

Physics 122 – Class #21 – (3/31/15)

- Homework/Readings
- Videos
- Capacitance is a geometric property
- Capacitance in Networks
- Energy in Capacitors
- Resistors
- Current
- Resistance and Ohm's Law
- Power and Ohm's Law
- Resistance and Resistivity
- Drude Model of Conduction

Homework

MP Assignment 9a

29.1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 25, 28, 29, 64, 65
[due Saturday night 4/4]

MP Assignment 10

30.9, 13, 19, 20, 21, 34, 43, 44, 46, 48
[Due Thursday 4/9]

MP Assignment 11 (really two assignments)

Problem 30.53, 30.55, 31.7, 31.8, 10, 11, 19, 20,
21, 22, 28, 30, 31, 32, 34, 35, 58, 62, 32.9, 10, 14,
33, 34, 35, 36

[Due Sunday 4/19]

Reworked Exams / Written Homework

Reading Assignments

This week – Chapter 30 – Currents. Read all of it (it's less mathematical than most), but you may skip pages 873-874.

Next week – Chapter 31 – Circuits.

Lab next week is on series/parallel and RC circuits (Ch. 31).

Chapter 29 – Summary

- 29.5 $Q=CV$

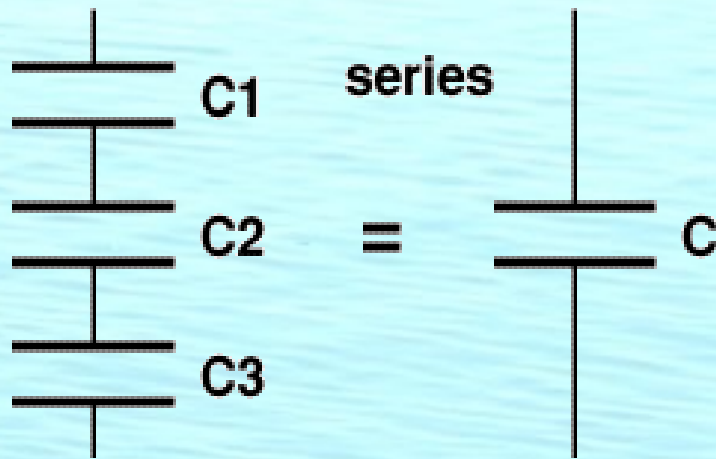
Capacitances in Parallel Add.

You need a reciprocal formula for capacitances in series.

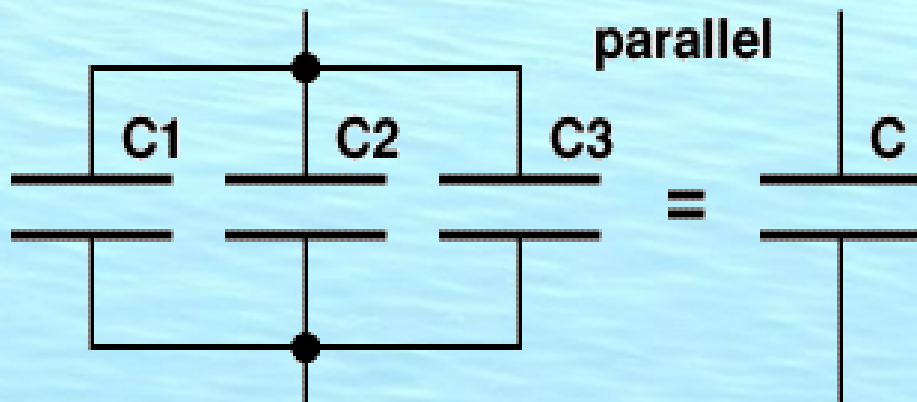
- 29.6 Capacitors store energy and charge

- 29.7 Dielectric materials let you store more energy and charge in the same space than using air or a vacuum.

Capacitors in Series and Parallel



Caps in series have the
“bottom wire” or one
Connected to the “top wire”
Of the next.



Caps in parallel have all
Their tops connected to
One wire and all their
Bottoms also connected to
A second wire.

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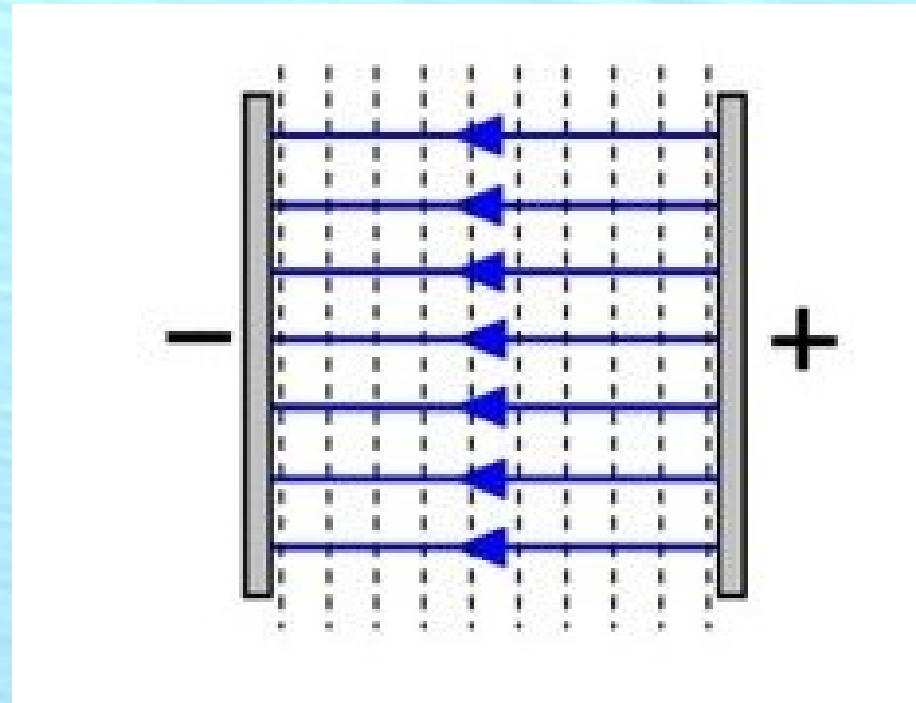
Electric potential of TWO large plates
(Area “A” separation “d”) ...

Electric field is doubled because two charged
plates of opposite sign.

$$\vec{E} = \frac{-\sigma}{\epsilon_0} \hat{i}$$

$$V = -\vec{E} \cdot \Delta \vec{r} = E d$$

$$V = \frac{\sigma}{\epsilon_0} x = \frac{\sigma}{\epsilon_0} d$$



Deriving Parallel Plate Formula

(Co-axial capacitor in recitation)

$$[1] Q = C V$$

$$[2] V = \frac{\sigma}{\epsilon_0} d$$

$$[3] \sigma = \frac{Q}{A}$$

$$[4] V = \frac{Q}{\epsilon_0 A} d$$

$$[5] Q = \frac{\epsilon_0 A}{d} V$$

$$[6] C = \frac{\epsilon_0 A}{d}$$

Deriving Parallel Plate Formula

(Co-axial capacitor in recitation)

$$[1] Q = C V$$

Definition of Capacitance

$$[2] V = \frac{\sigma}{\epsilon_0} d$$

Shown on last page

$$[3] \sigma = \frac{Q}{A}$$

Definition of surface charge density

$$[4] V = \frac{Q}{\epsilon_0 A} d$$

Plug 3 into 2

$$[5] Q = \frac{\epsilon_0 A}{d} V$$

Solve for Q

$$[6] C = \frac{\epsilon_0 A}{d}$$

How many capacitors are in your dorm room or house?

(A) None – I do not own a time machine or a DeLorean.



(B) About a dozen – Capacitors are energy storage and signal filtering devices used in power supplies for all electronics.

(C) Billions

You reposition the two plates of a capacitor so that the capacitance doubles.

If the charges $+Q$ and $-Q$ on the two plates are kept constant in this process, what happens to the potential difference V_{ab} between the two plates?

- A. V_{ab} is multiplied by a factor of 4
- B. V_{ab} is multiplied by a factor of 2
- C. V_{ab} is unchanged
- D. V_{ab} is multiplied by a factor of $1/2$
- E. V_{ab} is multiplied by a factor of $1/4$

$$Q = CV$$

Note that the parallel plate formula only has “geometric” variables. Capacitance is a property of a set of conductors and does not depend on charge or voltage (or any other electric variables)

$$C = \frac{\epsilon_0 A}{d}$$

[Ex. 1] Find the capacitance of a parallel plate capacitor consisting of circular plates 10 cm in radius separated by 1.5 mm.

$$\epsilon_0 = 8.86 \times 10^{-12} \text{ F/m}$$

$$C = \epsilon_0 \frac{A}{d}$$

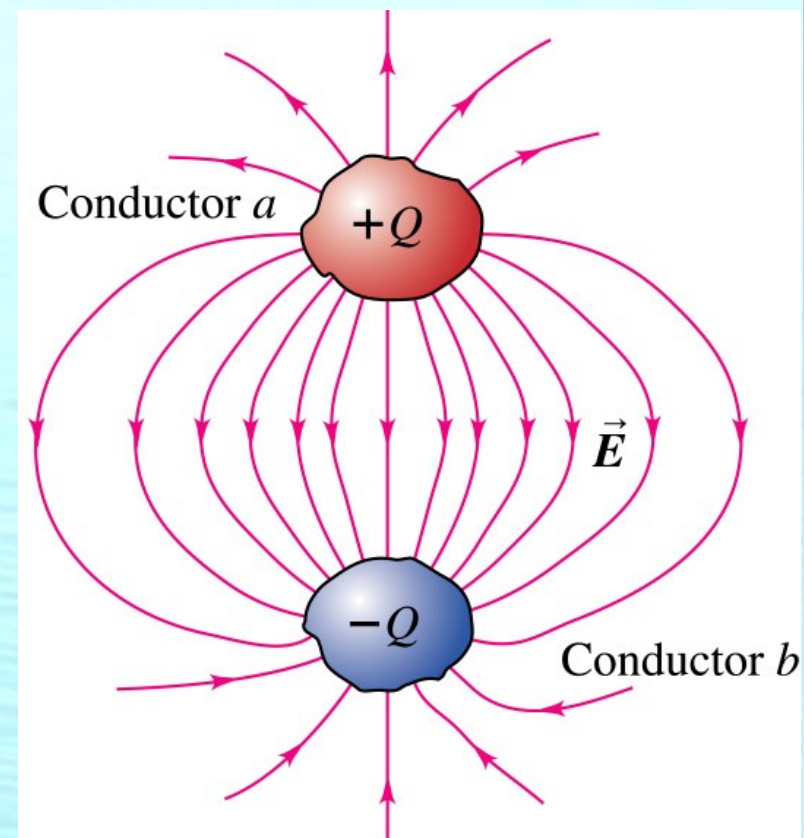
$$C = \frac{Q}{V}$$

$$U = \frac{1}{2} C V^2$$

The two conductors a and b are insulated from each other, forming a capacitor. You increase the charge on a to $+2Q$ and increase the charge on b to $-2Q$, while keeping the conductors in the same positions.

What effect does this have on the capacitance C ?

- A. C is multiplied by a factor of 4
- B. C is multiplied by a factor of 2
- C. C is unchanged
- D. C is multiplied by a factor of $1/2$
- E. C is multiplied by a factor of $1/4$



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Network of capacitors:

Replace parallel elements with an equivalent cap. Combine that equivalent cap in series with what is left.

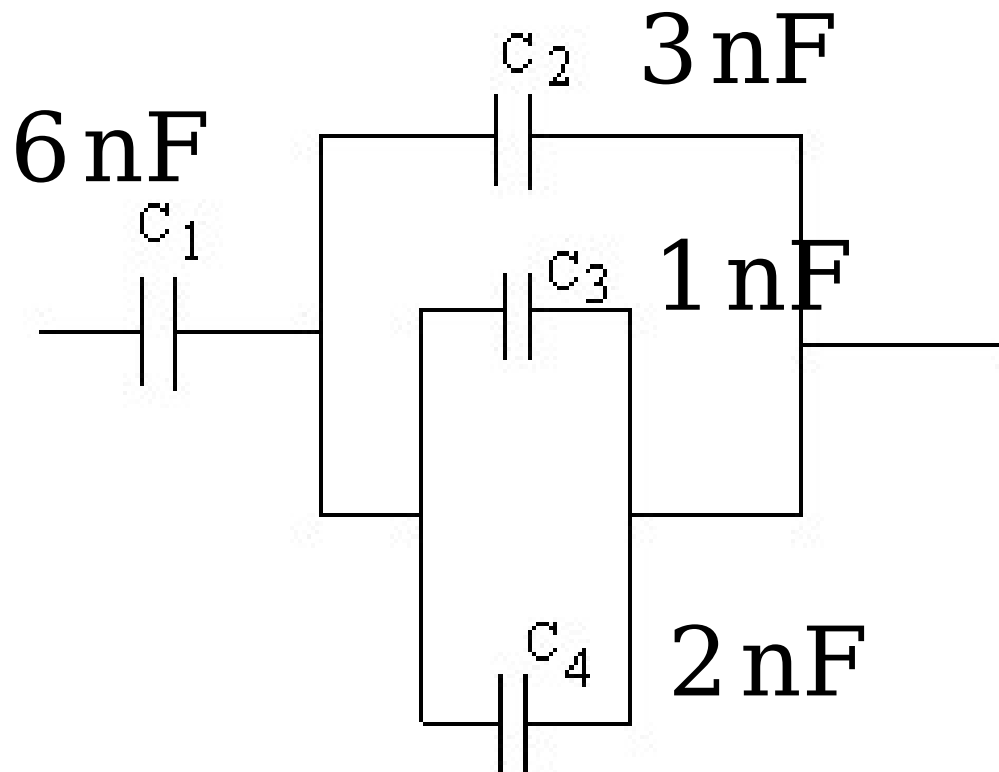
[A] 1 nF

[B] 2 nF

[C] 3 nF

[D] 6 nF

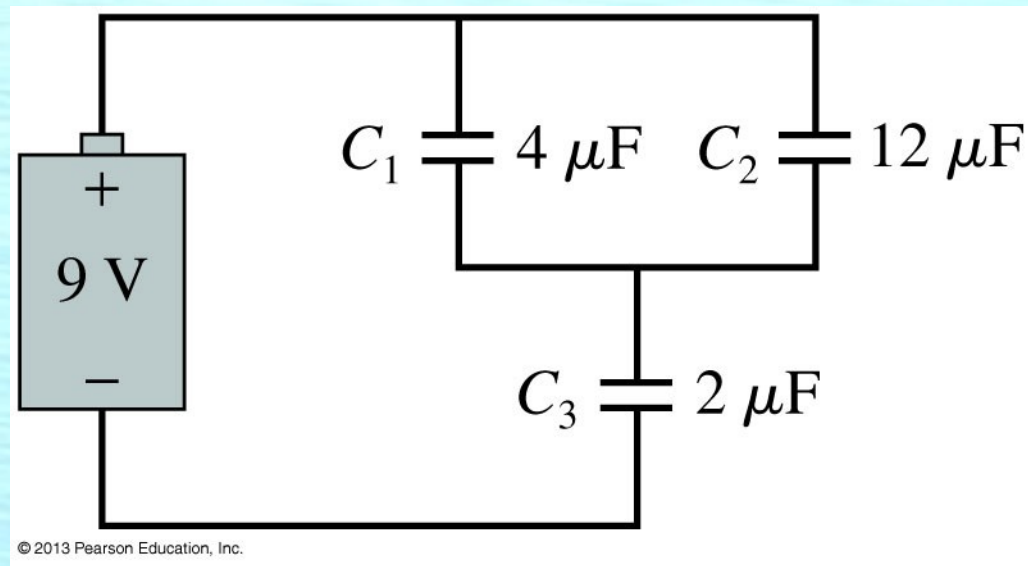
[E] 12 nF



What is the equivalent capacitance?

What is the charge on each capacitor?

What is the voltage across each capacitor?

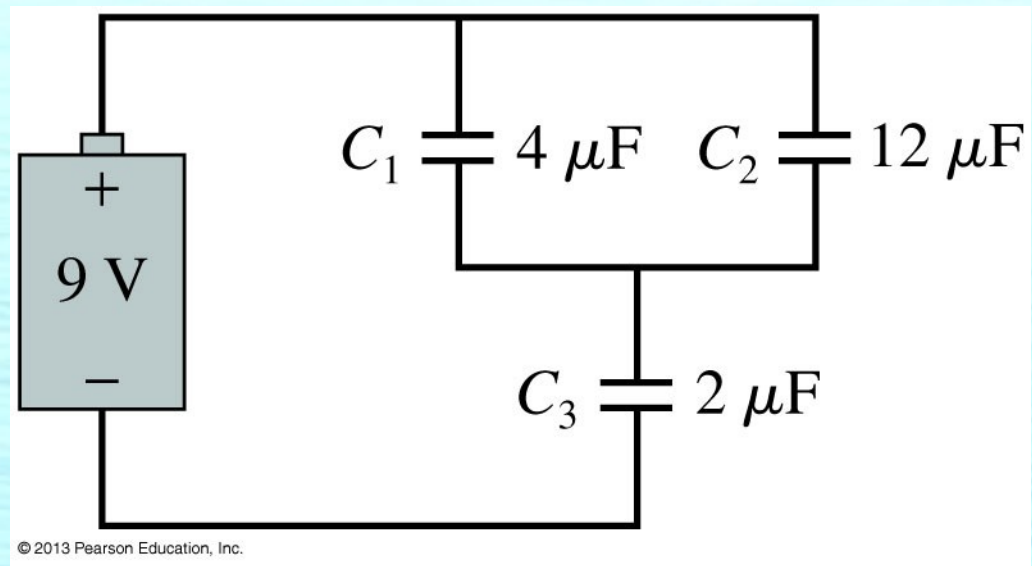


Caps in parallel have equal voltages.

Caps in series have equal charges.

Voltages in series add

Homework 29.54



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**[Ex. 3] A stereo receiver contains a $2500 \mu\text{F}$ capacitor charged to 35 V .
How much energy does it store?**

$$C = \epsilon_0 \frac{A}{d} \qquad C = \frac{Q}{V} \qquad U = \frac{1}{2} C V^2$$

... there is also energy in the electric field

$$u = \frac{1}{2} \epsilon_0 E^2$$

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Ohm's Law

Not as universal as $P=IV$ or $U=qV$
Or $F=ma$ or Coulomb's law – but
works for things called “resistors”.



You can buy a “resistor”, but any wire
at all does have some resistance.

Resistance, measured in Ohms (Ω)

$$V=IR \quad \text{Ohm's Law}$$

Resistors and Capacitors in Series & Parallel

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

$$R_{\text{series}} = R_1 + R_2 + R_3 + \dots$$

$$C_{\text{parallel.}} = C_1 + C_2 + C_3 + \dots$$

$$\frac{1}{R_{\text{parallel.}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Resistors(and capacitors) in parallel have equal voltages.

Resistors (and capacitors) in series have equal currents.

Resistors

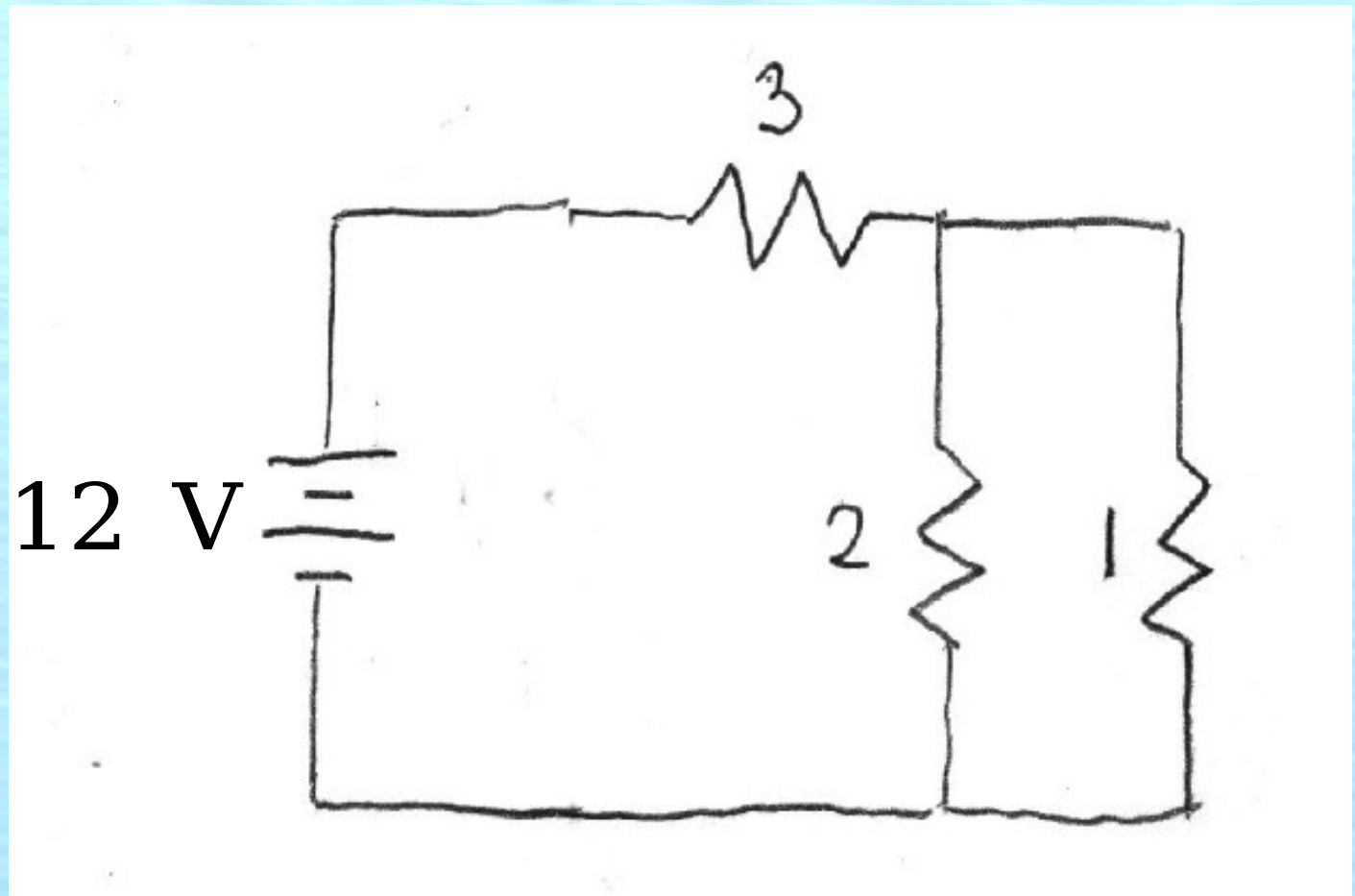


What is the equivalent resistance of this circuit?

What is the voltage across the 2-Ohm resistor?

What is the voltage across the 3-Ohm resistor?

$$V_B = 12 \text{ V}$$



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Water current is total mass that passes an observer per second.

Electrical current is charge flow rate past a fixed point.

Units (C/s)

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$



Hydraulic Analogy

http://en.wikipedia.org/wiki/Hydraulic_analogy

Mass of water (M)

Charge (Q)

Water current
(dM/dt)

Current ($I = dQ/dt$)

Water pressure (P)

Voltage (V)



Wire “W” carries 1.5 Amperes. A current of one Ampere means a coulomb of electrons pass a point every second.

How many electrons pass through W every second?

(A) 1.5

(B) 1.5×10^{-19}

(C) 9×10^{18}

(D) 6.02×10^{23}

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

The charge on a capacitor is given

by $Q = 12 - \frac{4}{t^2} + \frac{1}{t}$

What is the current after 1 second?

- (A) 9 Amps**
- (B) 7 Coulombs**
- (C) 3 Amps**
- (D) -9 Amps**
- (E) 7 Amps**

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

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Power, Current, and Voltage

Voltage is Energy per unit Charge

$$V = \frac{U}{Q} \rightarrow U = Q V$$

Power is Work per unit time
(or Energy expended per Unit time)

$$P = \frac{W}{\Delta t} = \frac{\Delta U}{\Delta t}$$

For constant voltage (e.g.
Current coming from a battery
Or an electric outlet)

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t} V = I V$$

$$P = I V$$

The voltage coming out of a wall outlet in North America is 120 V. How much current does it take to operate a 1800 Watt hair dryer?

(A) 15 Amps

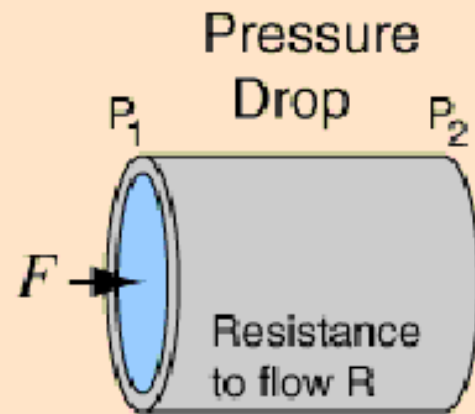
(B) $\frac{1}{15}$ Amps

(C) 9×10^{18} Amperes

(D) 1800 A

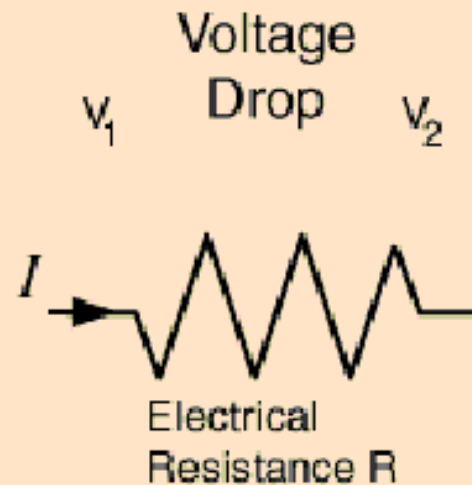
Ohm's Law-Poiseuille's Law

[Ohm's law](#) for electric current flow and [Poiseuille's law](#) for the smooth flow of fluids are of the same form.



$$F = \frac{P_1 - P_2}{R}$$

*Poiseuille's law
for fluids*



$$I = \frac{V_1 - V_2}{R}$$

*Ohm's law
for electric circuits*

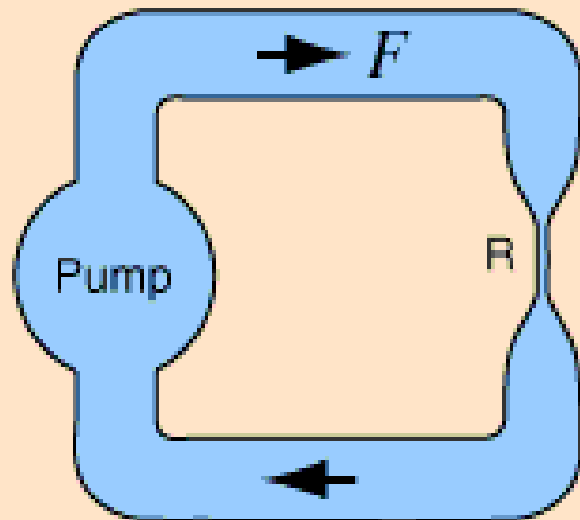
[Water analogy to DC circuits](#)

[Index](#)

[DC
Circuits](#)

Current Law and Flowrate

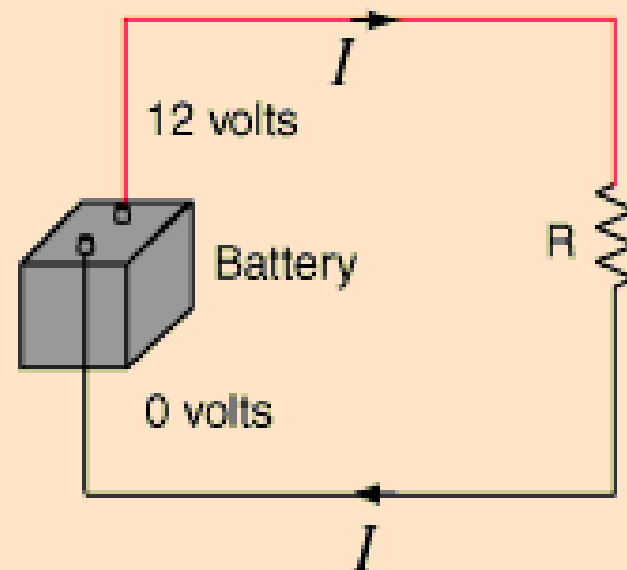
volume flowrate e.g., cm^3/sec



With continuous circulation around the pipe system, the volume flowrate must be the same at any cross-section of the pipe system.

Conservation of liquid

charge flowrate = current = $\frac{\text{coulombs}}{\text{second}} = \text{amperes}$



The electric current is the charge flowrate and it must be the same at any cross-section of the circuit. This is a general principle called the current law.

Conservation of charge

What's a resistor ... Why do I want one?

Resistors are devices that are meant to resist current flow. They obey Ohm's law. They make a good way of turning electricity into heat (hair-dryers, electric stoves, space-heaters)

They stop batteries from discharging all at once.

They are unavoidable – because even if you don't want one, all wires have some resistance (except superconductors!).

Body tissues have resistance.

Resistance is one of the “big three” basic electrical concepts (capacitance and inductance are the other two)

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Power is the product
of voltage and current
(True for ALL devices)

$$P = I V$$

Ohm's law (resistors only)

$$V = I R$$

Resistor power
dissipation

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

Resistance in terms of
resistivity

$$R = \rho \frac{L}{A} = \frac{L}{\sigma A}$$

Find the resistance of a heating coil that draws 4.8 A when the voltage across it is 120 V. What power is dissipated in this coil?

What current flows when a 45 V potential difference is imposed across a 1.8 k Ω resistor? What power is dissipated in this resistor?

An electric burner with $35\ \Omega$
resistance consumes 1.5 kilowatts.
At what voltage does it operate?

- (A) 120 V
- (B) 230 V
- (C) 52,500 V
- (D) 14,400 V
- (E) 42.8 V

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Microscopic and Human-scale view of electricity

Human-scale concepts Voltage,
Current, Resistance, and Power

Small-scale (Microscopic) concepts
Electric field, Current density,
Resistivity

Ohm's Law can be viewed both ways

Conductors

To conduct electricity, you need a conductor.

Conductors have charges that are free to move. (They are still usually electrically neutral).

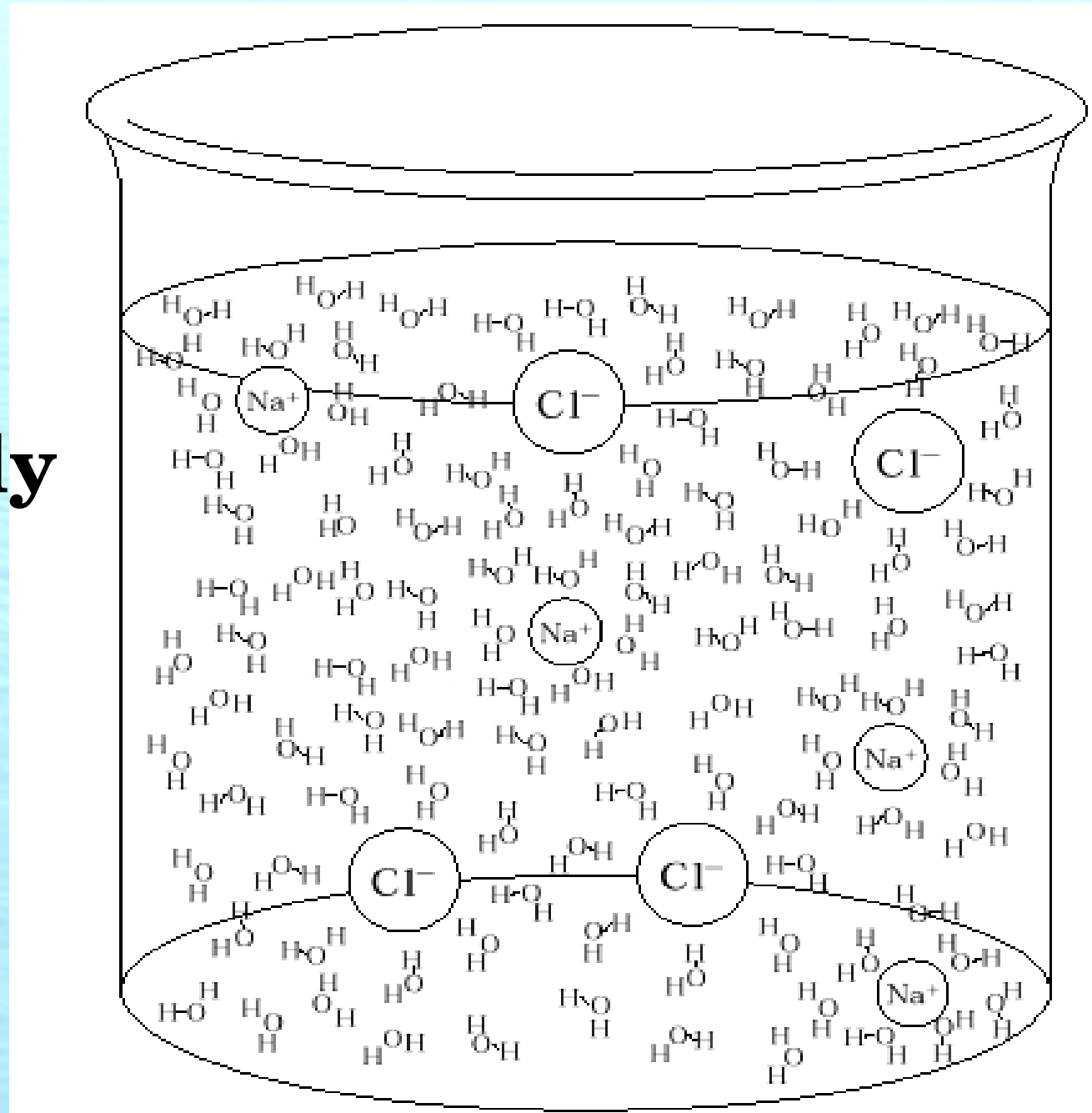
Salt-water, blood and copper are conductors.

Plastics and gasses are (usually) insulators.

Ionic Conductors

Salt water
Is electrically
Neutral ...

The ions
Can move.



Why are metals conductors of electricity while most gasses, oxides and plastics are insulators?

Metals have charges that are free to move.

Insulators bind all their electrons tightly to their atoms.

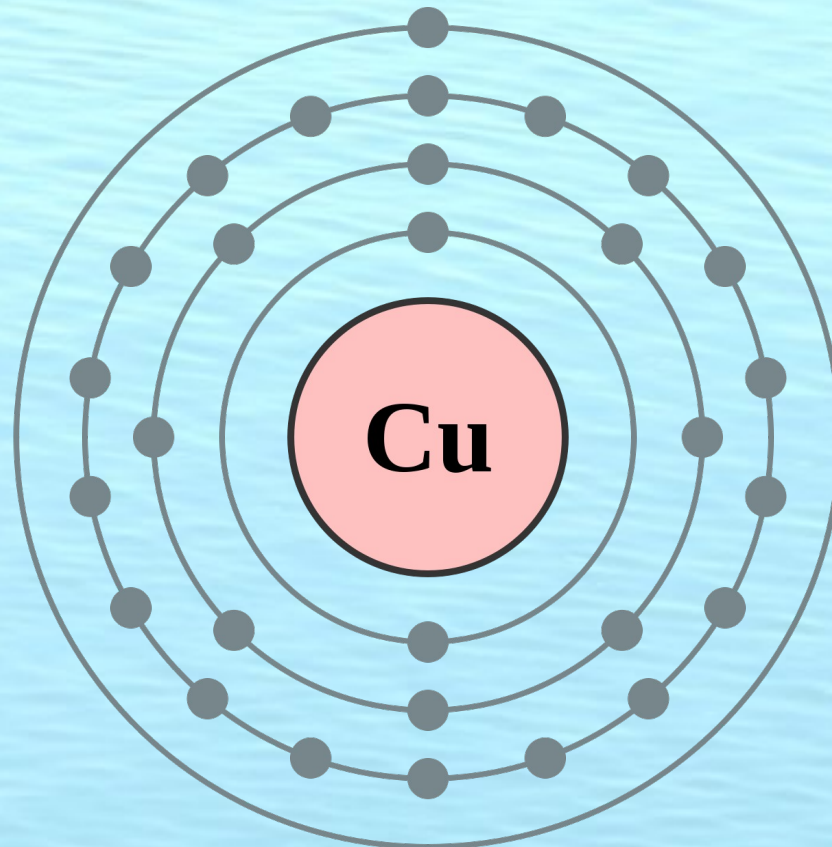
You can “break down” an insulator by ripping its electrons from its atoms.

Copper conducts electricity because its lone 4s electron is free to “wander”

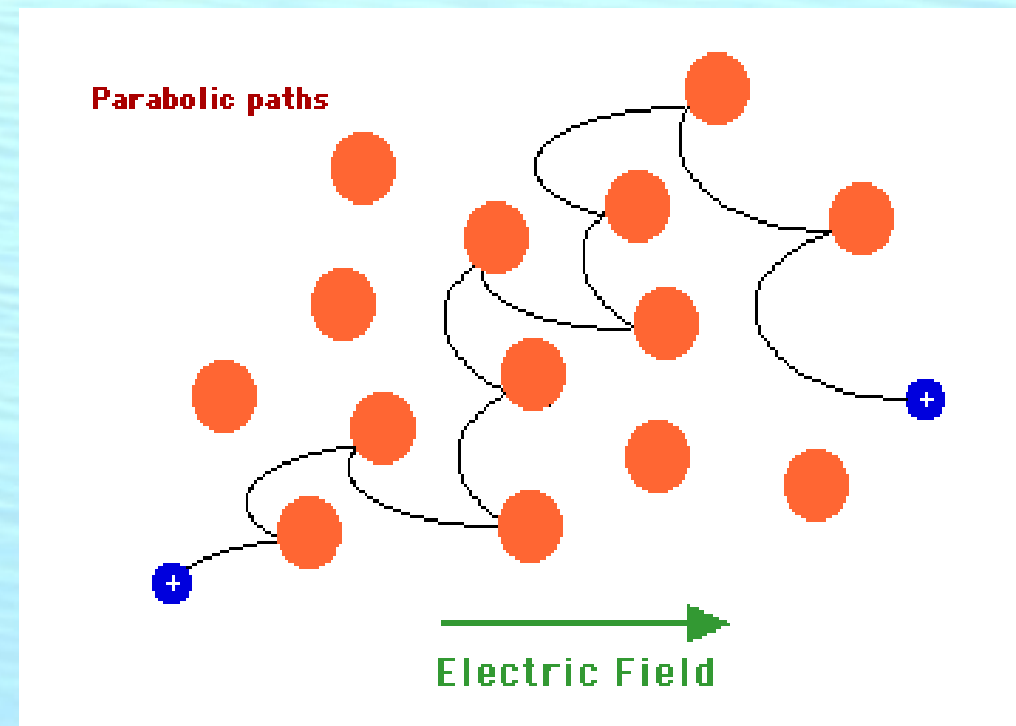
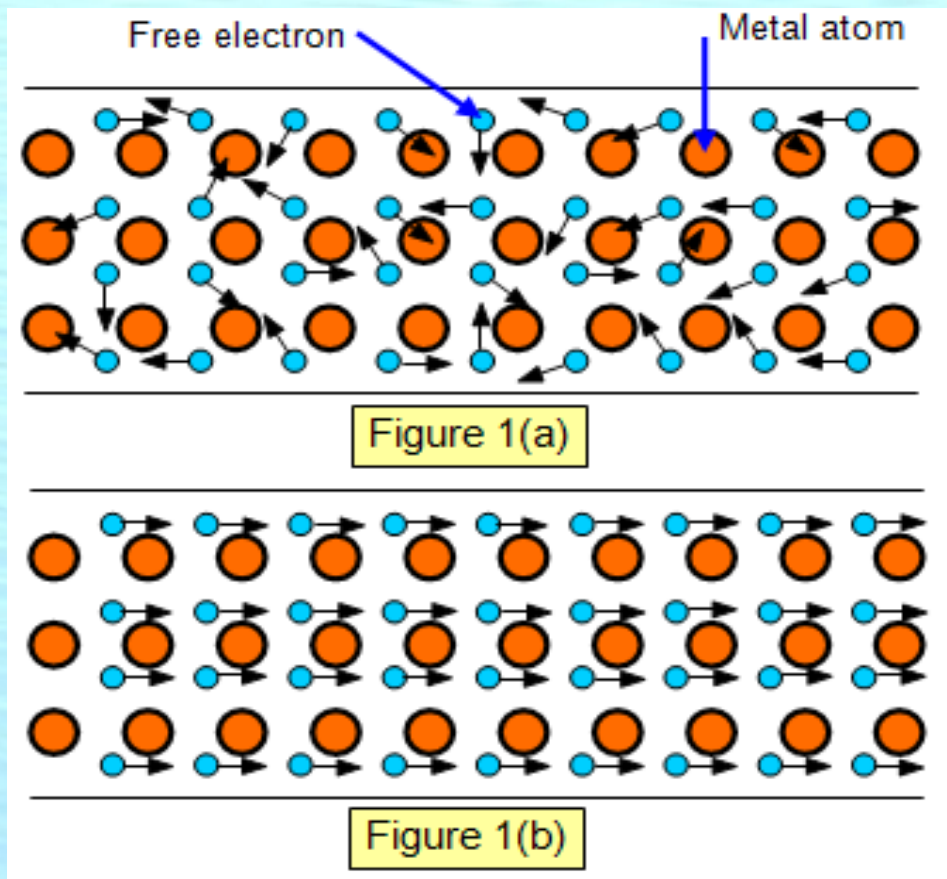
Conductivity (PHET)

29: Copper

2,8,18,1



Metal wires are actually “tubes” for electrons



The requirements for a material to Be a conductor are:

- (A) Electrons or ions are free to move from atom to atom.
- (B) There must be a net charge on the material
- (C) The material must be ductile, like a metal.
- (D) Both A and B
- (E) All of the above

Material	Resistivity ($\Omega \cdot m$)
Metallic conductors (20°C)	
Aluminum	2.65×10^{-8}
Copper	1.68×10^{-8}
Gold	2.24×10^{-8}
Iron	9.71×10^{-8}
Mercury	9.84×10^{-7}
Silver	1.59×10^{-8}
Ionic solutions (in water, 18°C)	
1-molar CuSO_4	3.9×10^{-4}
1-molar HCl	1.7×10^{-2}
1-molar NaCl	1.4×10^{-4}
H_2O	2.6×10^5
Blood, human	0.70
Seawater (typical)	0.22
Semiconductors (pure, 20°C)	
Germanium	0.47
Silicon	23.0
Insulators	
Ceramics	$10^{11}-10^{14}$
Glass	$10^{10}-10^{14}$
Polystyrene	$10^{15}-10^{17}$
Rubber	$10^{13}-10^{16}$
Wood (dry)	10^8-10^{14}

Resistance in terms of resistivity

$$R = \rho \frac{L}{A} = \frac{L}{\sigma A}$$

A piece of copper wire is 5 meters long and aluminum wire is 10 meters long. If the radius of the copper wire is 1 mm, what is the radius of the aluminum to have the same resistance?

$$R = \rho \frac{L}{A}$$

The femoral artery is the large artery that carries blood to the leg. What is the resistance of a 20-cm-long column of blood in a 1.0 cm diameter femoral artery? The conductivity of blood is

$$\sigma_{\text{blood}} = 0.63 \frac{1}{(\Omega \cdot \text{m})}$$

$$R = \rho \frac{L}{A} = \frac{L}{\sigma A}$$

$$\rho = \frac{1}{\sigma}$$

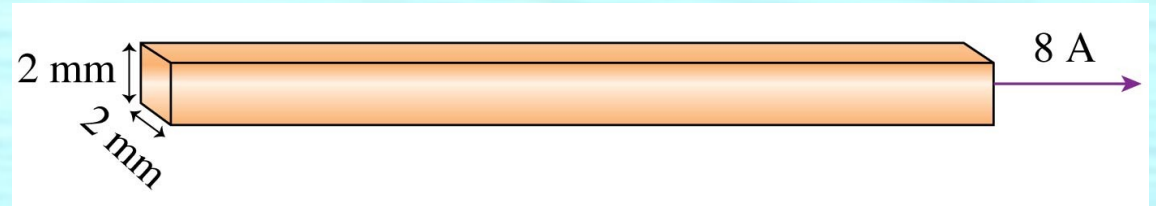
If you make a resistor with a length of copper wire of square cross-section with side 1 mm, how long a wire do you need to make a 3.2 Ohm resistor? $\rho_{\text{copper}} = 1.6 \times 10^{-8} \Omega \cdot \text{m}$

$$R = \rho \frac{L}{A}$$

- (A) 200,000 m
- (B) 160,000 m
- (C) 3.2 m
- (D) 100 m
- (E) 200 m

Clicker

The current density in this wire is



- A. $4 \times 10^6 \text{ A/m}^2$.
- B. $2 \times 10^6 \text{ A/m}^2$.
- C. $4 \times 10^3 \text{ A/m}^2$.
- D. $2 \times 10^3 \text{ A/m}^2$.
- E. Some other value.

The Current Density in a Wire

The **current density** J in a wire is the current per square meter of cross section:

$$J = \text{current density} \equiv \frac{I}{A} = n_e e v_d$$

The current density has units of A/m^2 .

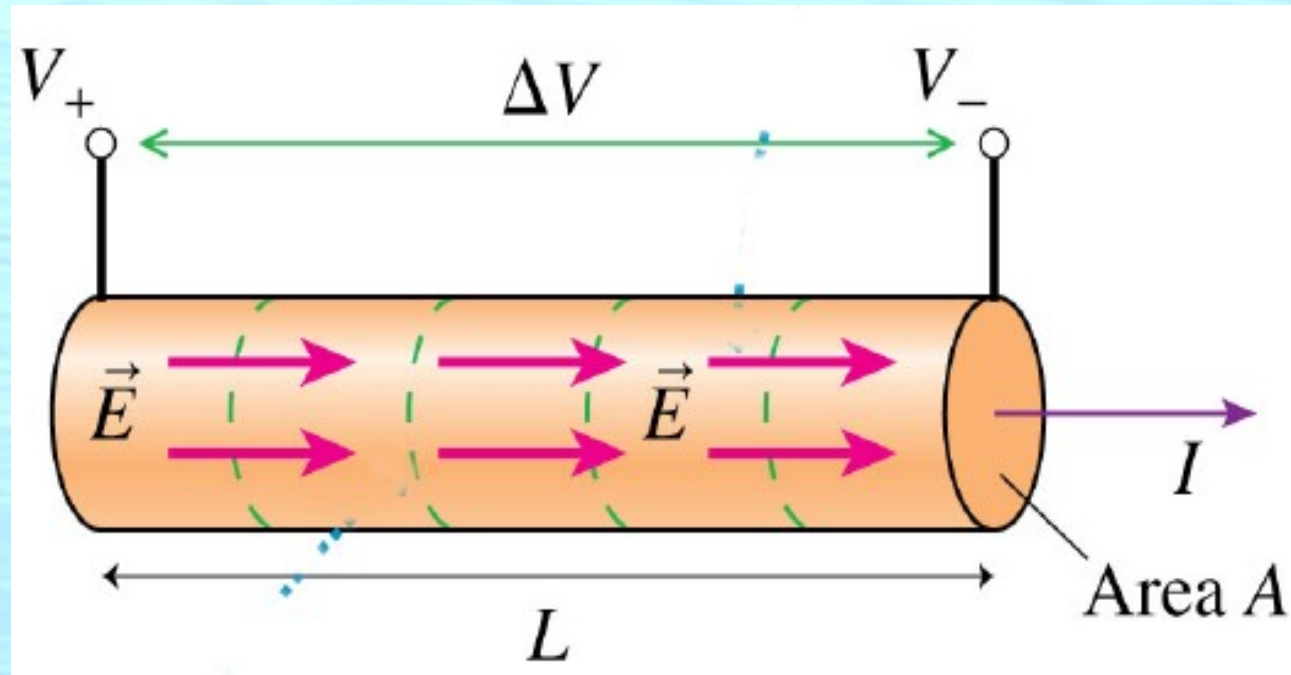
A cylindrical tube of seawater carries a total electric current of 350 mA. If the electric field in the water is 21 V/m, what is the diameter of the tube?

$$I = J A$$

$$J = \sigma E$$

Going from Microscopic to Macroscopic

$$\vec{J} = \sigma \vec{E} \quad E = \frac{V}{L} \quad J = \frac{I}{A} \quad R = \frac{L}{\sigma A} \rightarrow \sigma = \frac{L}{RA}$$



$$\frac{I}{A} = \frac{L}{RA} \frac{V}{L}$$

$$I = \frac{1}{R} V \rightarrow V = IR \quad \text{Ohm's Law}$$

Electric field in home wire?

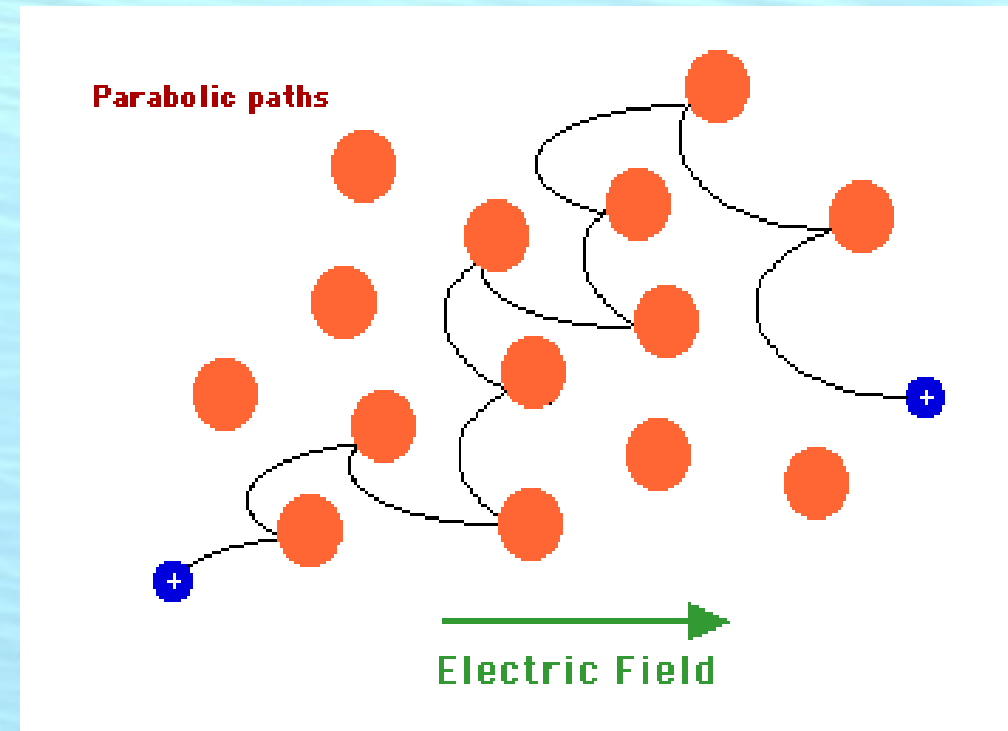
You are using a cheap extension cord made of #14 copper wire (which has a 2 mm^2 cross sectional area). What is the electric field in the wire when you light a 100 Watt bulb?

$$\rho_{\text{copper}} = 1.6 \times 10^{-8} \Omega \cdot \text{m}$$

Microscopic view of resistivity

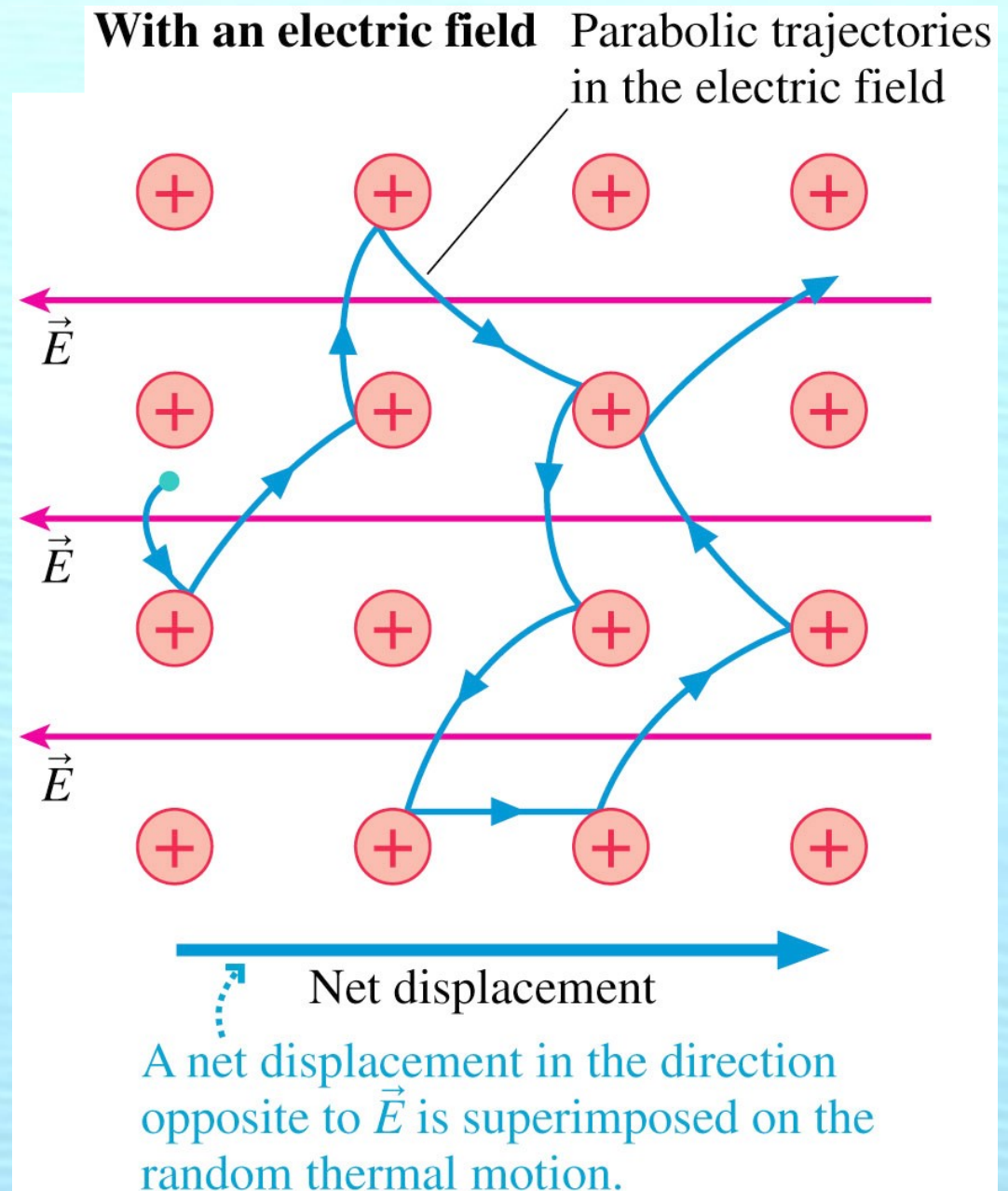
”Free electron gas” model (also called Drude model) of a metal.

You can derive Ohm's law by assuming a metal is a box full of loose electrons that bump into “scattering centers” every τ seconds.



Microscopic view of resistivity

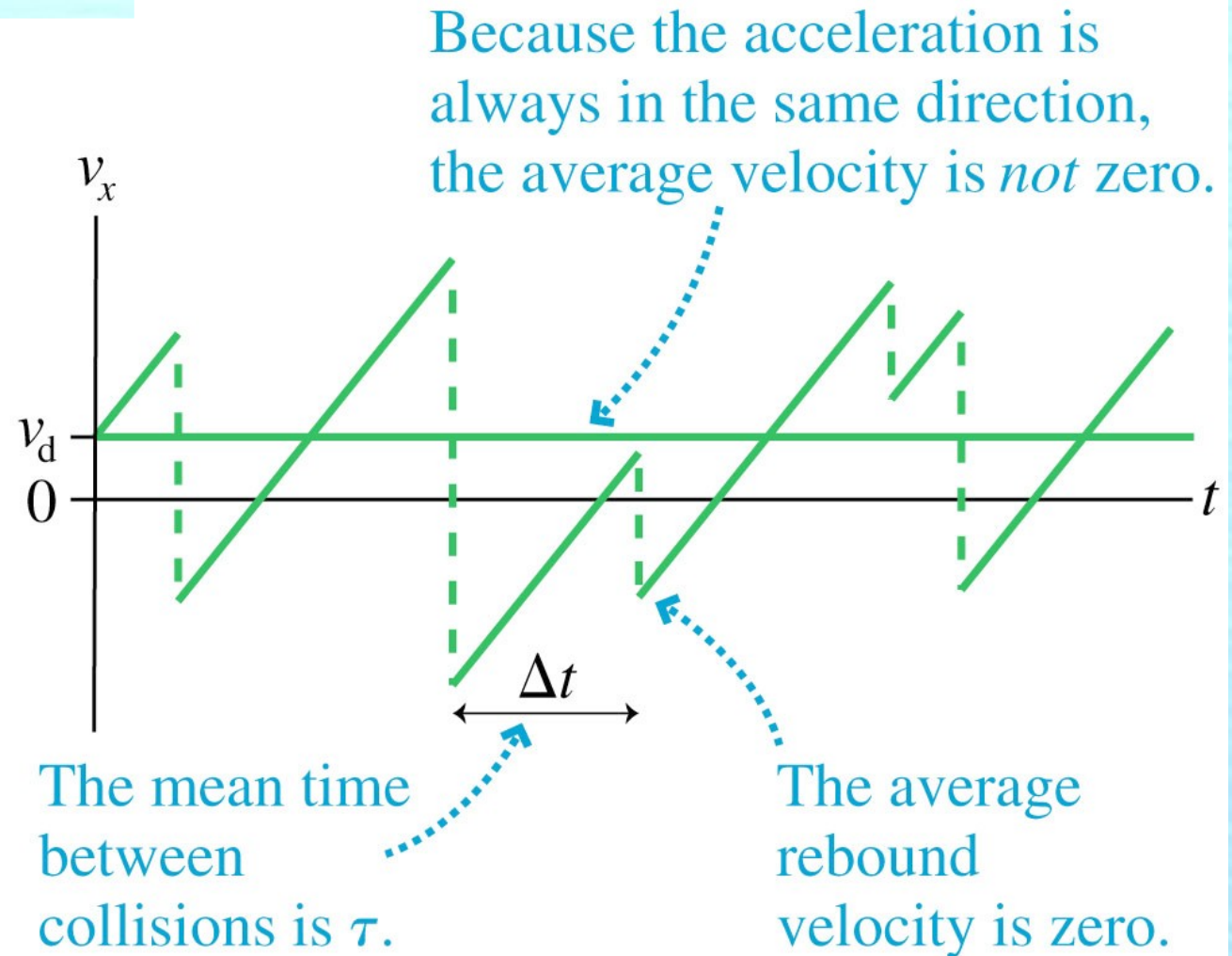
- In the presence of an electric field, the electric force causes electrons to move along parabolic trajectories between collisions.
- Because of the curvature of the trajectories, there is a slow net motion in the “downhill” direction.



Drude Model of Resistance

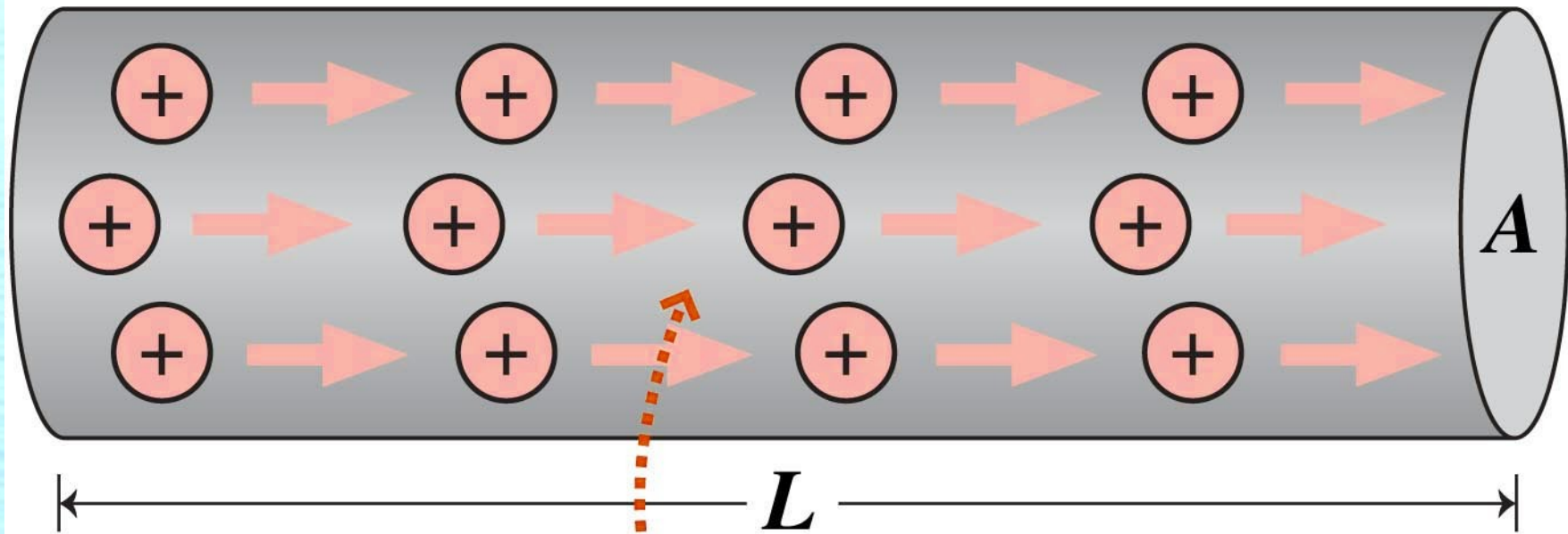
- The graph shows the speed of an electron during multiple collisions.
- The average drift speed is

$$v_d = \frac{e\tau}{m} E$$



$$\vec{v}_d \rightarrow$$

n charges/unit volume, each charge q



This volume contains charge $\Delta Q = nALq$.

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$$\Delta Q = n A \vec{v}_d \Delta t q$$

$$I = n q \vec{v}_d A$$

$$\vec{J} = n q \vec{v}_d$$

$$I = \vec{J} \cdot \vec{A}$$

Drude Model of Conductance

You can derive Ohm's law by assuming a metal is a box full of loose electrons that bump into “scattering centers” every τ Seconds. (Tau is < 1 picosecond for most solids at room temp.)

$$\vec{J} = n q \vec{v}_d = n q a \tau = \frac{n q^2 \tau}{m} \vec{E} = \sigma \vec{E}$$

Derived electrical resistance from Classical Mechanics!

$$\sigma = \frac{n q^2 \tau}{m}$$

In semiconductor, n is small, so ρ is larger than a conductor.
In insulator, n is nearly zero.

$$\rho = \frac{m}{n q^2 \tau}$$

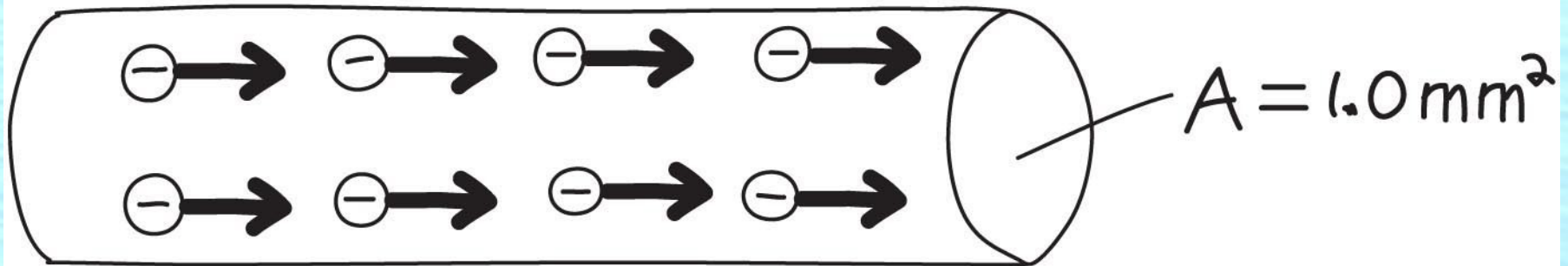
In ionic conductor, m is larger (ions not electrons) so conductivity is Lower.

A semiconductor may be “doped”

Typical drift velocity

$$I = 5.0 \text{ A}$$

$$n = 1.1 \times 10^{29} \text{ m}^{-3}$$



$$v_d = ?$$

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$$I = nq \vec{v}_d A$$