Physics 122 – Class #17 – Outline

- Announcements
- Field of continuous charge distributions (by symmetry)
- Electric Potential Energy
- Electric Potential
- Examples
- Equipotentials and your lab

Announcements

Test Covers Ch 25/26. Material up through today's lecture.
Review materials – sample test, omit problem 11.
More short problems online (Free MP)

Index card – One side. Equations. Pictures.
Subscripts, no words … MUST attach to exam.

Read Chapter 28 this week. Omit 28.3
Homework (MP) due this Saturday.
Extra time being spent on problems in recitation.
No quiz … attendance

Field near a “large” vertical uniform sheet of charge

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n} \qquad \sigma = \frac{Q}{A}$$

Field outside of a “long” line of charge.

$$\vec{E} = \frac{\lambda}{2\pi r\epsilon_0}\hat{r} = \frac{2k\lambda}{r}\hat{r} \qquad \lambda = \frac{Q}{L}$$

Field outside of a uniform sphere of charge Q.

$$\vec{E} = \frac{Q}{4\pi r^2\epsilon_0}\hat{r} = \frac{kQ}{r^2}\hat{r}$$

Direction of field from a sphere of charge

Because of spherical symmetry, the field must be radial (like a point charge)

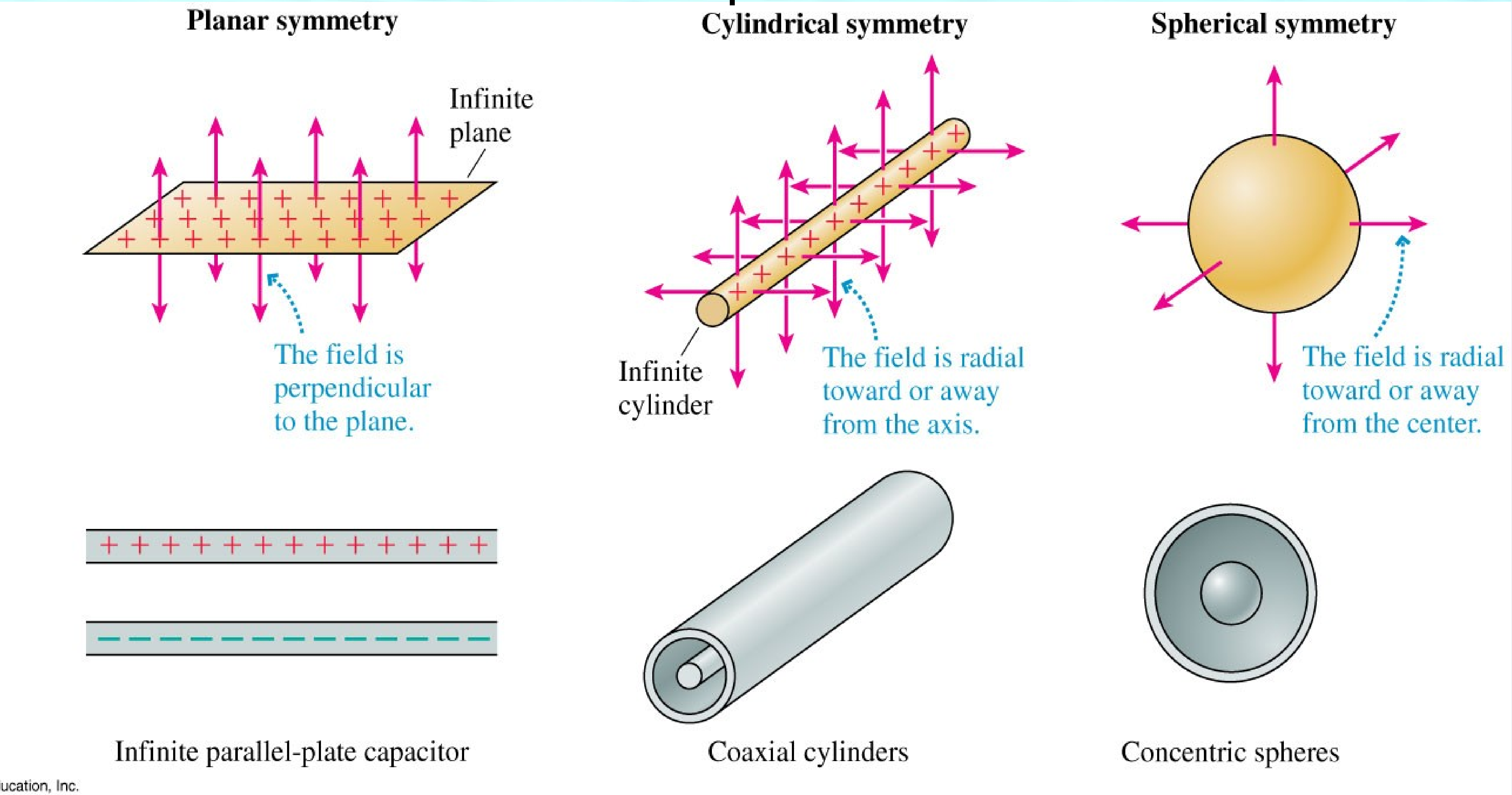
$$\vec{E}=\frac{\sigma}{2\epsilon_0}\hat{j}$$

$$\vec{E}=\frac{\lambda}{2\pi r\epsilon_0}\hat{r}$$

$$\vec{E}=2\frac{k}{r}\hat{r}$$

$$\vec{E}=\frac{Q}{4\pi r^2\epsilon_0}\hat{r}$$

$$\vec{E}=k\frac{Q}{r^2}\hat{r}$$

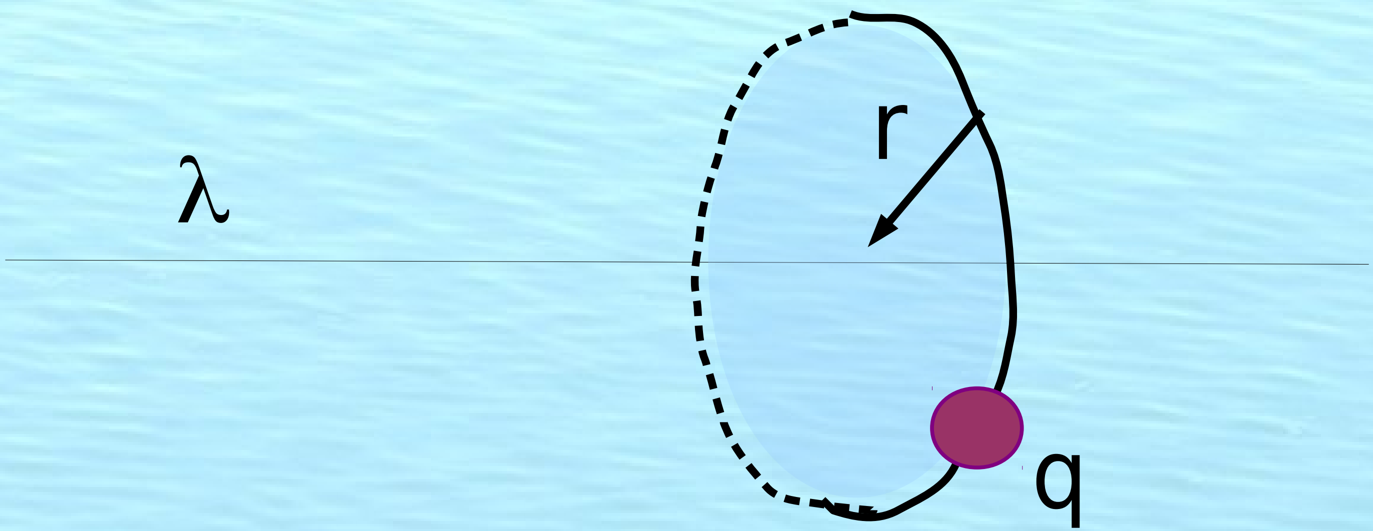


Problem

What is line charge density on a long wire if a 10 microgram particle carrying 3 nC orbits at 300 m/s?

$$\overrightarrow{E_{wire}} = 2k\frac{\lambda}{r}\hat{r}$$

$$\lambda = \frac{Q}{L}$$



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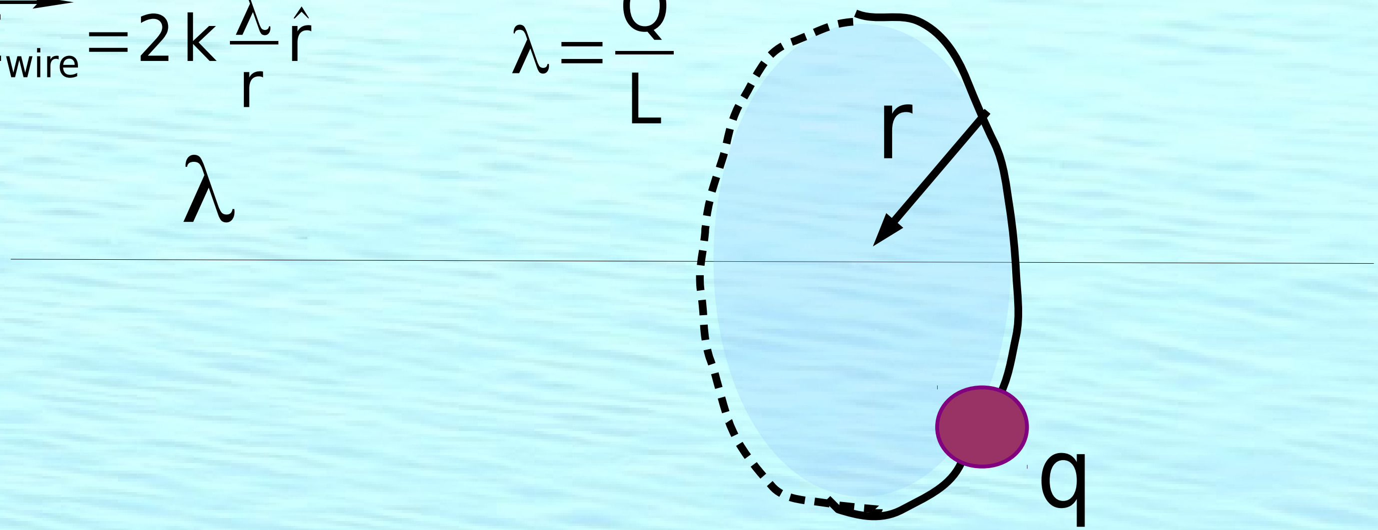
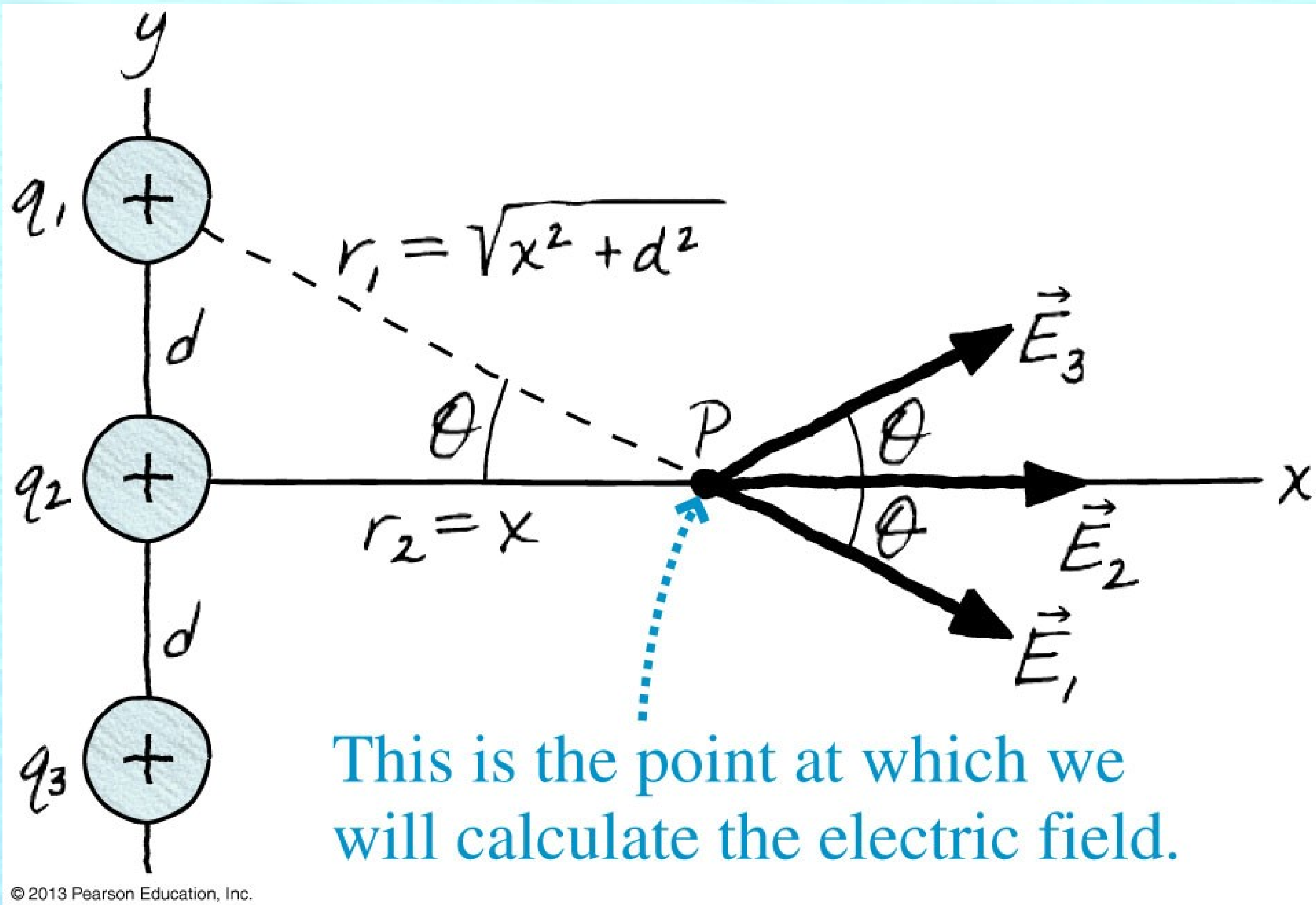


Physics 122 – Class #16 – Outline

- Announcements
- Field of continuous charge distributions (by symmetry)
- Worked Problems
- Derivation of field of a line
- Equipotentials (next week's lab)

Electric field of three charges



Electric field of three charges

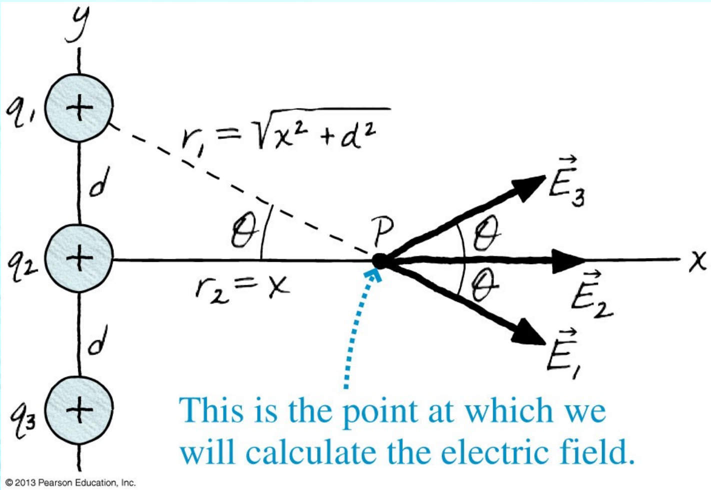
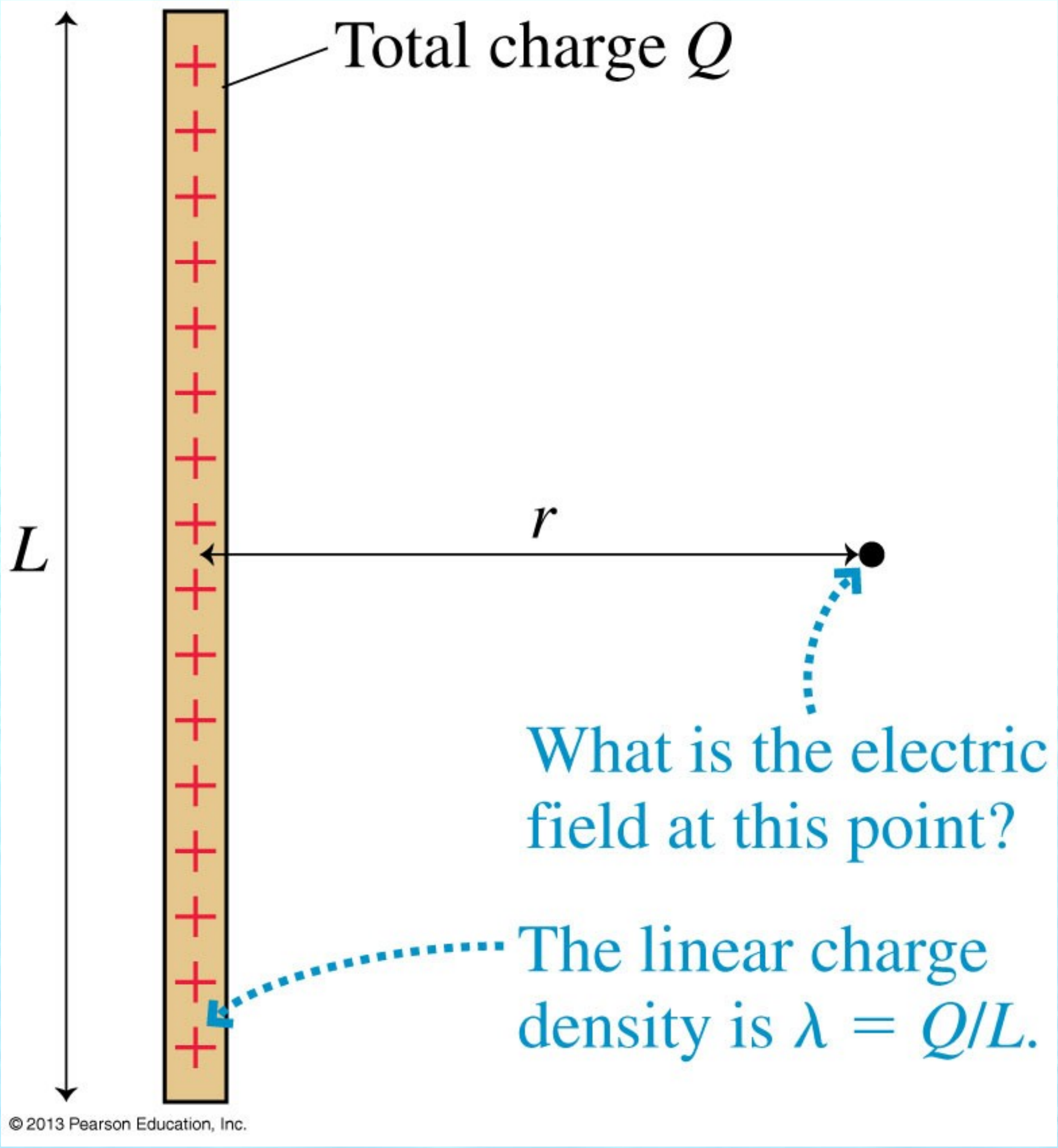
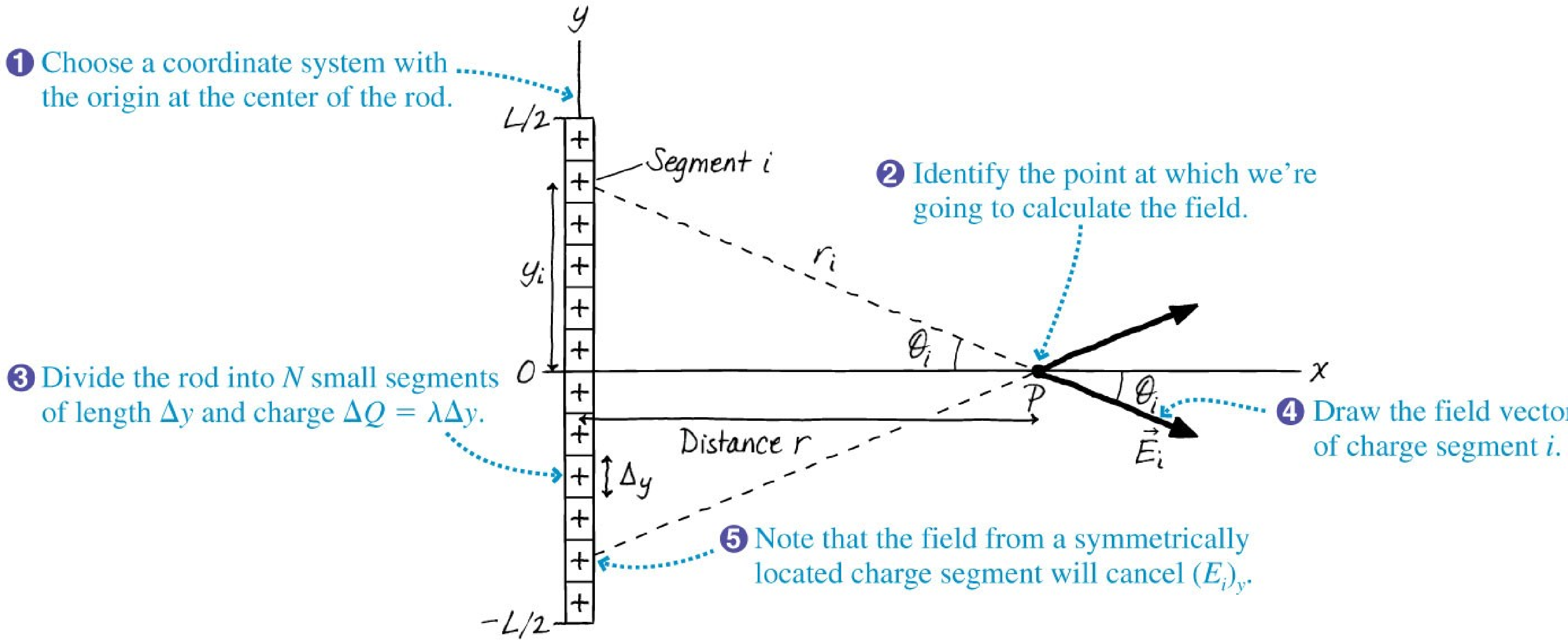


Figure 26.11

Electric field of charged wire





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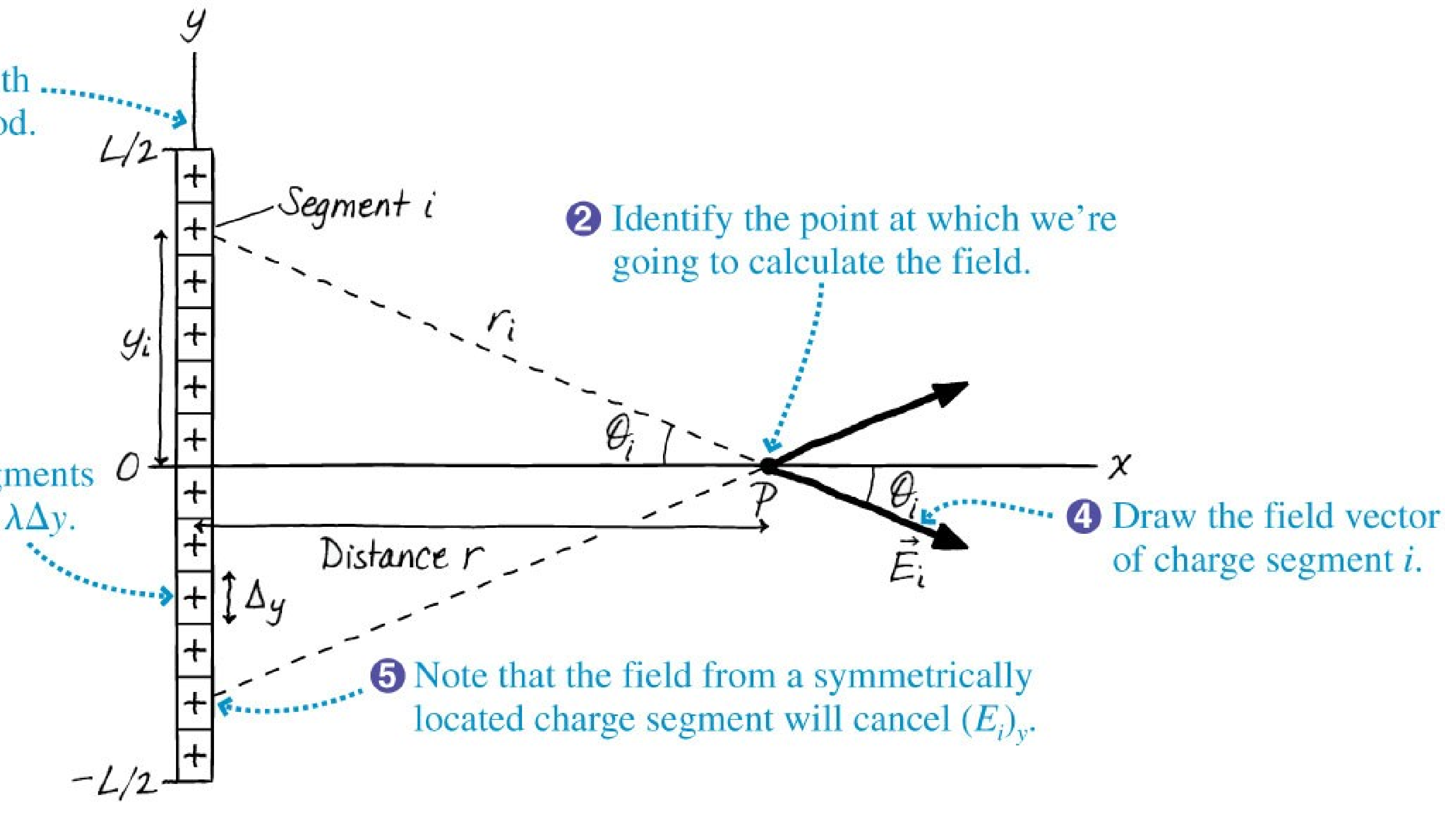


th
od.
y
L/2
Segment i
y_i
r_i
2 Identify the point at which we're going to calculate the field.
θ_i
0
x
gments
$\lambda\Delta y$.
P
Distance r
θ_i
$\vec{E}_i$
4 Draw the field vector of charge segment i.
Δy
5 Note that the field from a symmetrically located charge segment will cancel $(E_i)_y$.
−L/2

Electric field of a charged wire

The following material will NOT be on the exam … but you need it for lab and homework.

Energy

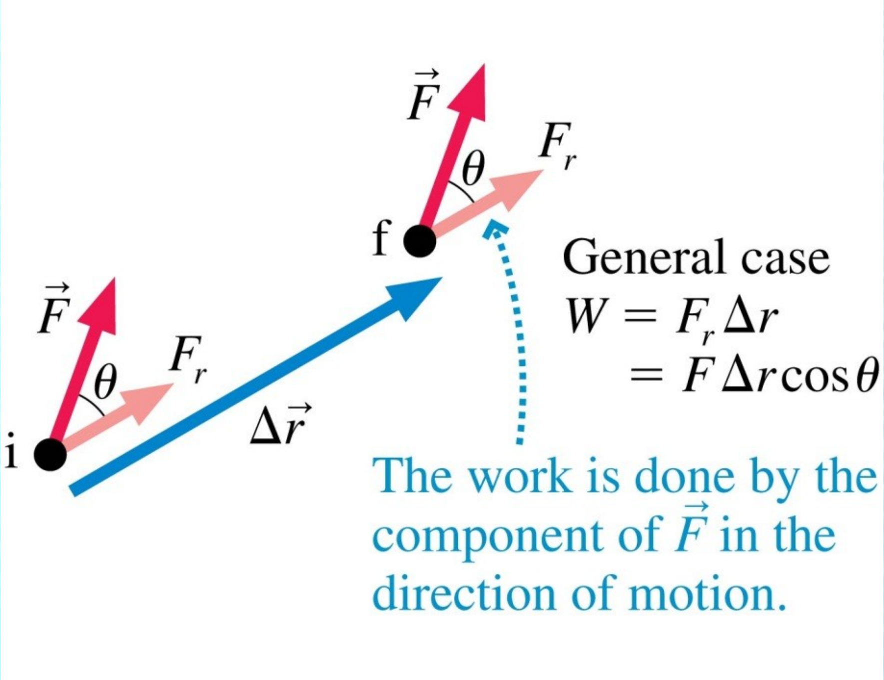
- The kinetic energy of a system, K, is the sum of the kinetic energies $K_i = 1/2 m_i v_i^2$ of all the particles in the system.
- The potential energy of a system, U, is the *interaction energy* of the system.
- The change in potential energy, ΔU, is -1 times the work done by the interaction forces:

$$\Delta U = U_\mathrm{f} - U_\mathrm{i} = -W_\text{interaction forces}$$

- If all of the forces involved are *conservative forces* (such as gravity or the electric force) then the total energy $K + U$ is *conserved*; it does not change with time.
- (Friction is a non-conservative Force … it converts Work into heat)

Work Done by a Constant Force

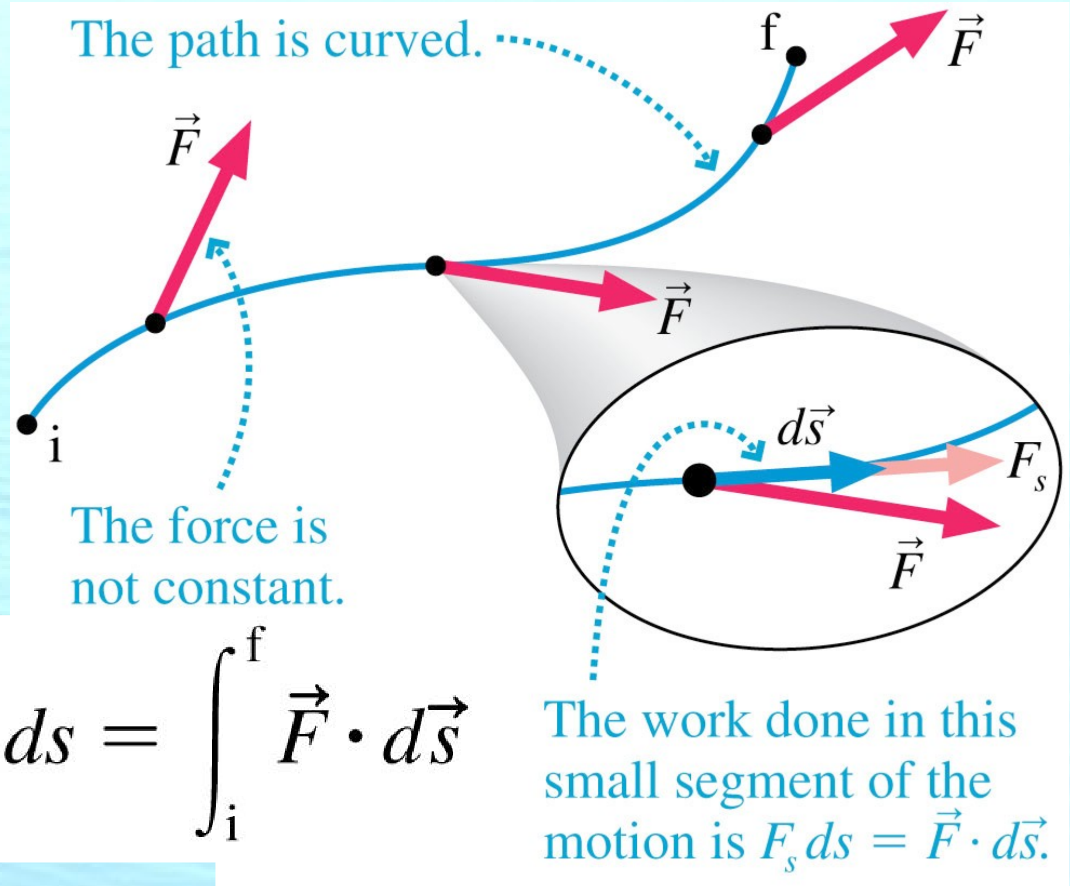
- Recall that the work done by a constant force depends on the angle θ between the force F and the displacement Δr.



- If $\theta = 0°$, then $W = F\Delta r$.
- If $\theta = 90°$, then $W = 0$.
- If $\theta = 180°$, then $W = -F\Delta r$.

Work

If the force is *not* constant or the displacement is *not* along a linear path, we can calculate the work by dividing the path into many small segments.



$$W = \sum_j (F_s)_j\,\Delta s_j \rightarrow \int_{s_i}^{s_f} F_s\,ds = \int_i^f \vec{F}\cdot d\vec{s}$$

Note that **dr** and **ds**

Both mean "small displacement"

Books and MP are inconsistent on which they use.

Gravitational Potential Energy

- Every conservative force is associated with a potential energy.
- In the case of gravity, the work done is:

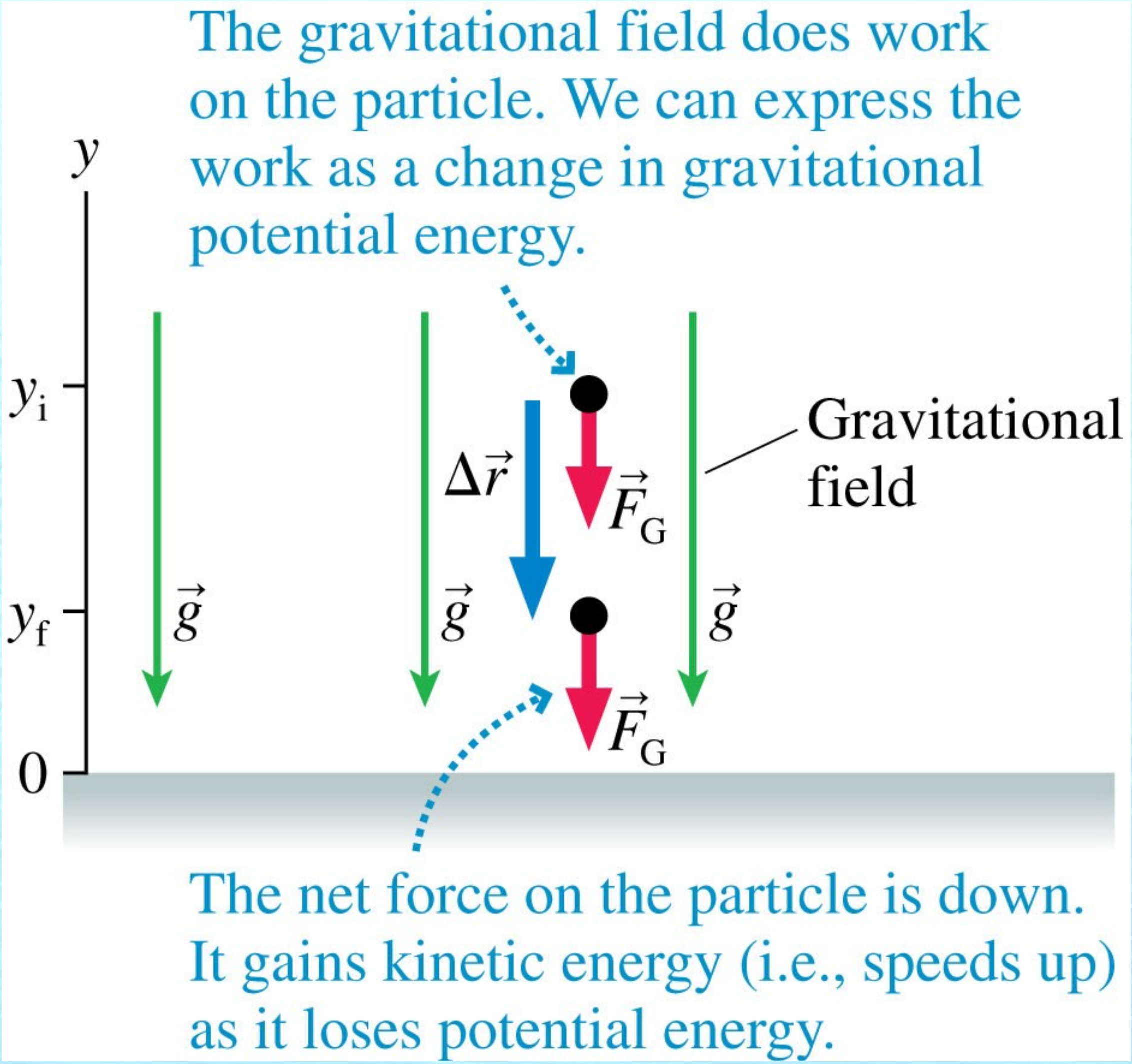
$$W_{\text{grav}} = mgy_{\text{i}} - mgy_{\text{f}}$$

- The change in gravitational potential energy is:

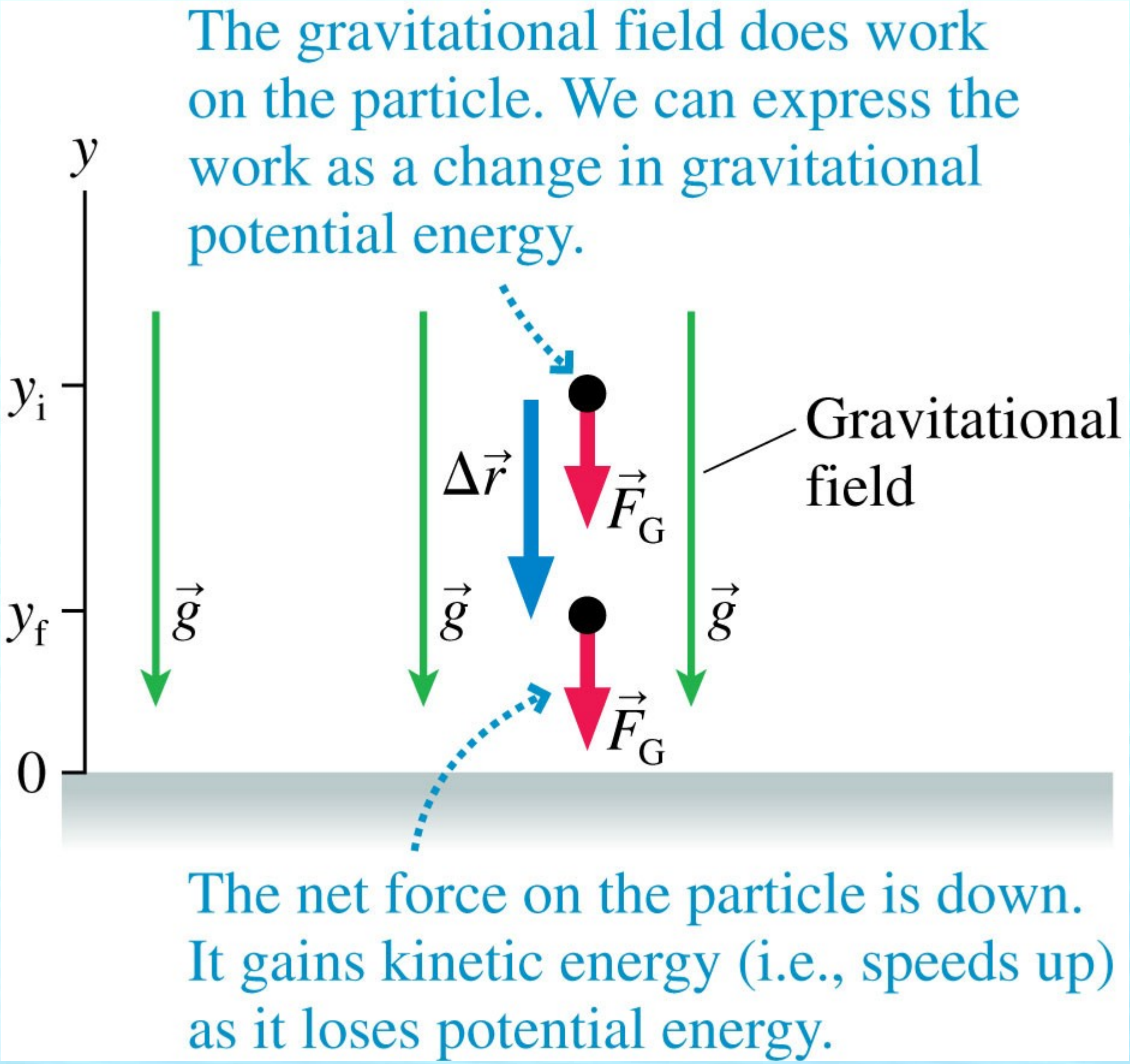
$$\Delta U_{\text{grav}} = -W_{\text{grav}}$$

where

$$U_{grav} = U_0 + mgy$$

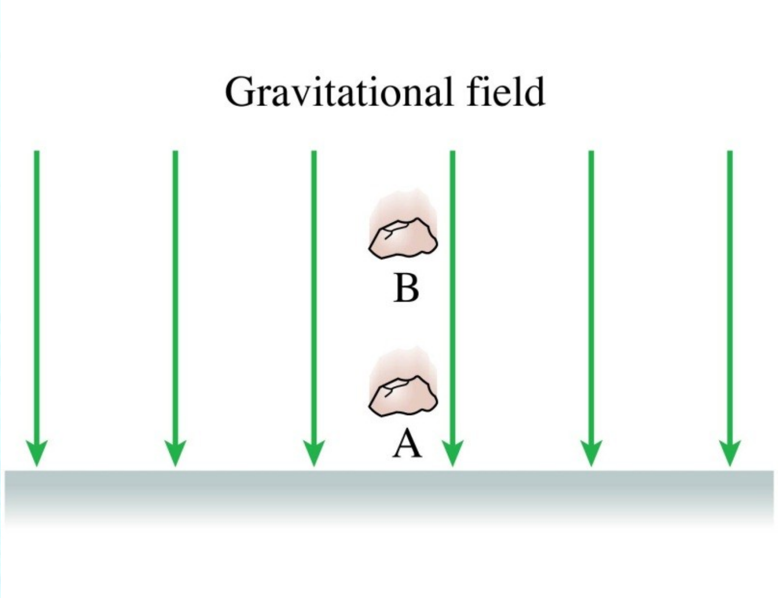


Gravitational Potential Energy –
Reminding you of why $U = mgy$



Clicker

Two rocks have equal mass. Which has more gravitational potential energy?



A. Rock A.

B. Rock B.

C. They have the same potential energy.

D. Both have zero potential energy.

Electric Potential Energy in a Uniform Field

- A positive charge q inside a capacitor speeds up as it "falls" toward the negative plate.
- There is a constant force $F = qE$ in the direction of the displacement.
- The work done is:

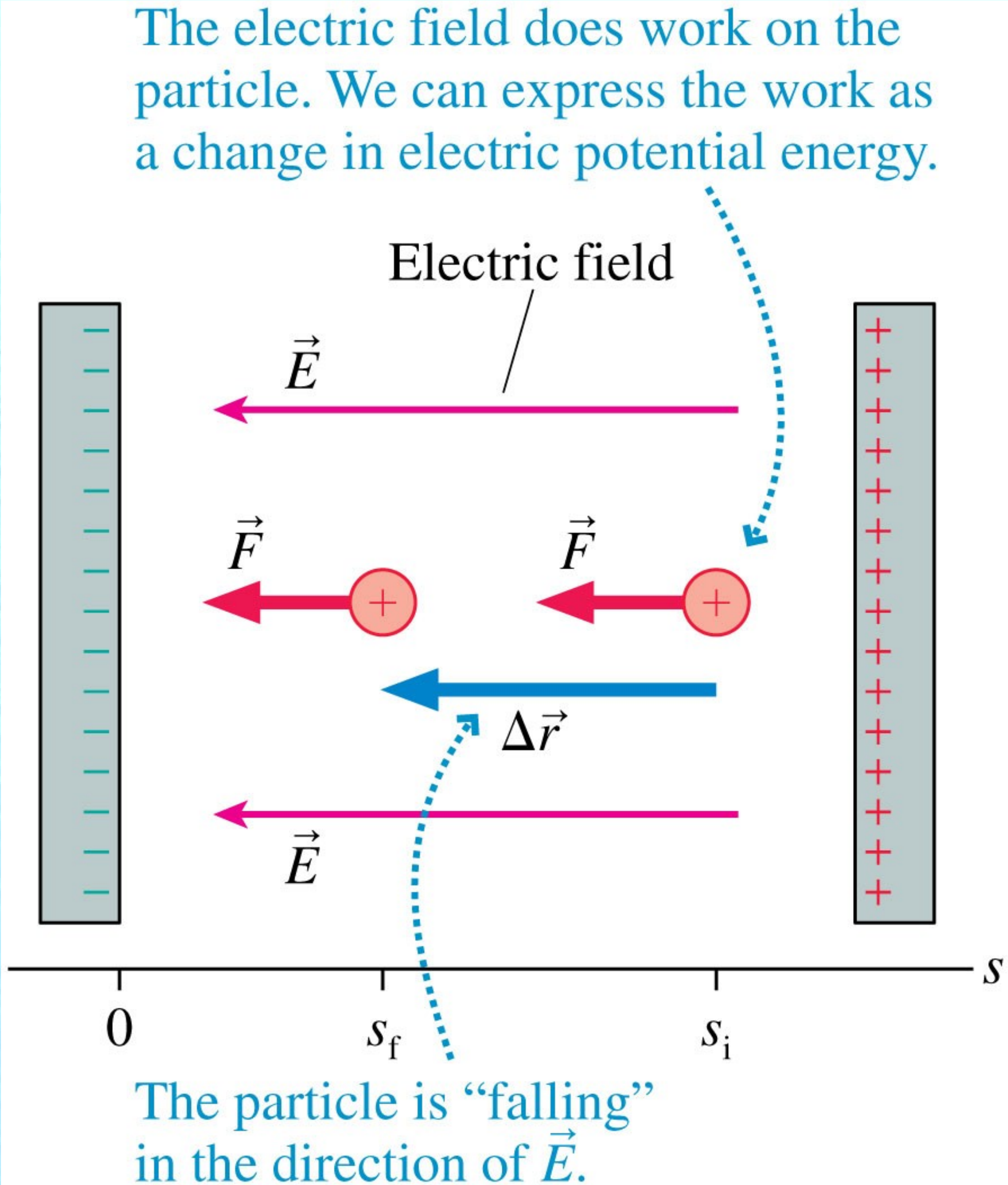
$$W_{\text{elec}} = qEs_{\text{i}} - qEs_{\text{f}}$$

- The change in **electric potential energy** is:

$$\Delta U_{\text{elec}} = -W_{\text{elec}}$$

where

$$U_{\text{elec}} = U_0 + qEs$$



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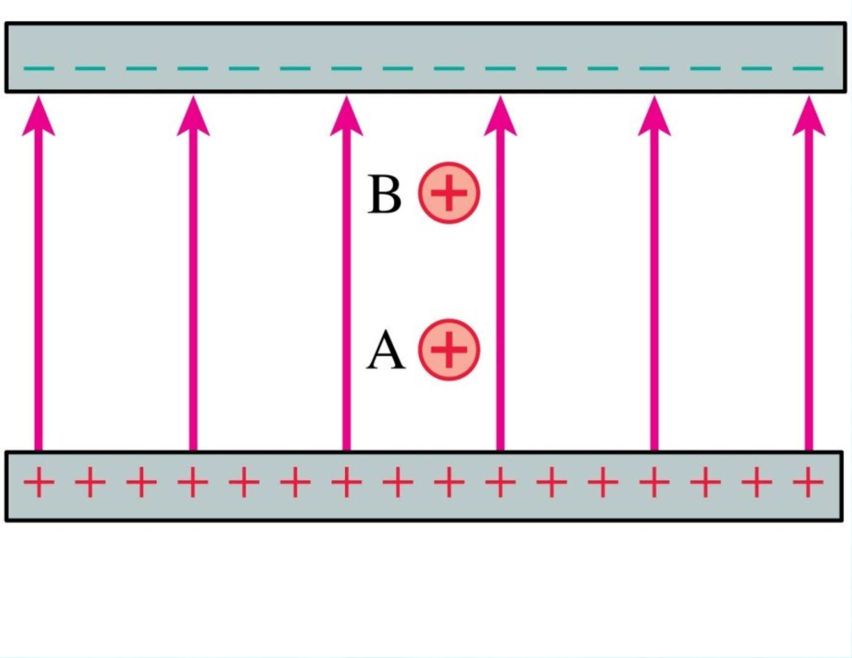
Two positive charges are equal. Which has more electric potential energy?



A. Charge A.

B. Charge B.

C. They have the same potential energy.

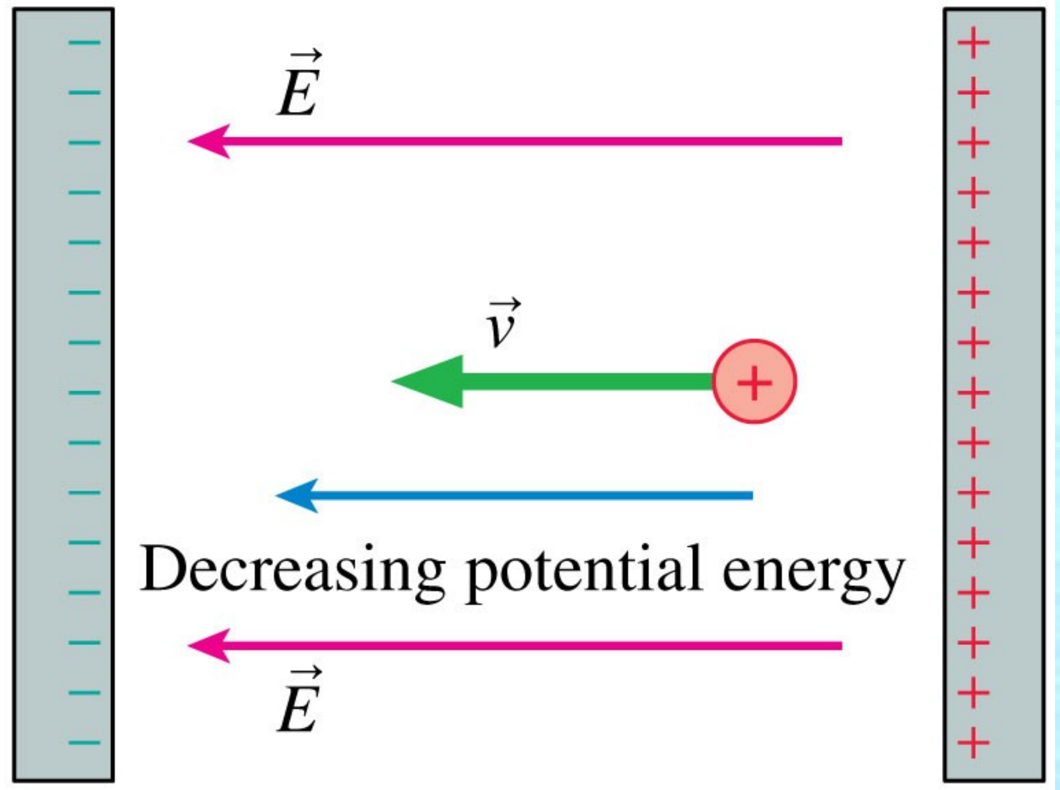
D. Both have zero potential energy.

HW Problem 28.13

What potential difference is needed to accelerate a He+ ion to a speed of $v = 2.0 \times 10^{6}\,\mathrm{m/s}$?

Electric Potential Energy in a Uniform Field

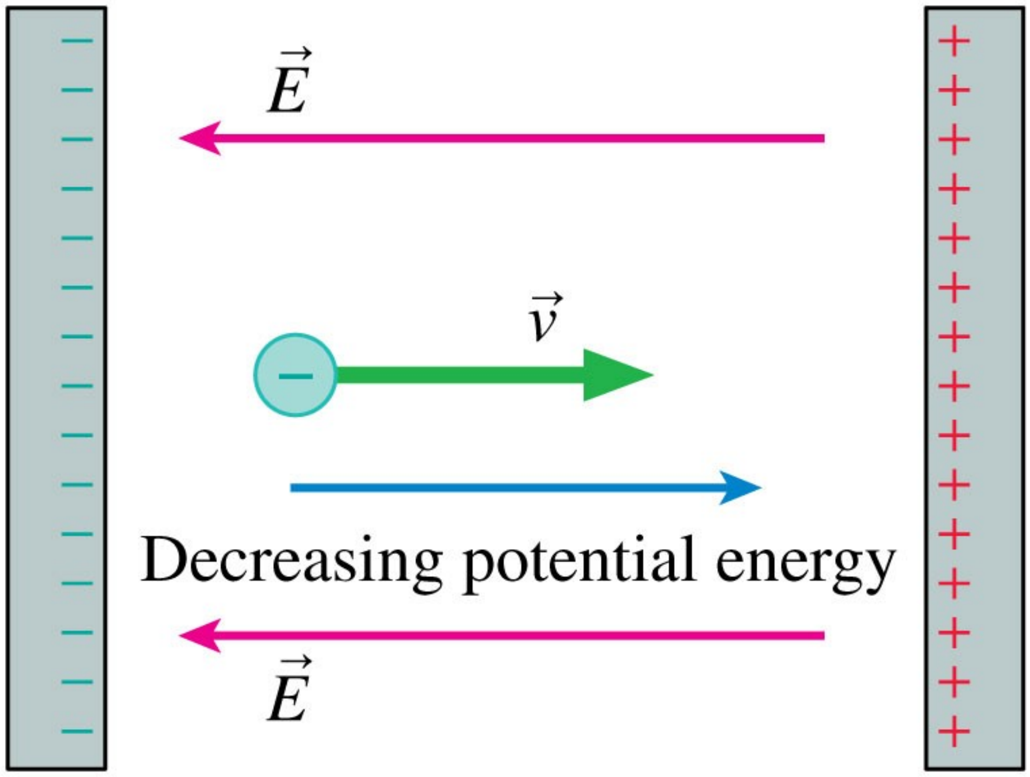
A positively charged particle gains kinetic energy as it moves in the direction of decreasing potential energy.



The potential energy of a positive charge decreases in the direction of $\vec{E}$. The charge gains kinetic energy as it moves toward the negative plate.

Electric Potential Energy in a Uniform Field

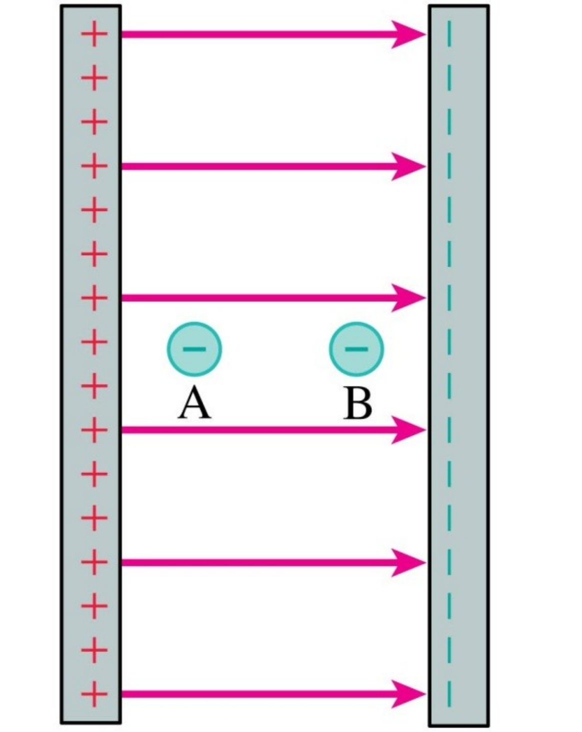
A negatively charged particle gains kinetic energy as it moves in the direction of decreasing potential energy.



The potential energy of a negative charge decreases in the direction opposite to $\vec{E}$. The charge gains kinetic energy as it moves away from the negative plate.

Clicker

Two negative charges are equal. Which has more electric potential energy?



A. Charge A.

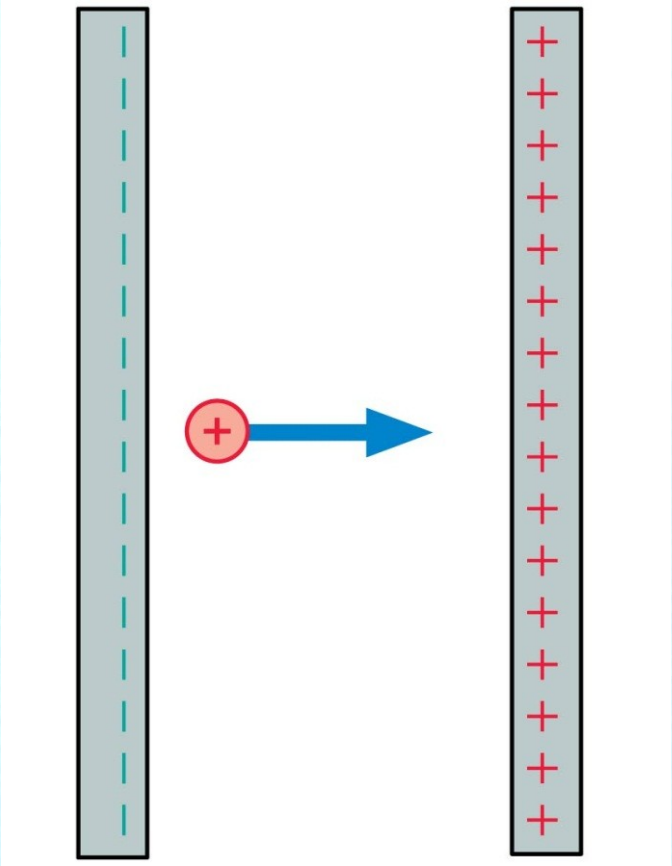
B. Charge B.

C. They have the same potential energy.

D. Both have zero potential energy.

Clicker

A positive charge moves as shown. Its kinetic energy



A. Increases.

B. Remains constant.

C. Decreases.

Potential Energy (Unit: Joule)

The energy an object gains as it moves from point A to point B for a given applied force. An object with high potential energy will spontaneously convert it to kinetic energy if released.

Potential energy is INDEPENDENT of path from A to B.

$$\vec{F} = q\vec{E}$$

$$U = qV$$

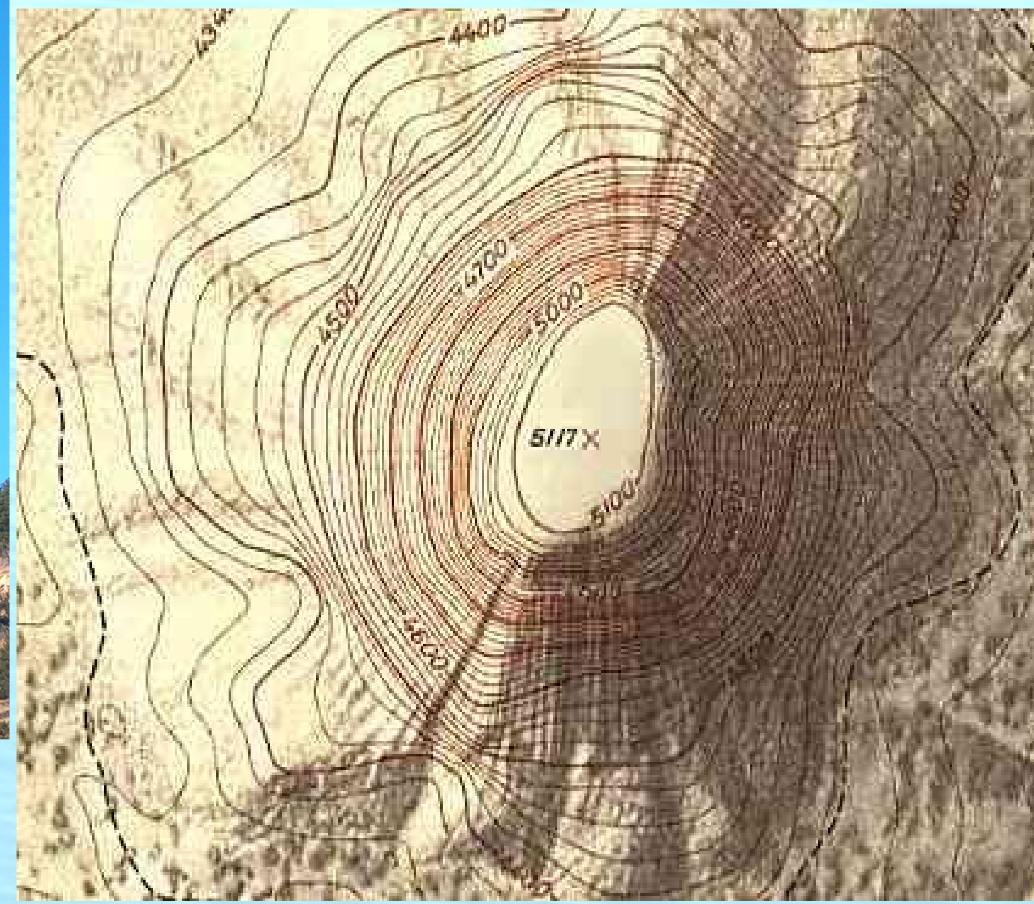
Electric Potential (Unit: Volt)

The energy a one Coulomb charge would gain as it moves from point A to B in an electric field. A positive charge at high voltage will spontaneously convert it to kinetic energy if released.

An "equipotential line" is one at which the potential is the same everywhere (and at which identical charges would have identical potential energies). It takes NO WORK to move a charge from one part of an equipotential line to another.

Electric Potential is like Geopotential

$$U = m(gh)$$



$$U = q(Ez) = qV$$

HW Problem 28.13

What potential difference is needed to accelerate a He+ ion to a speed of

$$v = 2.0 \times 10^{6}\,\text{m/s}$$

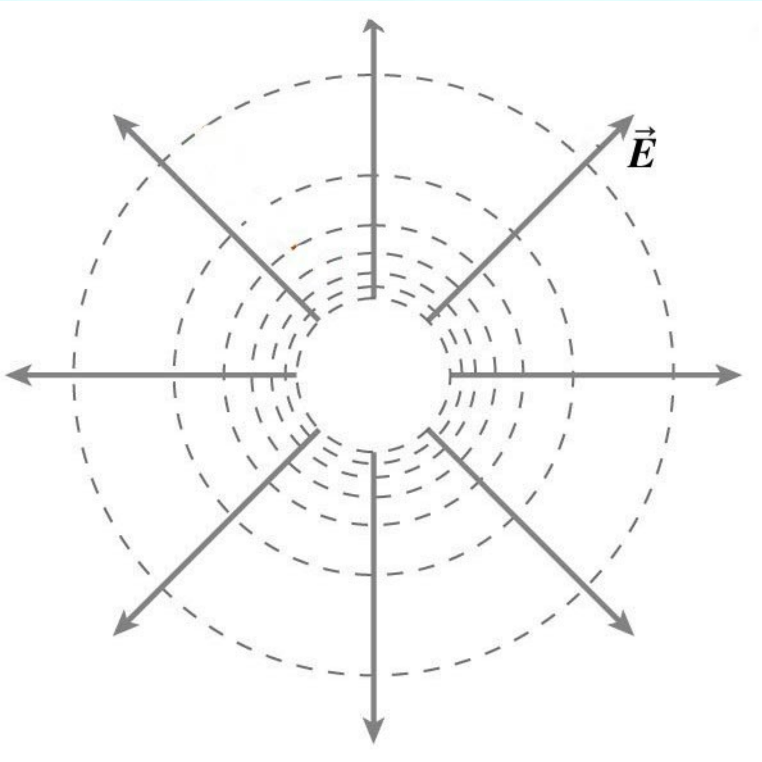
$$m_{He} = 4 \times m_p = 4 \text{ x } 1.67 \times 10^{-27}\,\text{kg}$$

$$q = 1.6 \times 10^{-19}\,\text{C}$$

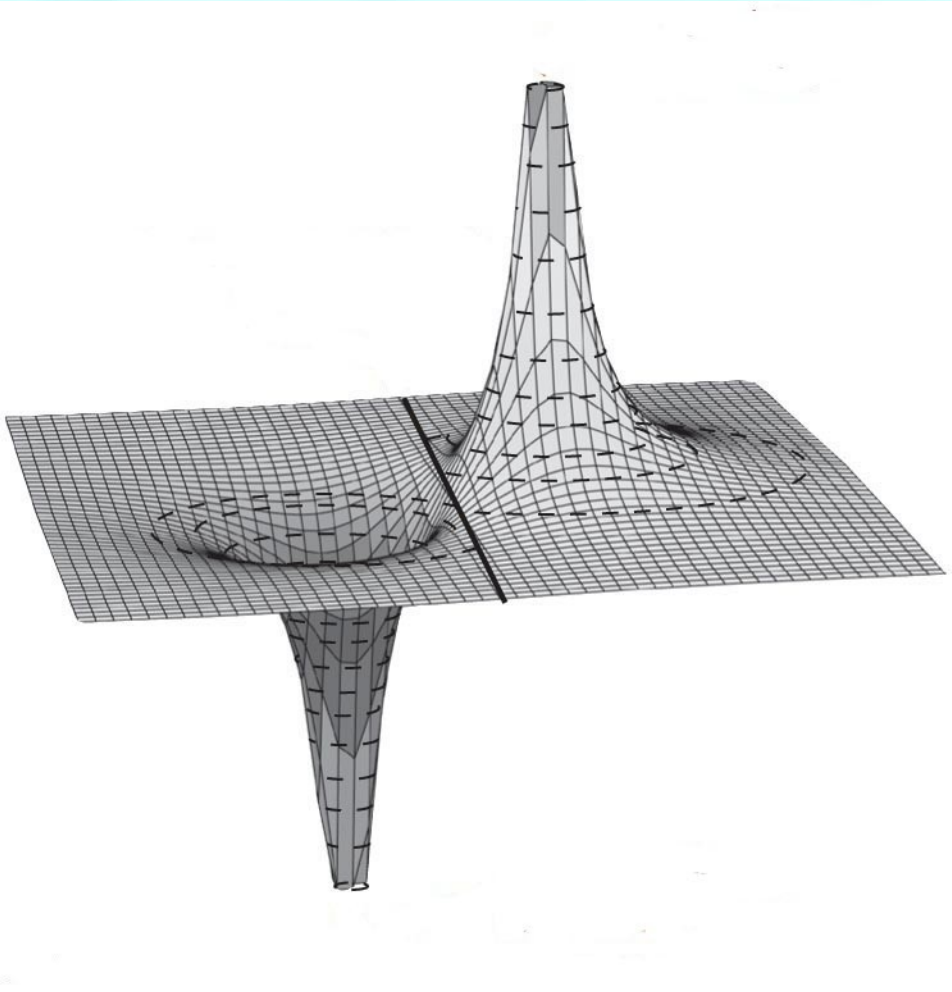
$$U = qV$$

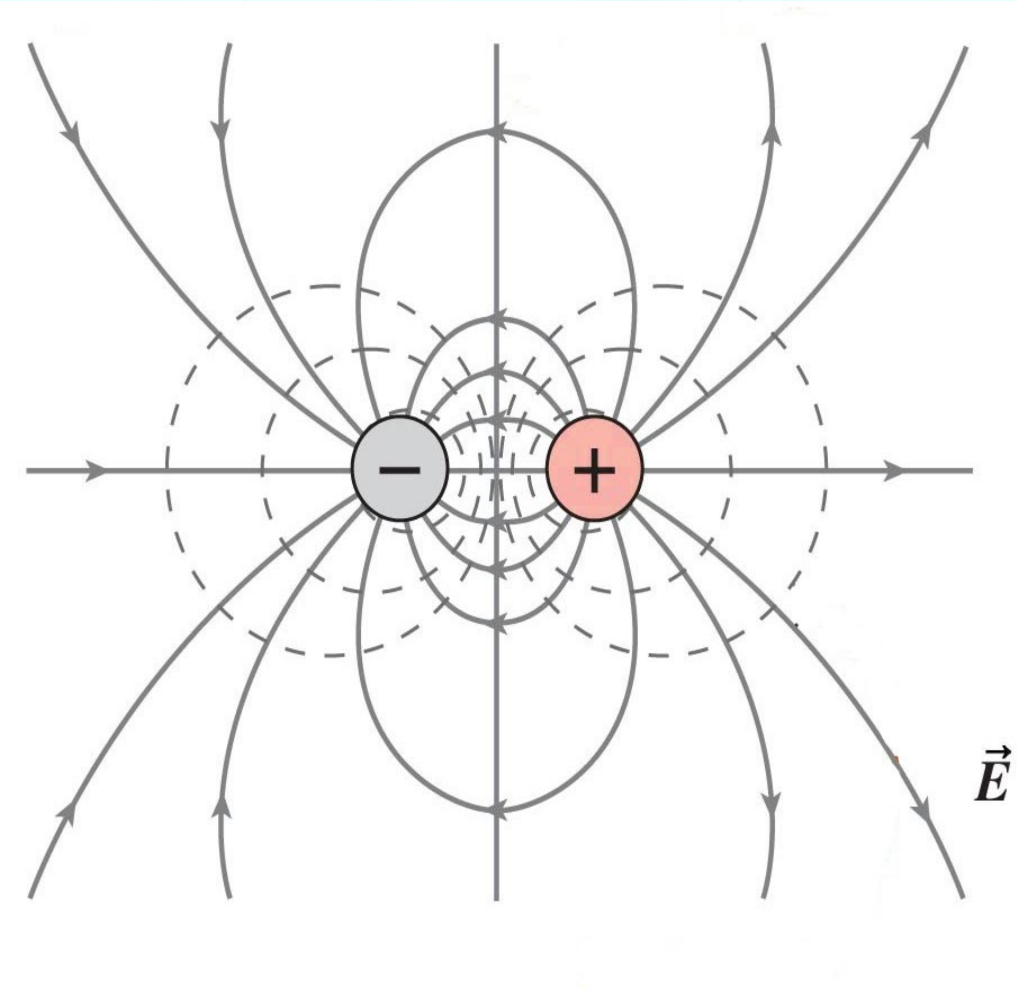
$$K_{initial} + U_{initial} = K_{final} + U_{final}$$

Potential of a charged conducting sphere



Potential surface of a dipole





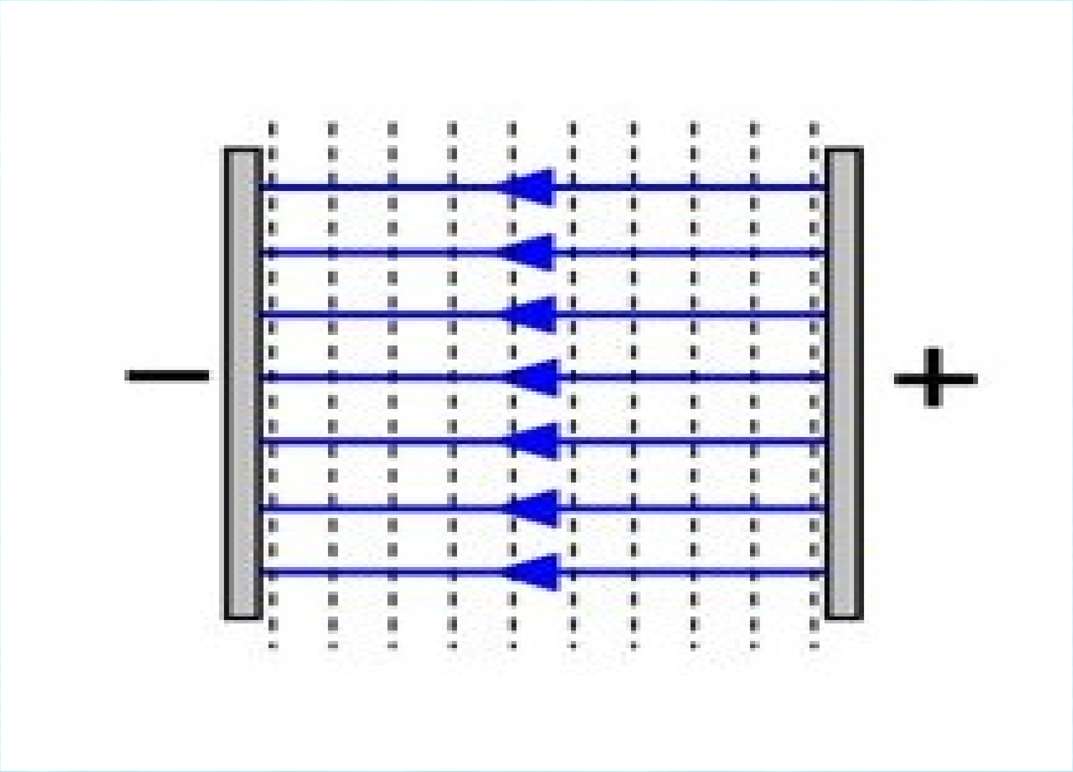
Parallel Plates

Equipotentials are equally spaced lines.

$$V = \frac{\sigma}{\epsilon_0} x$$

Electric field is constant in magnitude.

$$\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$$



Coaxial Cylinders

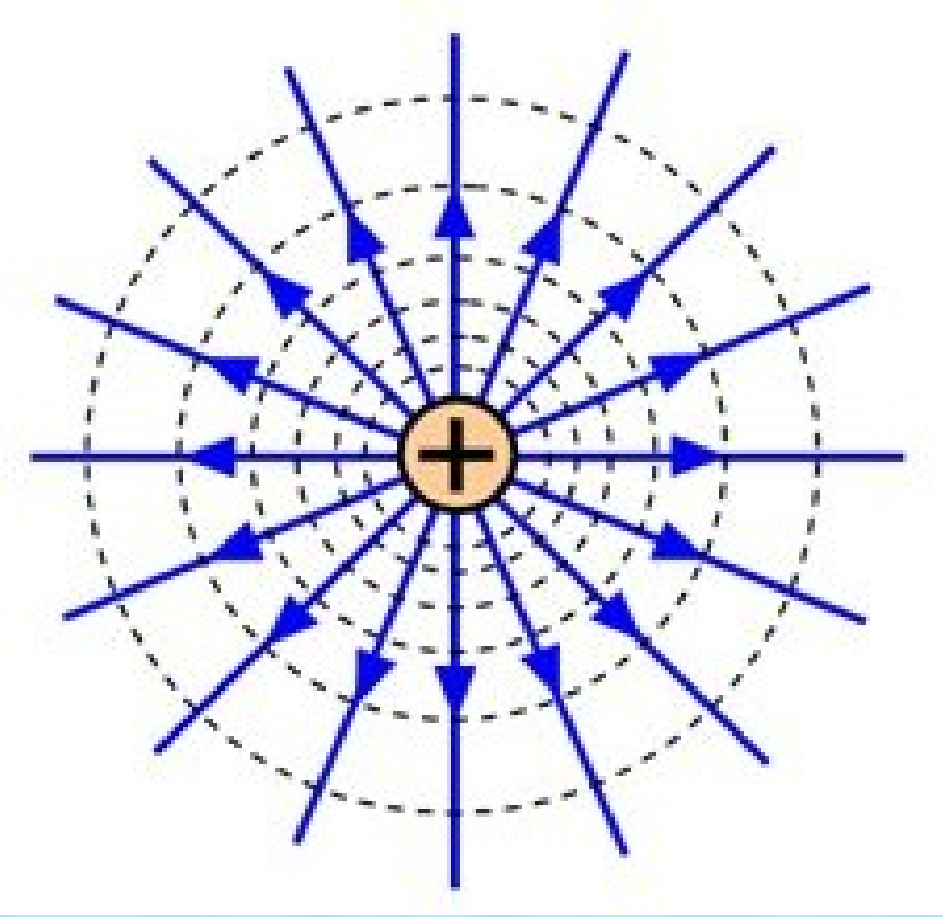
Equipotentials are circles and closer together near center electrode

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r}{r_1}\right)$$

Electric field is radial

And decreases with distance

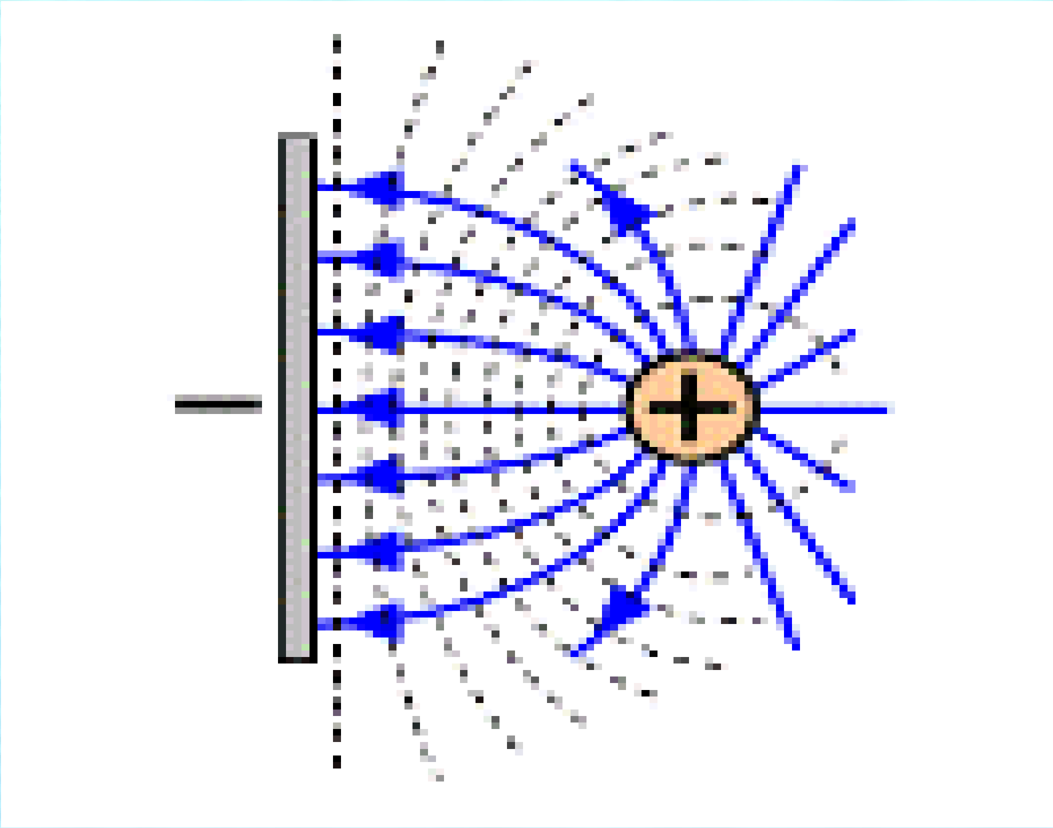
$$\vec{E} = \frac{\lambda}{2\pi r \epsilon_0} \hat{r}$$

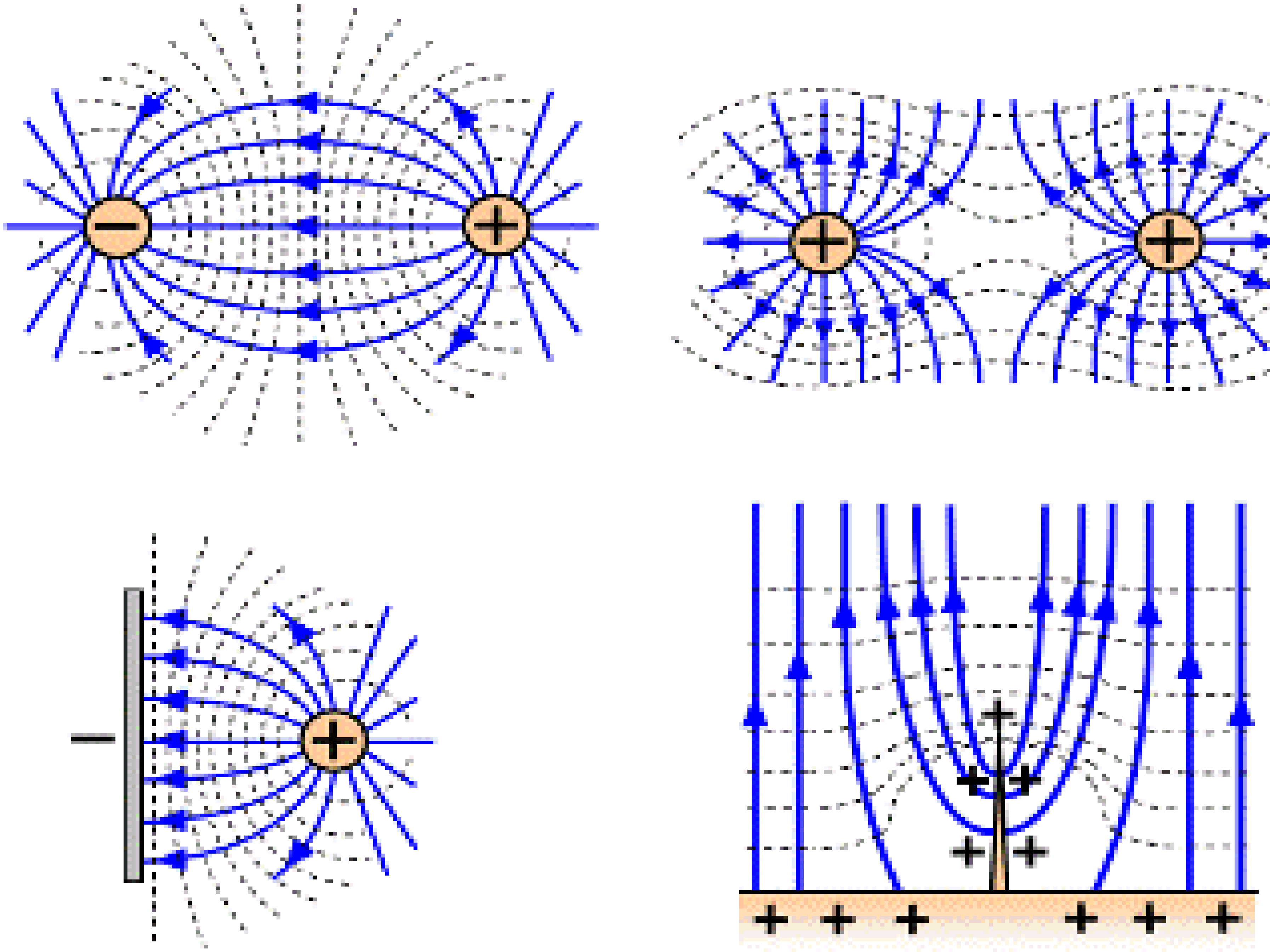


Cylinder over plane

Near plane, equipotentials are equally spaced.

Near cylinder, equipotentials look more like Coax.



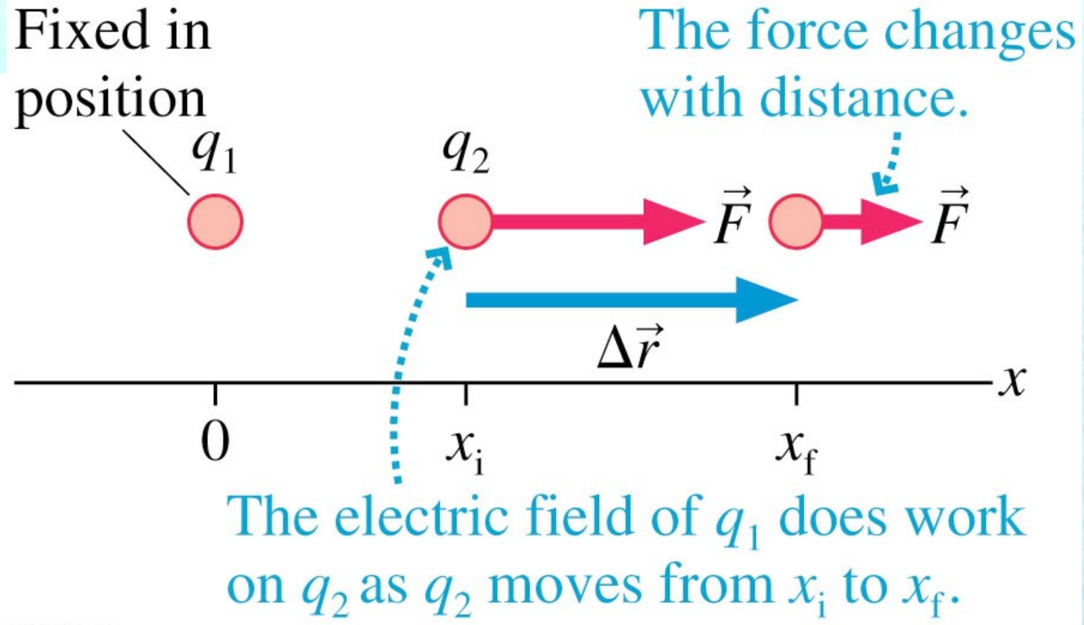


(resourcefulphysics.org)

Field Lines	Equipotentials
Originate at positive charges. Terminate at negative.	The voltage is the same everywhere on an equipotential
Closer spacing means larger fields.	Closer spacing means larger fields.
Never cross.	Never cross.
Are at right angles to conductors.	Are at right angles to field lines. (Are parallel to conductors)

The Potential Energy of Two Point Charges

- Consider two like charges q_1 and q_2.
- The electric field of q_1 *pushes* q_2 as it moves from x_i to x_f.
- The work done is:



$$W_{\text{elec}} = \int_{x_i}^{x_f} F_{1\text{ on }2}\,dx = \int_{x_i}^{x_f} \frac{Kq_1q_2}{x^2}\,dx = Kq_1q_2 \frac{-1}{x}\Bigg|_{x_i}^{x_f} = -\frac{Kq_1q_2}{x_f} + \frac{Kq_1q_2}{x_i}$$

- The change in electric potential energy of the system is $\Delta U_{\text{elec}} = -W_{\text{elec}}$ if:

$$U_{\text{elec}} = \frac{Kq_1q_2}{x}$$

The Potential Energy of Two Point Charges

Consider two point charges, q_1 and q_2, separated by a distance r. The electric potential energy is

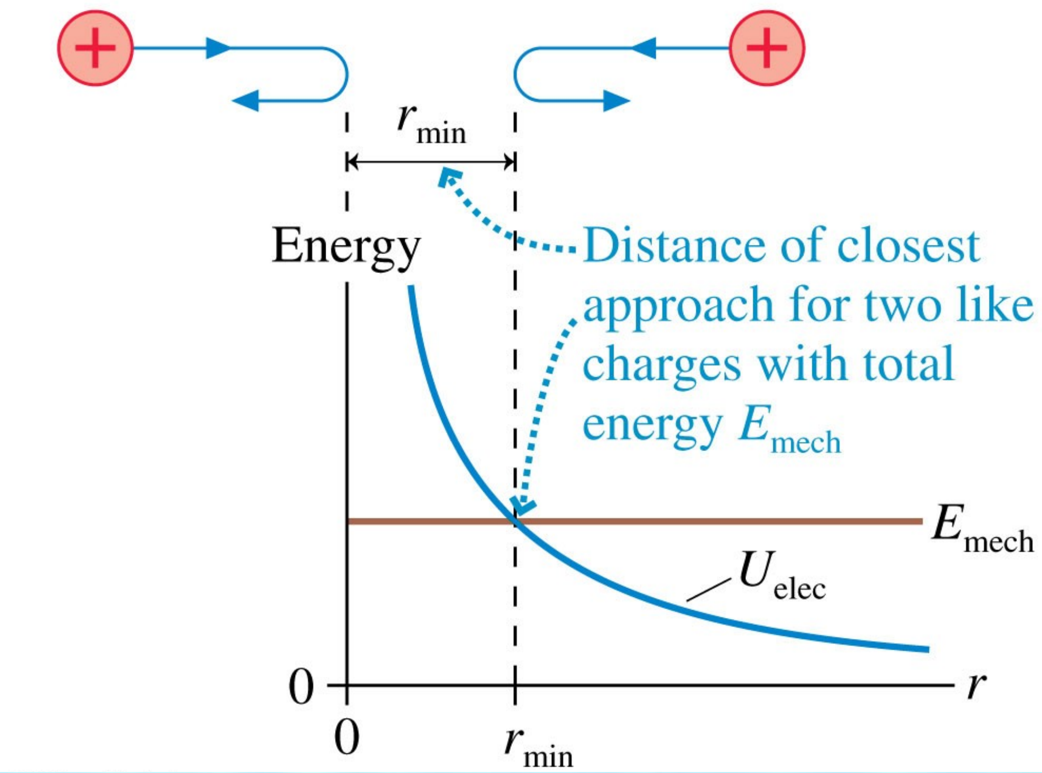
$$U_{elec} = \frac{K q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} \qquad \text{(two point charges)}$$

This is explicitly the energy of *the system*, not the energy of just q_1 or q_2.

Note that the potential energy of two charged particles approaches zero as $r \to \infty$.

The Potential Energy of Two Point Charges

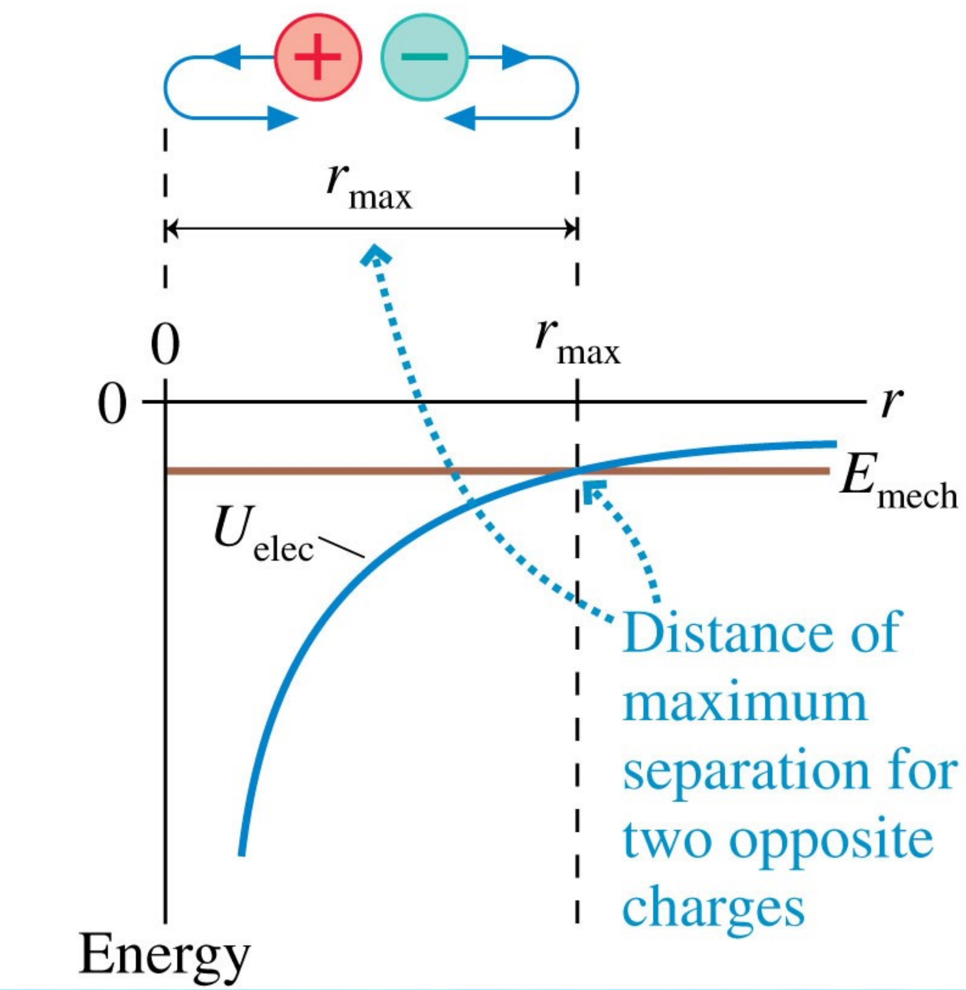
- Two like charges approach each other.
- Their total energy is $E_{mech} > 0$.
- They gradually slow down until the distance separating them is r_{min}.
- This is the *distance of closest approach.*



$$U_{elec} = \frac{Kq_1q_2}{r}$$

The Potential Energy of Two Point Charges

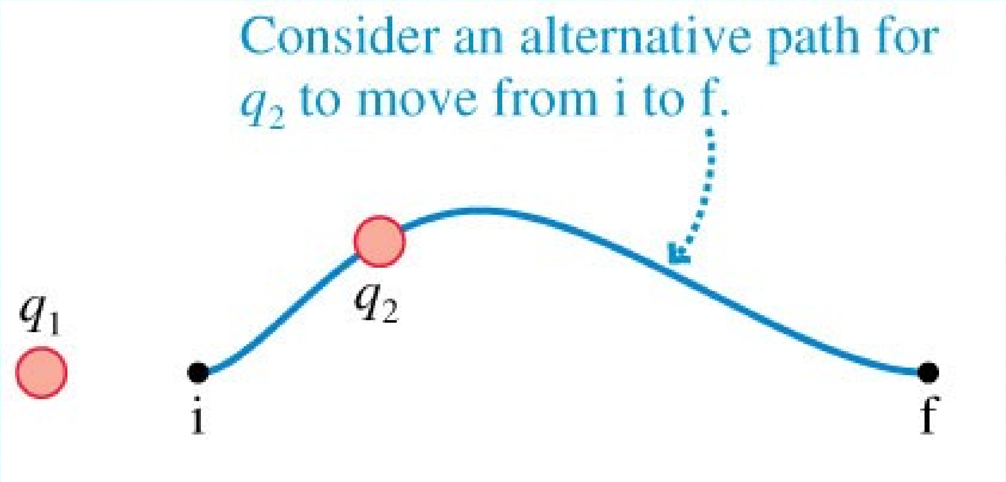
- Two opposite charges are shot apart from one another with equal and opposite momenta.
- Their total energy is $E_{mech} < 0$.
- They gradually slow down until the distance separating them is r_{max}.
- This is their *maximum separation.*

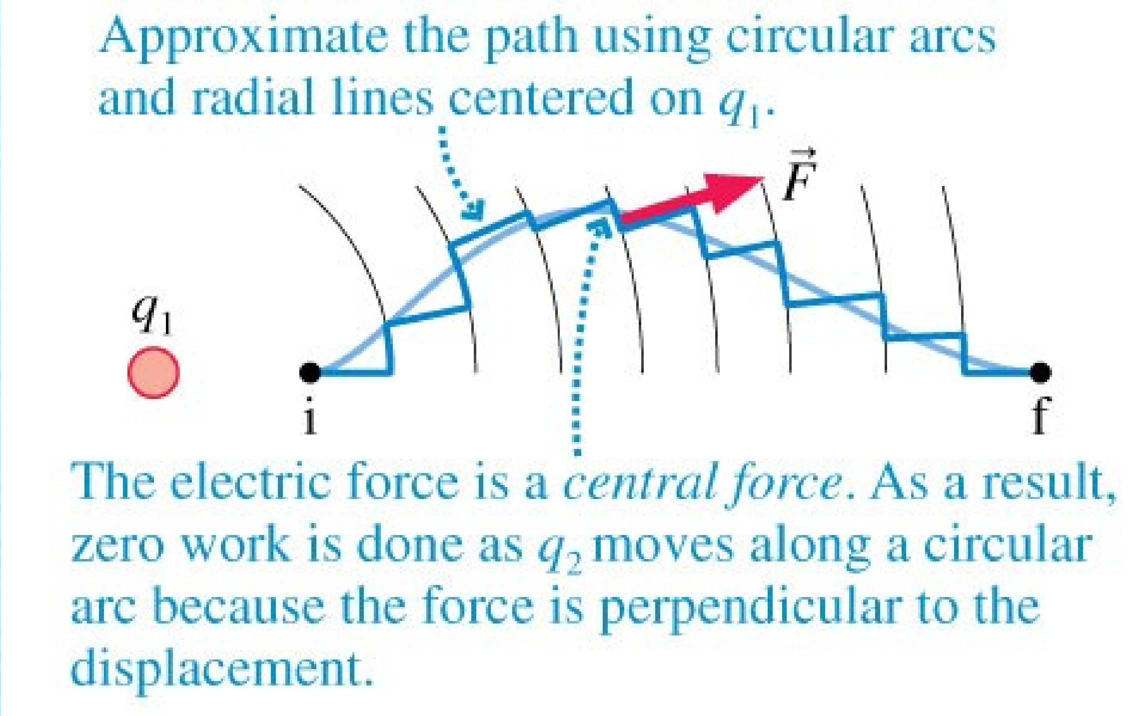


$$U_{elec} = \frac{Kq_1q_2}{r}$$

The Electric Force Is a Conservative Force

- Any path away from q_1 can be approximated using circular arcs and radial lines.
- All the work is done along the radial line segments, which is equivalent to a straight line from i to f.
- Therefore the work done by the electric force depends only on initial and final position, not the path followed.





Example 28.2 Approaching a Charged Sphere

EXAMPLE 28.2 Approaching a charged sphere

A proton is fired from far away at a 1.0-mm-diameter glass sphere that has been charged to +100 nC. What initial speed must the proton have to just reach the surface of the glass?

MODEL Energy is conserved. The glass sphere can be treated as a charged particle, so the potential energy is that of two point charges. The proton starts "far away," which we interpret as sufficiently far to make $U_i \approx 0$.

Example 28.3 Escape Velocity

EXAMPLE 28.3 **Escape velocity**

An interaction between two elementary particles causes an electron and a positron (a positive electron) to be shot out back to back with equal speeds. What minimum speed must each have when they are 100 fm apart in order to escape each other?

MODEL Energy is conserved. The particles end “far apart,” which we interpret as sufficiently far to make $U_f \approx 0$.

Problem 28.39

Problem 28.26