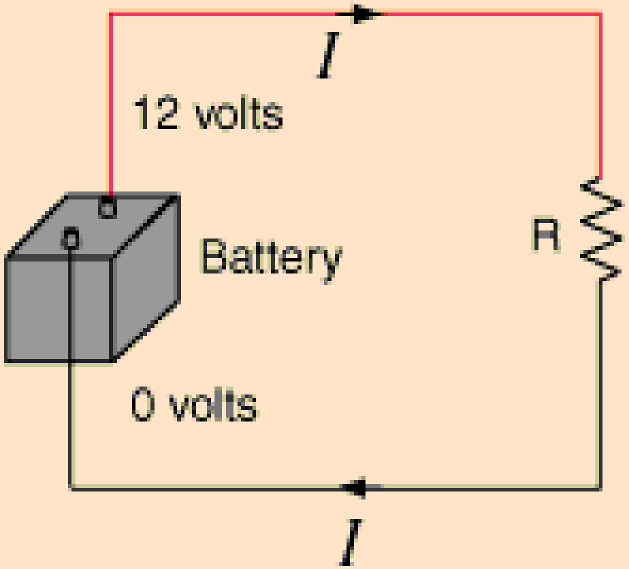
DC Circuits

Current Law and Flowrate

volume flowrate e.g., $\mathrm{cm}^3/\mathrm{sec}$

charge flowrate $= \text{current} = \dfrac{\text{coulombs}}{\text{second}} = \text{amperes}$





With continuous circulation around the pipe system, the volume flowrate must be the same at any cross-section of the pipe system.

The electric current is the charge flowrate and it must be the same at any cross-section of the circuit. This is a general principle called the current law.

Conservation of liquid

Conservation of charge

What's a resistor … Why do I want one?

Resistors are devices that resist current flow (like thin tubes resist water).

They are “controlled conductors”.

They make a good way of turning electricity into heat (hair-dryers, electric stoves, space-heaters)

They stop batteries from discharging all at once.

They can protect you from electrocution.

They are unavoidable – because even if you don't want one, all wires have some resistance (except superconductors!).

Body tissues have resistance.

They relate voltage and current

Resistors is one of the *three basic electrical devices* (capacitors and inductors are the other two)

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Microscopic and Human-scale view of electricity

Human-scale concepts Voltage, Current, Resistance, and Power

Small-scale (Microscopic) concepts Electric field, Current density, Resistivity

Ohm's Law can be viewed both ways

Conductors

To conduct electricity, you need a conductor.

Conductors have charges that are free to move. (They are still usually electrically neutral).

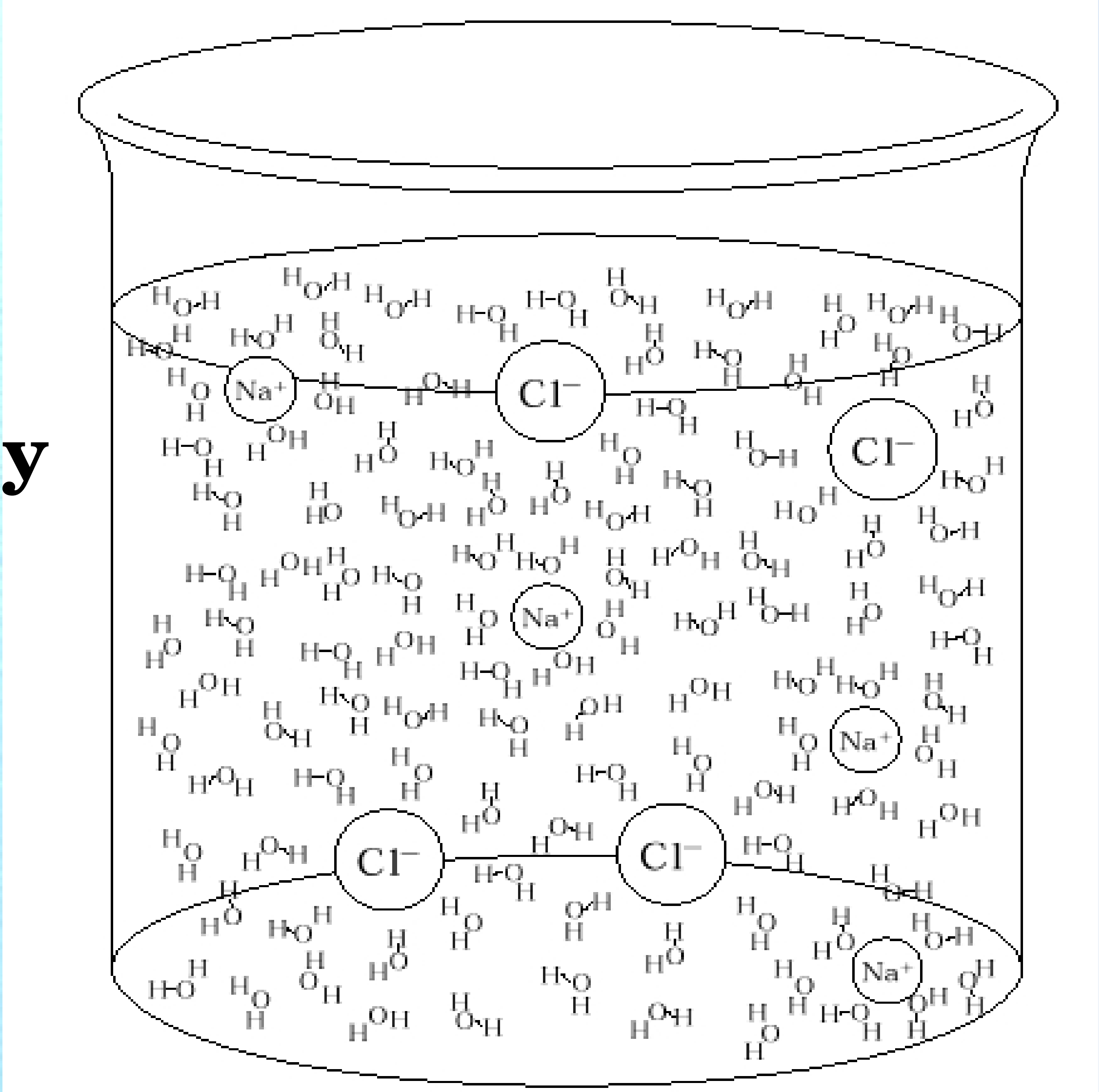
Salt-water, blood and copper are conductors.

Plastics and gasses are (usually) insulators.

Ionic Conductors

Salt water
Is electrically
Neutral …

The ions
Can move.



Why are metals conductors of electricity while most gasses, oxides and plastics are insulators?

Metals have charges that are free to move.

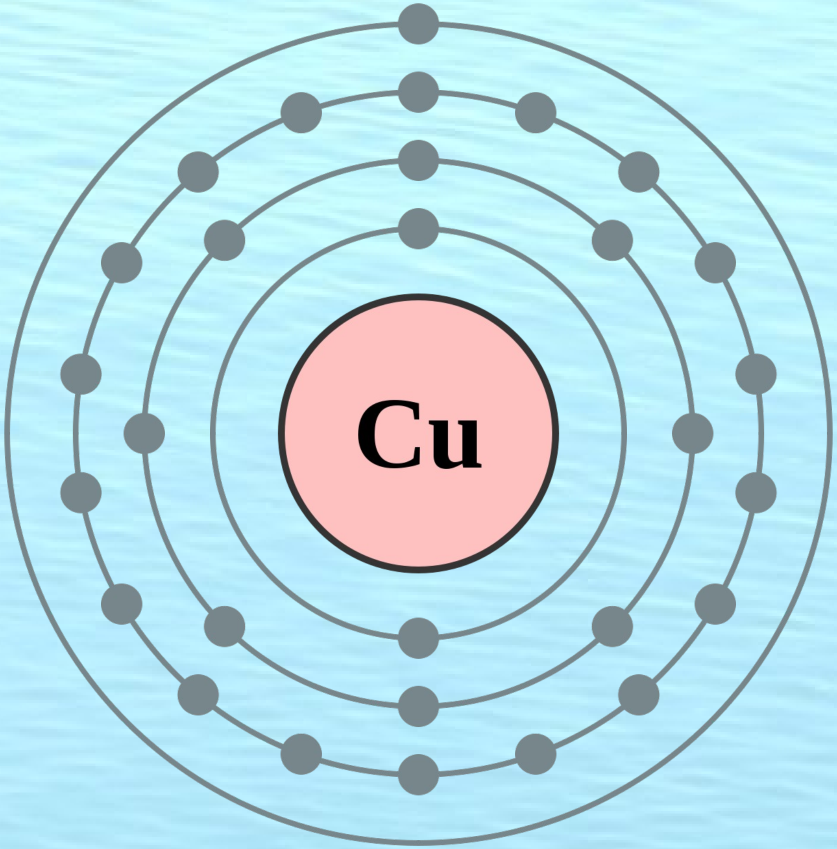
Insulators bind all their electrons tightly to their atoms.

You can “break down” an insulator by ripping its electrons from its atoms.

Copper conducts electricity because its lone 4s electron is free to "wander"

Conductivity (PHET)

29: Copper **2,8,18,1**



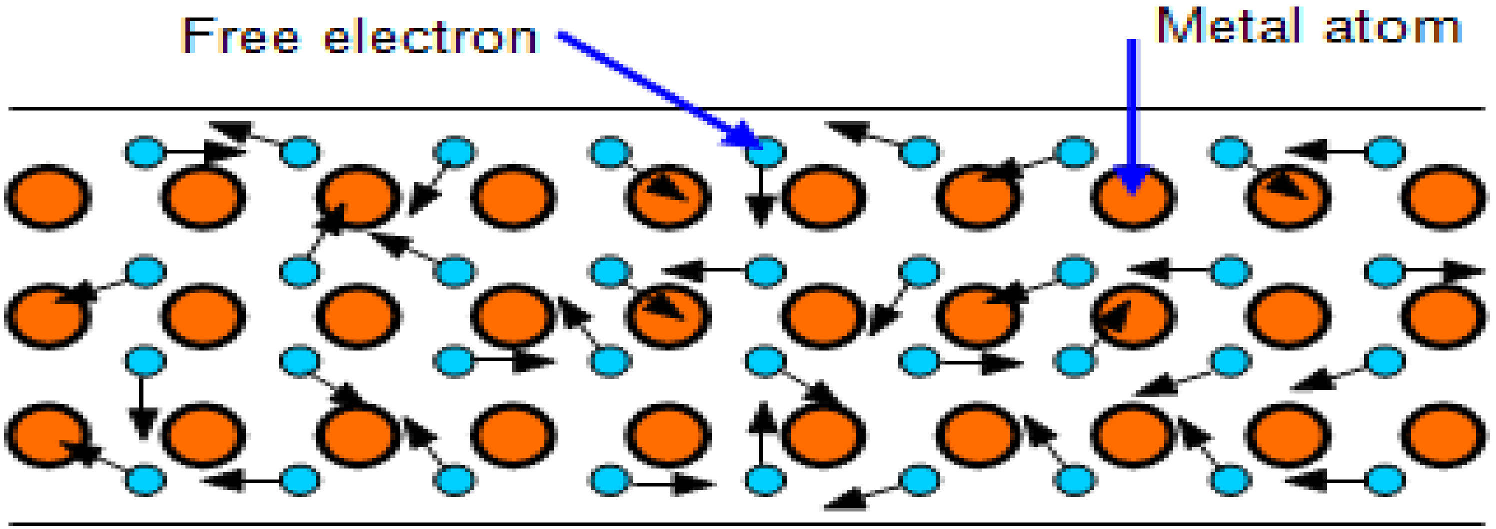


Figure 1(a)

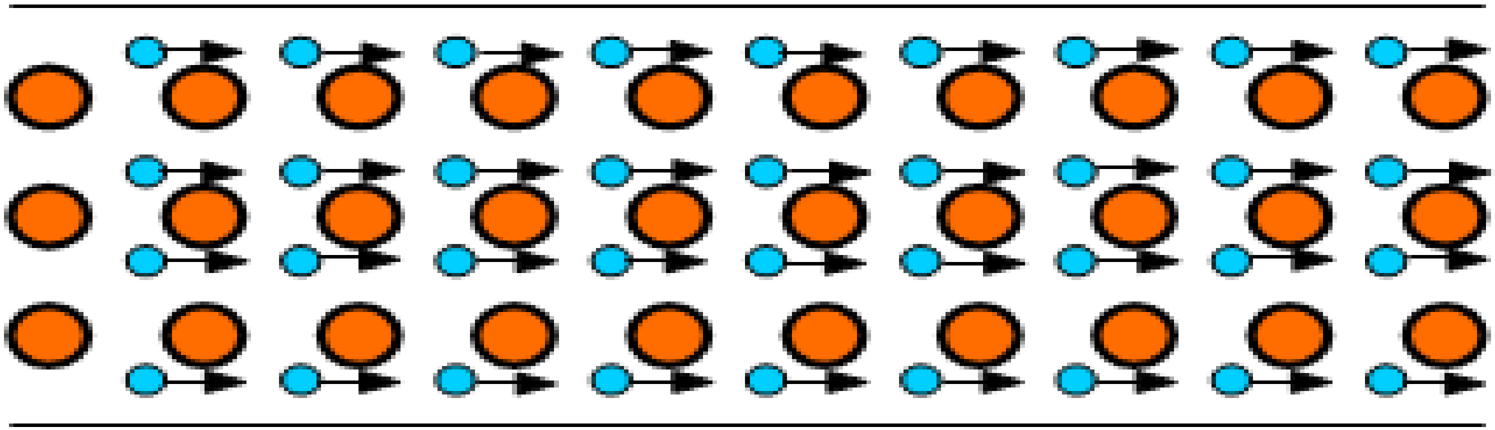
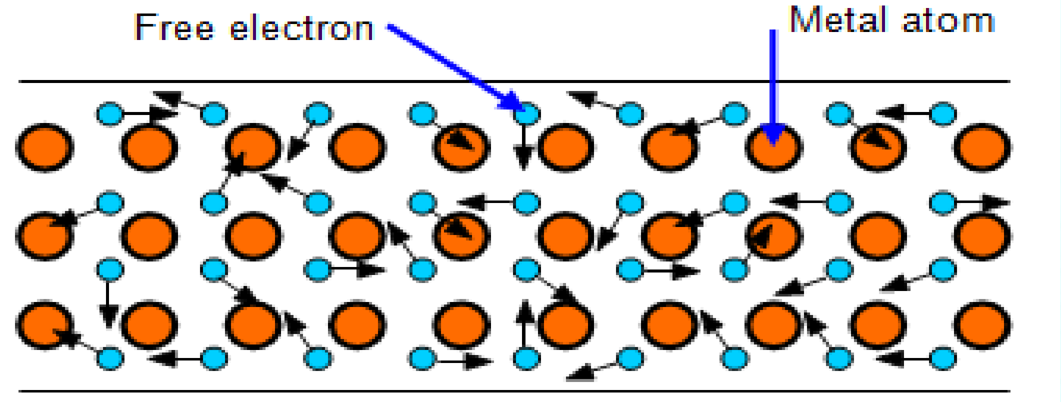


Figure 1(b)


Free electron
Metal atom


Wires are a series of tubes ... for electrons!

Slightly more realistic view of electrons “drifting” through a wire.

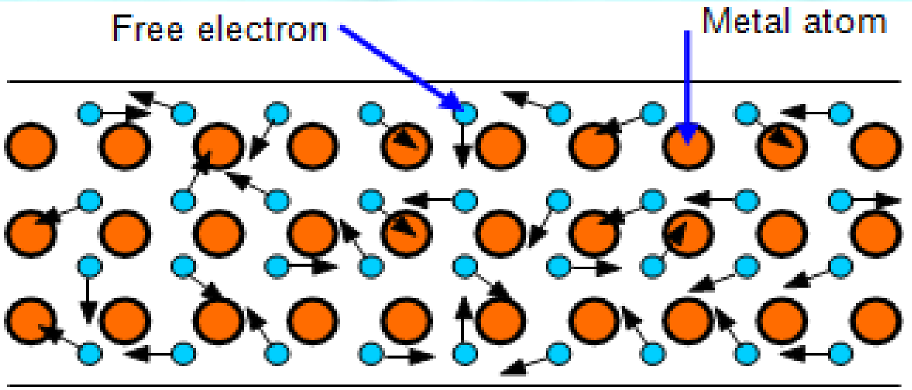


Figure 1(a)

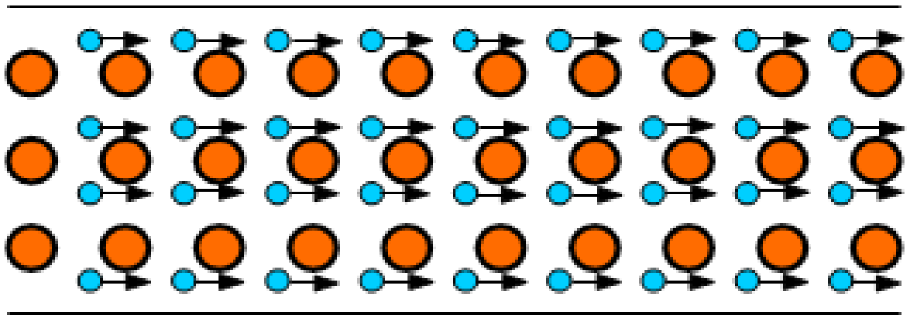
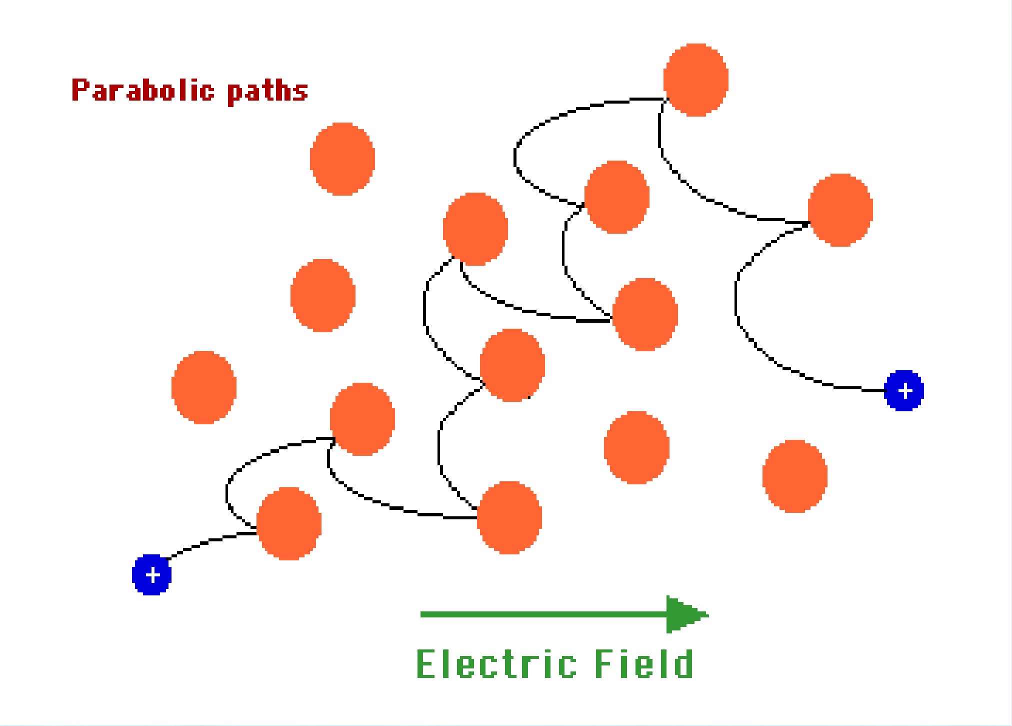


Figure 1(b)



The requirements for a material to
Be a conductor are:

(A) Electrons or ions are free to move from atom to atom.
(B) There must be a net charge on the material
(C) The material must be ductile, like a metal.
(D) Both A and B
(E) All of the above

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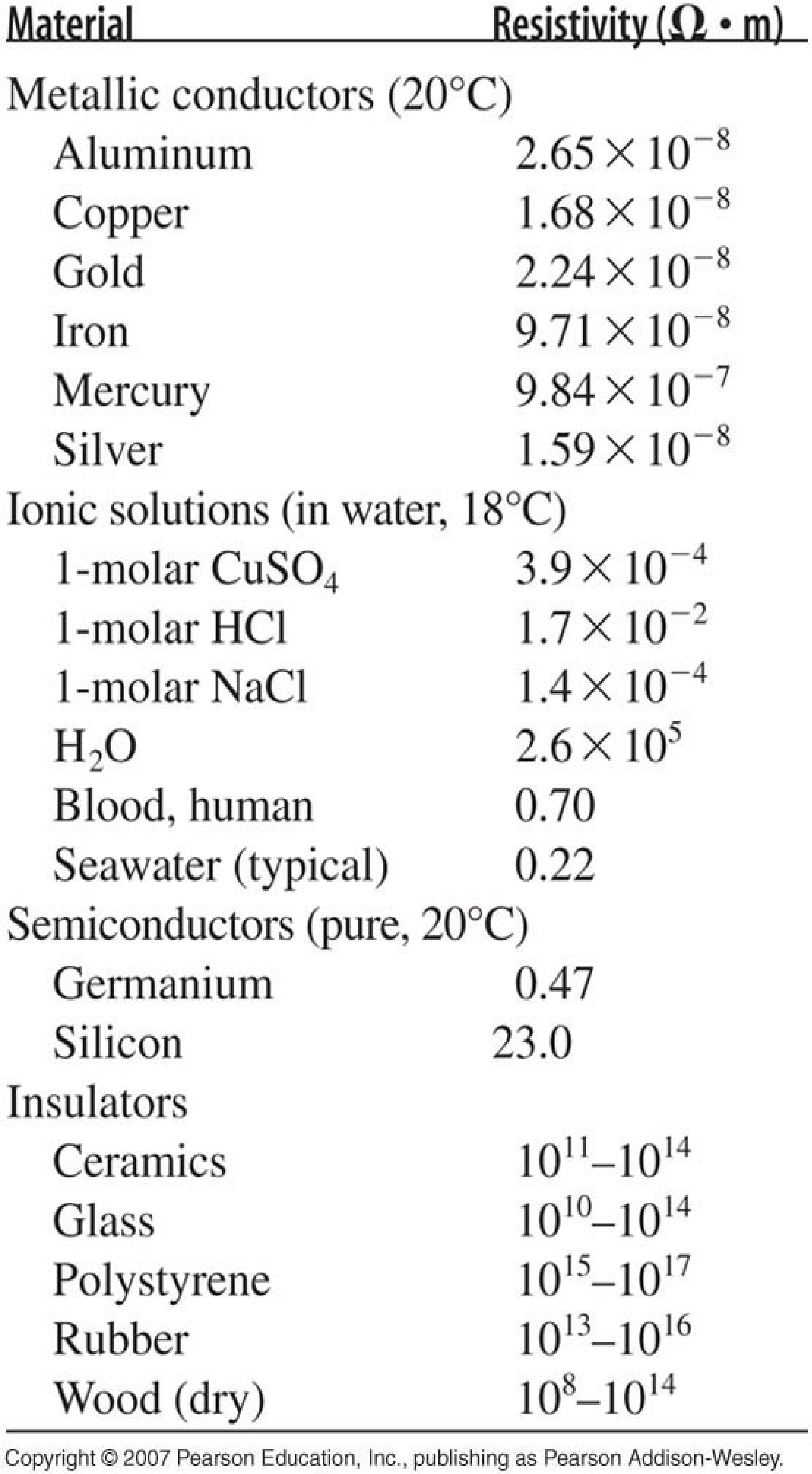


Material	Resistivity ($\Omega \cdot m$)
Metallic conductors (20°C)	
Aluminum	2.65×10^{-8}
Copper	1.68×10^{-8}
Gold	2.24×10^{-8}
Iron	9.71×10^{-8}
Mercury	9.84×10^{-7}
Silver	1.59×10^{-8}
Ionic solutions (in water, 18°C)	
1-molar $CuSO_4$	3.9×10^{-4}
1-molar HCl	1.7×10^{-2}
1-molar NaCl	1.4×10^{-4}
H_2O	2.6×10^{5}
Blood, human	0.70
Seawater (typical)	0.22
Semiconductors (pure, 20°C)	
Germanium	0.47
Silicon	23.0
Insulators	
Ceramics	10^{11}–10^{14}
Glass	10^{10}–10^{14}
Polystyrene	10^{15}–10^{17}
Rubber	10^{13}–10^{16}
Wood (dry)	10^{8}–10^{14}

Resistance in terms of Resistivity ρ

$$R = \rho \frac{L}{A} = \frac{L}{\sigma A}$$

$$\rho = \frac{1}{\sigma}$$

A piece of copper wire is 5 meters long and aluminum wire is 10 meters long. If the radius of the copper wire is 1 mm, what is the radius of the aluminum to have the same resistance?

$$R = \rho \frac{L}{A}$$

The femoral artery is the large artery that carries blood to the leg. What is the resistance of a 20-cm-long column of blood in a 1.0 cm diameter femoral artery? The conductivity of blood is

$$\sigma_{\text{blood}} = 0.63 \frac{1}{(\Omega \cdot \text{m})}$$

$$R = \rho \frac{L}{A} = \frac{L}{\sigma A}$$

$$\rho = \frac{1}{\sigma}$$

If you make a resistor with a length of copper wire of square cross-section with side 1 mm, how long a wire do you need to make a 3.2 Ohm resistor? $\rho_{\text{copper}} = 1.6\times10^{-8}\,\Omega\cdot\text{m}$

$$R = \rho\frac{L}{A}$$

(A) 200,000 m
(B) 160,000 m
(C) 3.2 m
(D) 100 m
(E) 200 m

The Current Density in a Wire

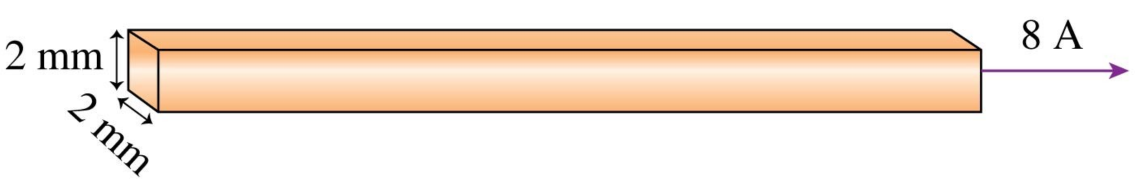
The **current density** J in a wire is the current per square meter of cross section:

$$J = \text{current density} \equiv \frac{I}{A} = n_\mathrm{e} e v_\mathrm{d}$$

The current density has units of $\mathrm{A/m^2}$.

Clicker

The current density in this wire is



A. 4×10^6 A/m^2.

B. 2×10^6 A/m^2.

C. 4×10^3 A/m^2.

D. 2×10^3 A/m^2.

E. Some other value.

A cylindrical tube of seawater carries a total electric current of 350 mA. If the electric field in the water is 21 V/m, what is the diameter of the tube?

$$I = J A$$

$$J = \sigma E$$

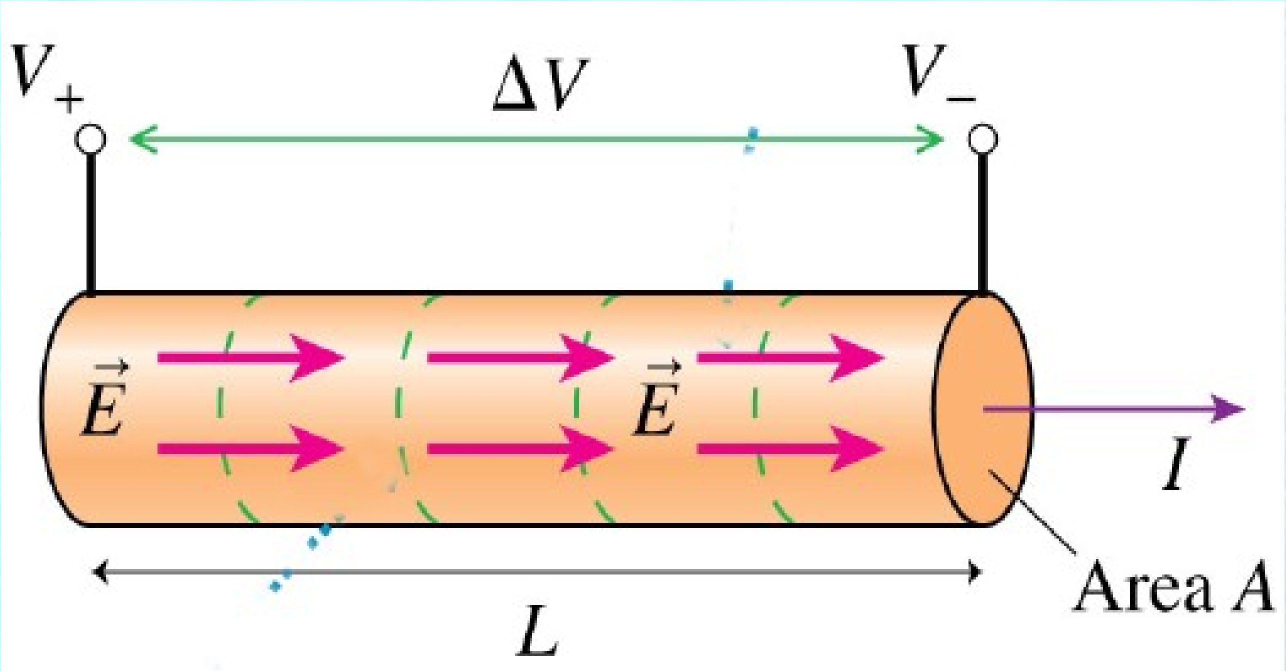
Electric field in home wire?

You are using a cheap extension cord made of #14 copper wire (which has a $2\ \mathrm{mm}^2$ cross sectional area). What is the electric field in the wire when you light a 100 Watt bulb?

$$\rho_{\mathrm{copper}} = 1.6\times 10^{-8}\,\Omega\cdot\mathrm{m}$$

Going from Microscopic to Macroscopic

$$\vec{J} = \sigma \vec{E} \qquad E = \frac{V}{L} \qquad J = \frac{I}{A} \qquad R = \frac{L}{\sigma A} \rightarrow \sigma = \frac{L}{RA}$$



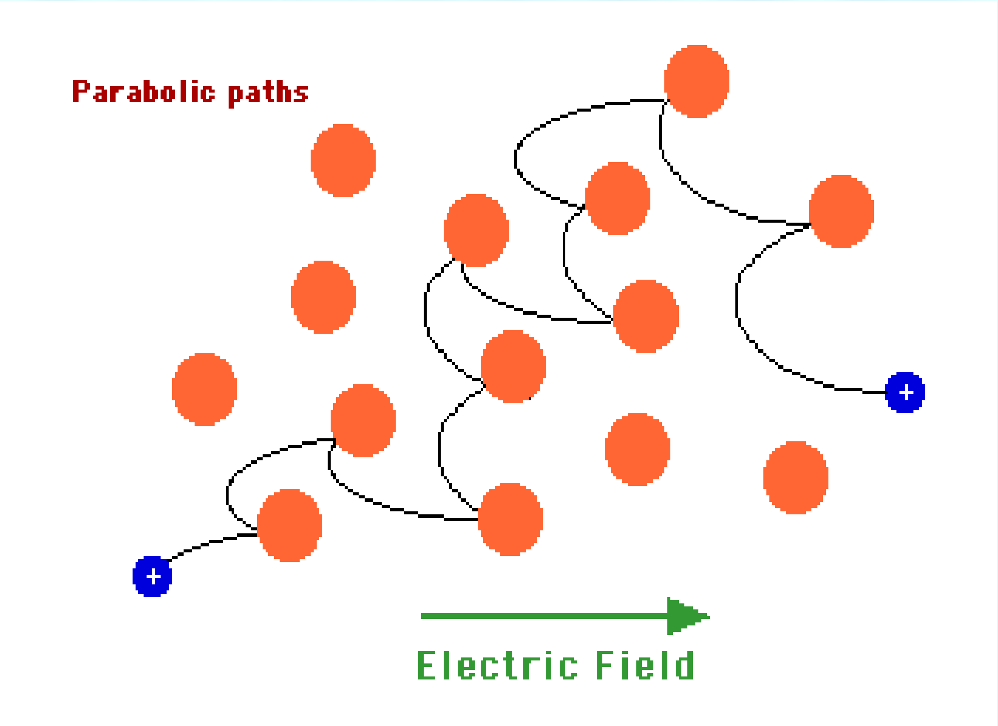
$$\frac{I}{A} = \frac{L}{RA}\frac{V}{L}$$

$$I = \frac{1}{R}V \rightarrow V = IR \quad \text{Ohm's Law}$$

Microscopic view of resistivity

”Free electron gas” model (also called Drude model) of a metal.

You can derive Ohm's law by assuming a metal is a box full of loose electrons that bump into “scattering centers” every τ seconds.



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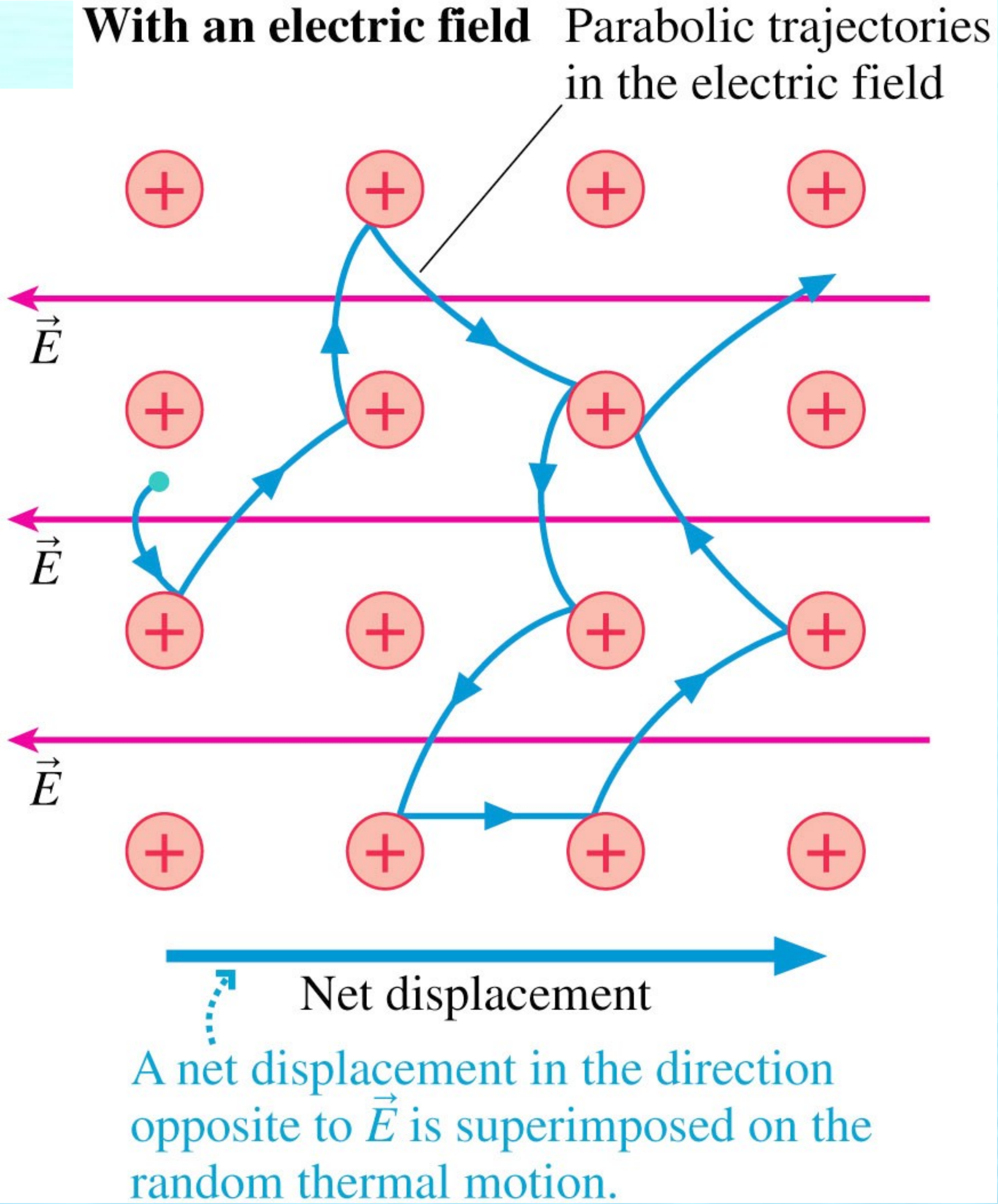
Microscopic view of resistivity

- In the presence of an electric field, the electric force causes electrons to move along parabolic trajectories between collisions.
- Because of the curvature of the trajectories, there is a slow net motion in the "downhill" direction.

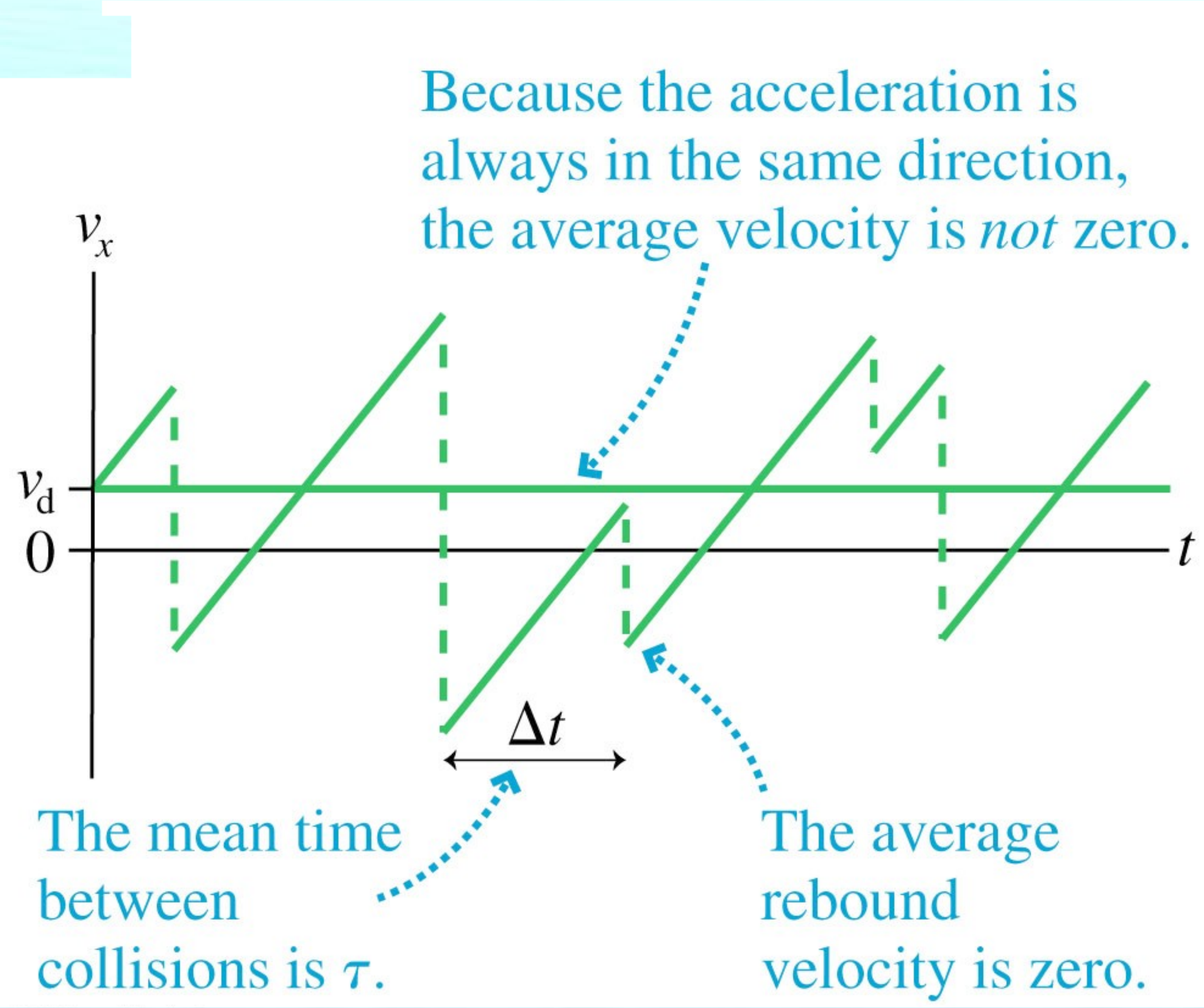


A net displacement in the direction opposite to $\vec{E}$ is superimposed on the random thermal motion.

Drude Model of Resistance

- The graph shows the speed of an electron during multiple collisions.
- The average drift speed is

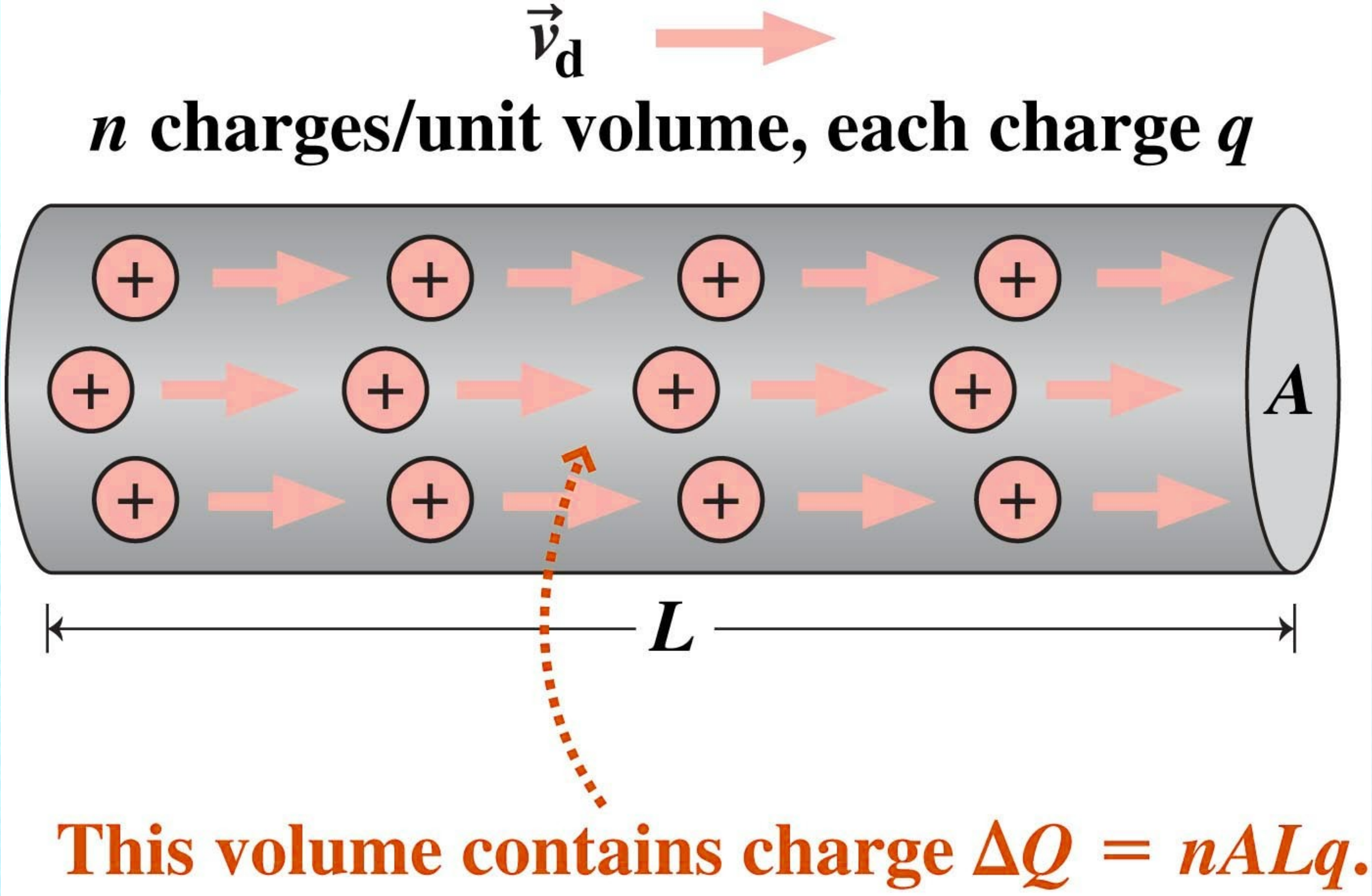
$$v_d = \frac{e\tau}{m}E$$



Because the acceleration is always in the same direction, the average velocity is *not* zero.

The mean time between collisions is τ.

The average rebound velocity is zero.



This volume contains charge $\Delta Q = nALq$.

$$\Delta Q = n A \vec{v}_d \Delta t \; q$$

$$I = n q \vec{v}_d A$$

$$\vec{J} = n q \vec{v}_d$$

$$I = \vec{J} \cdot \vec{A}$$

Drude Model of Conductance

You can derive Ohm's law by assuming a metal is a box full of loose electrons that bump into “scattering centers” every τ Seconds. (Tau is < 1 picosecond for most solids at room temp.)

$$\vec{J} = n q \vec{v}_d = n q a \tau = \frac{n q^2 \tau}{m} \vec{E} = \sigma \vec{E}$$

Derived electrical resistance from Classical Mechanics!

$$\sigma = \frac{n q^2 \tau}{m}$$

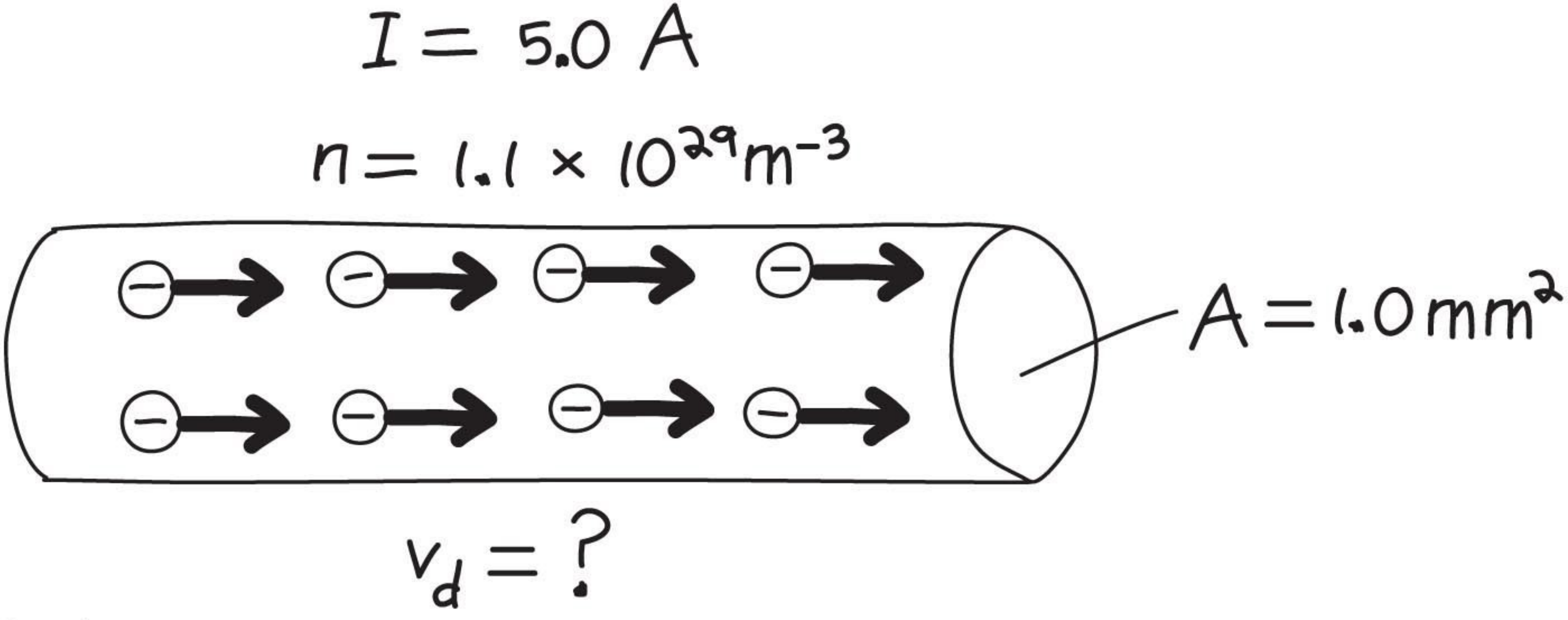
In semiconductor, n is small, so
τ
rho is larger than a conductor.
In insulator, n is nearly zero.

$$\rho = \frac{m}{n q^2 \tau}$$

In ionic conductor, m is larger (ions not electrons) so conductivity is Lower.

A semiconductor may be “doped”

Typical drift velocity



$$I = n\,q\,\vec{v}_d\,A$$