Go language convective ensemble model

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1 Introduction

This is a continuation of our toy convective ensemble modeling effort with a rewrite of the model in the Go programming language. Go can be made as fast as C and it is a safer, more expressive language. This rewrite will allow the application of accumulated knowledge to make the code faster and more readable.

2 Model details

This represents version 077 of the model.

2.1 Dynamical core

We write the dependent variables in a manner friendly to the flux form of the equations of motion, i.e., multiplied by the density of air ρ. The mass continuity equation for the dry air component is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{u} = 0 \tag{1}$$

and the momentum equation is

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}/\rho + p\mathbf{I}) + g(\rho + \rho_t + \rho_p)\mathbf{k} + f\mathbf{k} \times \mathbf{u} - \mu(z)[\rho \mathbf{v}_R(z) - \mathbf{u}_h] = \mathbf{S}_u, \tag{2}$$

where ρ_t and ρ_p are the densities of total advected water substance and precipitation. $\mathbf{v}$ is the air velocity, $\mathbf{u} = \rho\mathbf{v}$, and $p\mathbf{I}$ is the stress due to pressure p. $\mathbf{v}_R$ is a reference horizontal velocity profile, $\mathbf{u}_h$ is the horizontal component of $\mathbf{u}$, and

$$\mu(z) = \mu_{max}(z - z_{tp})/(z_{top} - z_{tp}) \tag{3}$$

where $\mu(z) = 0$ below the tropopause z_{tp}. The $\mu(z)$ term represents a stratospheric sponge layer. The quantity g is the acceleration of gravity, f is the Coriolis parameter, $\mathbf{S}_u$ is a source term to be discussed later, and $\mathbf{k}$ is an upward unit vector. Note that we have approximated the total air density by that for dry air in the definition of $\mathbf{u}$.

Constant	Value	Meaning
R_d	287 J K^{-1} kg^{-1}	Gas constant for air
R_v	461 J K^{-1} kg^{-1}	Gas constant for water vapor
ϵ	0.623	R_d/R_v
C_{pd}	1005 J K^{-1} kg^{-1}	Specific heat of air at const pres
C_{vd}	718 J K^{-1} kg^{-1}	Specific heat of air at const vol
C_{pv}	1850 J K^{-1} kg^{-1}	Specific heat of water vapor at const pres
C_{vv}	1390 J K^{-1} kg^{-1}	Specific heat of water vapor at const vol
C_l	4218 J K^{-1} kg^{-1}	Specific heat of liquid water
C_i	1959 J K^{-1} kg^{-1}	Specific heat of ice (-20°C)
μ_{BL}	3.15×10^6 J kg^{-1}	Binding energy for liquid water
μ_{BI}	2.86×10^6 J kg^{-1}	Binding energy for ice
L_{LF}	2.5008×10^6 J kg^{-1}	Latent heat of condensation at freezing
L_F	3.34×10^5 J kg^{-1}	Latent heat of freezing
e_{SF}	611 Pa	Saturation vapor pressure at freezing
T_F	273.15 K	Freezing point
p_0	10^5 Pa	Reference pressure
ρ_0	1.28 kg m^{-3}	Reference density $p_0/(R_d T_F)$
r_0	0.02 kg kg^{-1}	Reference vapor mixing ratio
κ_c	0.01 kg m^{-2}	Cloud water absorptivity

Table 1: Thermodynamic constants.

2.2 Thermodynamics

The thermodynamics of the model are defined by Raymond (2013). Table 1 defines constants.

The three primary thermodynamic variables are the total specific moist entropy $s = s_a + s_p$, where s_a the entropy of moist air plus advected condensate and s_p is the specific entropy of precipitation, the cloud water mixing ratio $r_t = r_v + r_c$, where r_v and r_c are the mixing ratios of water vapor and advected condensate, and the mixing ratio of precipitation r_p. Both the advected condensate and the precipitation are assumed to be liquid above freezing and ice below freezing. Precipitation falls relative to the air with a terminal fall speed of w_t. This fall speed is set equal to constant values of w_l above freezing and w_i below freezing. Densities rather than mixing ratios, e.g., $\rho_s = \rho s$, $\rho_t = \rho r_t$, etc., are used in the flux form of the governing thermodynamic equations, as with the dynamic equations.

The total moist entropy governing equation in flux form is

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\mathbf{u} s) = S_s \tag{4}$$

where S_s is the source term for moist entropy. The cloud water governing equation is

$$\frac{\partial \rho_t}{\partial t} + \nabla \cdot (\mathbf{u} r_t) = S_t \tag{5}$$

where S_t is the source term for advected condensate. The precipitation governing equation is

$$\frac{\partial \rho_p}{\partial t} + \nabla \cdot (\mathbf{u} r_p) = S_p \tag{6}$$

where ρ_p is the density of precipitation and S_p is the corresponding source term.

The specific entropy of precipitation is

$$s_p = r_p \left(C_x \ln \frac{T}{T_F} - \frac{L_X}{T_F} \right) \tag{7}$$

and the advected moist entropy written in terms of temperature and density is

$$s_a = (C_v + r_v C_{vv} + r_c C_x) \ln \frac{T}{T_F} - (R_d + r_v R_v) \ln \frac{\rho}{\rho_0} - r_v R_v \ln \frac{p_0 r_v}{\epsilon e_{SF}} + \frac{L_{LF} r_v - L_X r_c}{T_F} \tag{8}$$

where p_d is the partial pressure of dry air, C_v and C_{vv} are respectively the specific heats at constant volume for air and water vapor. C_x equals the specific heat of liquid water C_l above freezing and ice C_i below freezing. The quantity L_X equals zero above freezing and the latent heat of freezing L_F below freezing, while L_{LF} is the latent heat of condensation at freezing. The quantities p_0 and ρ_0 are respectively the reference pressure and density while e_{SF} is the saturation vapor pressure at freezing.

The pressure of air is diagnosed from the temperature and density of dry air and water vapor using the ideal gas law

$$p = p_d + p_v = R_d(1 + r_v/\epsilon)T\rho \tag{9}$$

where the partial pressure of water vapor $p_v = p_d r_v/\epsilon$ with $\epsilon = R_d/R_v$, the ratio of gas constants for dry air and water vapor. The temperature in turn is diagnosed from the density, the advected moist entropy, and the cloud water mixing ratio:

$$T = T(\rho, s_a, r_t). \tag{10}$$

There are two cases in this diagnosis, the unsaturated case in which $r_t = r_v$ and the saturated case in which $r_t = r_v + r_c = r_s(\rho, T) + r_c$, where r_s is the saturation mixing ratio and r_c is the mixing ratio of the advected condensate.

The theoretical equation for saturation vapor pressure over liquid water is used,

$$e_s(T) = e_{SF} \left(\frac{T_F}{T} \right)^{(C_l - C_{pv})/R_v} \exp\left(\frac{\mu_{BL}}{T_F} - \frac{\mu_{BL}}{T} \right) \tag{11}$$

where C_{pv} is the specific heat of air at constant pressure, C_l is the specific heat of liquid water, and μ_{BL} is a constant related to the latent heat of condensation. Over ice the same equation applies except that C_l is replaced by C_i and μ_{BL} is replaced by μ_{BI}.

2.3 Source terms

Equations (1) - (6) describe the fast dynamical core of the model. The source terms on the right sides, $\mathbf{S}_u$, S_s, S_t, and S_p represent quantities such as turbulent frictional terms, heat, moisture, and momentum fluxes, precipitation formation, fall, and evaporation, and radiative heating/cooling, that change less rapidly with time.

First, the eddy mixing coefficient:

$$K = C\rho E \Delta z^2 \tag{12}$$

where C is a dimensionless scale factor, Δz is the vertical grid size, and E is related to the strain rate and the Brunt frequency

$$E = (D^2 - 2N^2)^{1/2} \tag{13}$$

where we set $E = 0$ if $D^2 - 2N^2 < 0$.

The strain rate is

$$\mathbf{D} = D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{14}$$

and

$$D^2 = D_{ij}D_{ij} \tag{15}$$

with implicit summation on repeated indices assumed.

The Brunt frequency is a blend of dry and moist values depending on the relative humidity:

$$N^2 = \frac{g}{C_p}\left(\nu\frac{\partial s_a}{\partial z} + (1-\nu)\frac{\partial s_d}{\partial z}\right) \tag{16}$$

where s_a is the specific advected moist entropy (8),

$$s_d = (C_v + r_v C_{vv})\ln\frac{T}{T_F} - (R_d + r_v R_v)\ln\frac{\rho}{\rho_0} - r_v R_v \ln\frac{p_0 r_v}{\epsilon e_{SF}} \tag{17}$$

is the specific dry entropy, and

$$\nu = 0.5[(Z-1)/\Delta Z + 1] \tag{18}$$

where $Z = r_t/r_s$ is the relative humidity and $\Delta Z = 0.02$ produces a smooth switch between a saturated and unsaturated environment. ν is limited to the range $[0, 1]$.

The source terms are functions of the eddy fluxes $\mathbf{F}_{...}$ (defined below) and various other quantities:

$$\mathbf{S}_u = -\nabla\cdot\mathbf{F}_u + \mathbf{S}_{uwtg} \tag{19}$$

for velocity,

$$S_s = -\nabla\cdot(\mathbf{F}_s - \rho s_p w_t \mathbf{k}) + \rho R/T + S_{swtg} \tag{20}$$

for entropy, where R is the radiative heating per unit mass with C_{pd} being the specific heat of dry air at constant pressure. The source term for cloud water mixing ratio is

$$S_t = -\nabla\cdot\mathbf{F}_t - \rho P + S_{twtg} \tag{21}$$

where P is the precipitation formation rate per unit mass, and the precipitation source term is

$$S_p = -\nabla\cdot(\mathbf{F}_p - \rho r_p w_t \mathbf{k}) + \rho P \tag{22}$$

where w_t is the hydrometeor terminal velocity. Different values of w_t can be assigned above and below the freezing level. The terms $\mathbf{S}_{uwtg}$, S_{swtg}, and S_{twtg} are weak temperature gradient approximation source terms, as described below. The first term for all variables is minus the divergence of the eddy flux of the quantity in question.

The precipitation formation rate is given by

$$P = C_{precip} r_c - C_{evap}(r_s - r) r_p \tag{23}$$

where r_c is the mixing ratio of advected condensate and $r = r_t - r_c$ is the water vapor mixing ratio. The constant C_{precip} equals C_{rain} or C_{snow} depending on whether the temperature is above or below freezing.

The eddy fluxes for each variable are defined

$$\mathbf{F}_u = -K\mathbf{D} - \rho(K_h \mathbf{D}_h + K_z \mathbf{D}_z) \tag{24}$$

where $\mathbf{D}_h$ is $\mathbf{D}$ with vertical velocities and derivatives omitted and $\mathbf{D}_z$ has horizontal velocities and derivatives omitted,

$$\mathbf{F}_s = -\left(K\nabla + \rho K_h \nabla_h + \rho K_z \frac{\partial}{\partial z}\right) s \tag{25}$$

$$\mathbf{F}_t = \left(K\nabla + \rho K_h \nabla_h + \rho K_z \frac{\partial}{\partial z}\right) r_t \tag{26}$$

$$\mathbf{F}_p = -\left(K\nabla + \rho K_h \nabla_h + \rho K_z \frac{\partial}{\partial z}\right) r_p \tag{27}$$

Lower boundary conditions on the fluxes are defined by surface bulk fluxes except for precipitation. For this variable the lower boundary condition is just the surface precipitation rate $w_t \rho_p$.

Extra smoothing is needed in the horizontal for numerical reasons. This is provided by the horizontal mixing coefficient

$$K_h = \lambda_h(\Delta x^2 + \Delta y^2)/\Delta t \tag{28}$$

where $(\Delta x, \Delta y, \Delta z)$ and Δt are respectively the grid box dimensions and fine scale time step. Typically, $\lambda_h = 0.01$. The precipitation needs additional vertical smoothing for numerical reasons as well, with

$$K_z = \lambda_z \Delta z^2/\Delta t, \tag{29}$$

where $\lambda_z \approx 0.004$. All other variables have vertical smoothing with $\lambda_z = \lambda_w \approx 3 \times 10^{-5}$ at the surface, tapering to zero at the tropopause.

The surface fluxes for each variable are given by bulk flux formulas

$$\mathbf{F}_{su} = -\rho_{bl} C_D U_e \mathbf{u}_{bl} \tag{30}$$

$$F_{ss} = \rho_{bl} C_D U_e (s_{ss} - s_{bl}) - \rho_{bl} r_p s_h w_t \tag{31}$$

$$F_{st} = B \rho_{bl} C_D U_e (r_{ss} - r_{tbl}) \tag{32}$$

$$F_{sp} = -\rho_{bl} r_p w_t \tag{33}$$

where a subscripted bl indicates a boundary layer value of ρ, a subscripted ss indicates a saturated sea surface value, C_D is the surface drag coefficient and thermodynamics transfer coefficient, and $U_e = (|\mathbf{u}_{bl}|^2 + W^2)^{1/2}$ is the effective surface wind, where W is the gustiness correction. The Bowen parameter B in the mixing ratio equation equals 1 over the ocean and is $0 \leq B \leq 1$ over the land, depending on how moist the surface is. In addition to the above bulk fluxes, the entropy and precipitation equations have terms related to the flow of rain into the surface.

2.4 Radiation

Radiative heating is provided either by a fixed radiative heating profile or an extension to the toy radiative heating model of Raymond and Torres (1998) and Raymond (2000b).

2.4.1 Fixed radiation

The fixed radiation representation is very simple, with a constant radiative temperature tendency (generally negative) up to a fixed altitude, tapering linearly to zero at the tropopause.

2.4.2 Interactive radiation

Instead of working with sums of integrals over frequency intervals as is convectionally done in radiative transfer calculations, we sort the radiative contributions of water vapor in particular into bins representing different ranges of absorption coefficient. These ranges are small enough to assume that the Beer-Lambert law can applied separately for each bin using an average absorption coefficient for each bin. The vertical radiative flux is assumed to equal the difference between upward and downward streams:

$$F_r(z) = \sum_i (I^{i+} - I^{i-}) + S^{i+} - S^{i-} \tag{34}$$

where the sum is over all thermal infrared bins I^* plus separate terms S^* for solar radiation.

The radiative heating per unit mass R (see (20)) is given by

$$R = -\frac{1}{\rho}\frac{\partial F_r}{\partial z}. \tag{35}$$

The upward and downward fluxes for each thermal infrared bin satisfy the equations

$$\frac{dI^{i+}}{d\tau_i} = f_i \sigma_{SB} T^4 - I^{i+} \quad \frac{dI^{i-}}{d\tau_i} = f_i \sigma_{SB} T^4 + I^{i-} \tag{36}$$

where f_i is the fraction of the spectrum associated with each bin, σ_{SB} is the Stefan-Boltzmann constant, T is the absolute temperature, and τ_i is the optical depth for each bin, starting upward from the surface. The differential $d\tau_i$ is given by

$$d\tau_i = \rho(C_i \kappa_i + r_l \kappa_c) dz \tag{37}$$

where C_i is a constant depending on the bin, κ_i is the absorptivity of the gas associated with each bin, r_l is the mixing ratio of condensed cloud water, and κ_c is the cloud water absorptivity. (Precipitation particles are assumed to be large enough to have small radiative effect.)

We estimate $\kappa_c = 1/(\rho_c L) \approx 10$ m^2/kg, where we assume that the cloud water condensate density $\rho_c \approx 10^{-4}$ kg/m^3 results in an optical depth of unity over a geometrical distance of $L \approx 100$ m. This is roughly consistent with spherical cloud droplets of radius 10^{-5} m.

The equations in (36) are integrated from the bottom up for I^{i+} and from the top down for I^{i-}. The lower boundary condition on the former is $I^{i+surf} = f_i \sigma_{SB} T_s^4$ where T_s is

Bin	f_i	κ_i (m^2/kg)	Purpose
1	0.11	0.002	carbon dioxide
2	0.20	0.0005	windows and continuum
3	0.13	0.001	water vapor bands
4	0.11	0.00316	water vapor bands
5	0.09	0.01	water vapor bands
6	0.08	0.0316	water vapor bands
7	0.07	0.1	water vapor bands
8	0.06	0.316	water vapor bands
9	0.05	1.0	water vapor bands
10	0.04	3.16	water vapor bands
11	0.03	10.0	water vapor bands
12	0.02	31.6	water vapor bands
13	0.01	100.0	water vapor bands

Table 2: Values of constants for each thermal infrared bin.

the surface temperature and the surface is assumed to radiate as a black body. The upper boundary condition is $I^{i-top} = 0$.

The C_i have different meanings for different bins. For carbon dioxide, C_i represents the effect of pressure broadening and takes the form

$$C_i = C_{co2} = (\rho/\rho_0)(T/T_F)^{1/2}, \tag{38}$$

where ρ_0 and T_F are respectively density and temperature reference values. (See table 1.) For water vapor bands,

$$C_i = C_{h2o} = (r_v/r_0)(\rho/\rho_0)(T/T_F)^{1/2}, \tag{39}$$

where r_0 is a reference value of the water vapor mixing ratio r_v. For the water vapor continuum,

$$C_i = C_{cont} = (r_v/r_0)^2. \tag{40}$$

The current implementation of the toy radiative model incorporates 13 thermal infrared bins, a single bin each for carbon dioxide and for radiative windows and continuum, and 11 bins for water vapor bands. Constants used for each bin are listed in table 2.

Equations (36) are solved numerically on cell-edge levels

$$I^{i+}_{j+1} = I^{i+}_j \exp(-\delta_j) + [1 - \exp(-\delta_j)] f_i \sigma_{SB} T^4_{j+1/2} \tag{41}$$

and

$$I^{i-}_j = I^{i-}_{j+1} \exp(-\delta_j) + [1 - \exp(-\delta_j)] f_i \sigma_{SB} T^4_{j+1/2} \tag{42}$$

where $\delta_j = \tau_{j+1} - \tau_j$ and $T_{j+1/2}$ is the cell-centered temperature.

Solar radiation is confined to a single separate bin. The equations for the upward and downward fluxes of solar radiation are slightly different than for the thermal infrared because scattering by cloud particles between the upward and downward beams is important.

Cloud particles can also absorb a certain fraction of the solar radiation as can water vapor. Interaction of solar radiation with carbon dioxide is currently ignored. The upward (S^+) and downward (S^-) beams of solar radiation obey

$$\frac{dS^+}{dz} = \rho\kappa_c r_l (S^- - \epsilon S^+) - \rho\kappa_{wv} r_v S^+ \tag{43}$$

and

$$\frac{dS^-}{dz} = \rho\kappa_c r_l (S^+ + \epsilon S^-) + \rho\kappa_{wv} r_v S^- \tag{44}$$

where ϵ is the fraction of scattered solar radiation that is absorbed by cloud condensate. The last term in (43) and (44) represents absorption by the water vapor continuum. A reasonable value of $\kappa_{wv} = 0.0005$ m^2/kg. This gives about 1 K/day solar heating rate near the surface in tropical conditions.

Note that κ_c is the same parameter that occurs in (37). Note also that z rather than optical depth τ is used as the height variable. The upper boundary condition on (44) is that $S^{-top} = S_{const} \cos(\phi_z)$ is the downward component of the solar flux, equal to the product of the solar constant S_{const} and the cosine of the solar zenith angle ϕ_z. The surface is assumed to have zero albedo so that $S^{+surface} = 0$. Both equations are integrated from the surface up with $S^{-surface} = 1$. The solutions are then normalized so that S^{-top} takes on the assumed upper boundary condition value.

There are three options for the cosine of the solar zenith angle ϕ_z. Option 0 sets this to zero, which means that solar radiation is ignored. Option 1 sets it to $1/\pi$, which corresponds to a diurnal average value. Option 2 computes the actual value as a function of time, assuming that the sun is directly overhead at noon.

2.5 Weak temperature gradient approximation

The weak temperature gradient (WTG) approximation here implements a modified version of spectral WTG as documented by Herman and Raymond (2014). In this version the potential temperature is replaced by the specific dry entropy. The WTG vertical velocity is computed as a Fourier reconstruction over the model levels

$$w_{wtg}(z_k) = \sum_{j=1}^{n} C_j \sin(m_j z_k) \tag{45}$$

where the z_k are the grid cell levels. We assume that $w_{wtg} = 0$ at and above the tropopause. Note that n is defined so as to retain all vertical modes in the troposphere.

Since the application of WTG in the boundary layer is questionable, the value of w_{wtg} below the boundary layer top b is linearly interpolated to zero at the surface from the boundary layer top value:

$$w_{wtg}(z) = w_{wtg}(b)(z/b) \quad z < b. \tag{46}$$

If we set $b = 0$, this interpolation is eliminated.

The Fourier coefficients C_j are given by

$$C_j = \frac{2}{n} \sum_{k=1}^{n} \omega_j \delta(z_k) \sin(m_j z_k) \tag{47}$$

where n is the number of levels in the troposphere and the spectral relaxation rate is

$$\omega_j = \frac{\tau_j^2}{(f^2 + \tau_j^2)^{1/2}} \tag{48}$$

where f is the Coriolis parameter, j is the vertical mode number, and

$$\tau_j = \tau_t / j. \tag{49}$$

The quantity τ_t is taken to be the inverse of the time required for a fundamental mode inertial-gravity wave to cross the model domain

$$\tau_t = c/D \tag{50}$$

where c is the phase speed of the fundamental, non-rotating gravity wave mode and D is the domain size. The term on the right side of (48) divided by τ_t is the factor by which rotation reduces the group velocity of hydrostatic gravity waves from the non-rotating value. The quantity δ is defined

$$\delta(z) = \frac{\overline{s_d}(z) - s_{dR}(z)}{ds_{dR}/dz}, \tag{51}$$

where $\overline{s_d}(z)$ is the domain-mean dry entropy profile and s_{dR} is the reference profile of dry entropy.

The WTG vertical velocity also implies lateral inflow and outflow, together constituting a "virtual" large-scale flow that relaxes the dry entropy profile toward an assumed reference profile. This virtual flow produces real tendencies in the moist entropy and total cloud water which counter the tendencies produced by latent heat release, surface fluxes, and radiation, as represented in by S_{swtg} and S_{twtg} in (20) and (21).

The WTG entropy source is given by

$$S_{swtg} = -\rho_R w_{wtg} \frac{\partial \overline{s}}{\partial z} - (\overline{s} - s_R)\, \Theta \left(\frac{dM_{wtg}}{dz} + \kappa |\mathbf{u}_R| \right) \tag{52}$$

where $\overline{s}$ is the model domain mean moist entropy, ρ_R and s_R are the reference profiles for density and entropy, $M_{wtg} = \rho_R w_{wtg}$ is the vertical mass flux, $\mathbf{u}_R$ is the reference profile for horizontal momentum density, and $\Theta(x) = x$ for $x > 0$ and $\Theta(x) = 0$ for $x < 0$. A similar equation exists for total cloud water mixing ratio r_t. The first term in (52) represents the vertical advection of domain-mean entropy by the WTG vertical velocity, while the second represents the entrainment of reference profile entropy by the horizontal convergence associated with increasing M_{wtg} with height. The κ term adds the effect of ventilation by the ambient wind and is taken to be the inverse of the domain size D,

$$\kappa = 1/D, \tag{53}$$

implying that the time scale for relaxation of the domain-mean entropy to the reference profile value resulting from ventilation is the time for a parcel to cross the computational domain.

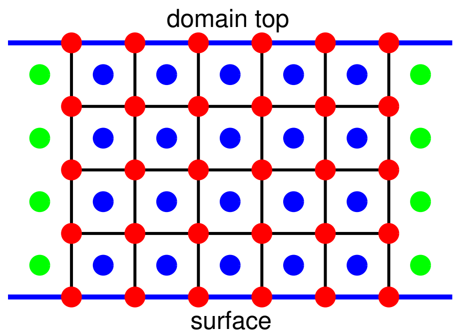

Figure 1: The computational domain for a single processor is illustrated. See the text for further explanation.

If the computational domain is assumed to be moving with some velocity $\mathbf{v}_{sys}$, then $\mathbf{u}_R$ is replaced by $\mathbf{u}_R - \rho_R \mathbf{v}_{sys}$ in (52). In this case corresponding compensations must be made in the bulk surface flux equations so that fluxes are always calculated using earth-relative winds.

The addition of WTG domain ventilation is new to this version of the model and does not appear in Herman and Raymond (2014).

A nudging term that relaxes the horizontal mean of the horizontal velocity toward a reference profile, as represented in (19), is included as a part of the WTG package:

$$\mathbf{S}_{uwtg} = \tau_d [\mathbf{u}_R - \overline{\mathbf{u}_h}(\rho_R/\overline{\rho})] \tag{54}$$

where τ_d is the dynamical relaxation rate, $\overline{\mathbf{u}_h}$ is the horizontal model domain-mean wind velocity times density, and $\mathbf{u}_R$ is the reference profile wind times density. The ratio of reference profile to mean densities in the last term insures that velocities rather than momentum densities are nudged.

2.6 Numerical issues

The code for this model is written in the Go language and is set up for parallel processing using the message passing interface (MPI) protocol. Go is a modern language which emphasizes clarity and safety of code as well as execution speed. Table 3 lists the output variables of the model along with their meaning and units.

Lax-Wendroff differencing is used to advance the computation in time. Executing one time step is a two-part process. First, fluxes of the prognostic variables are computed on the unstaggered grid. Flux divergences are then computed on the staggered grid using these data, advancing the calculation by half a time step. Fluxes are then computed on the staggered grid and the divergences of these fluxes are used to advance a full time step from the original unstaggered grid data. Lax-Wendroff is second-order in both space and time and is inherently flux-conservative, which means that thermodynamic variables are accurately conserved over long time periods.

Since the governing equations are fully compressible, sound waves are computed explicitly, and the time step is necessarily small using an explicit scheme such as Lax-Wendroff. For

Variable	Description	Units
v(x,y)sys	system horizontal velocity	m/s
wtgu(x,y,z)	WTG reference air momentum density	$kg/m^2/s$
wtgpres	WTG reference pressure	Pa
wtgrho	WTG reference density	kg/m^3
wtgdrt	WTG reference total cloud water density	kg/m^3
wtgddryent	WTG reference dry entropy density	$J/K/m^3$
wtgdent	WTG reference moist entropy density	$J/K/m^3$
wtgdsatent	WTG reference saturated entropy density	$J/K/m^3$
wtgrt	WTG reference mixing ratio	g/g
wtgdryent	WTG reference dry entropy	J/K/kg
wtgent	WTG reference moist entropy	J/K/kg
wtgsatent	WTG reference saturated entropy	J/K/kg
wtgv(x,y)	WTG reference horizontal air velocity	m/s
wtgsrcv(x,y)	WTG source of horizontal momentum	$kg/m^2/s^2$
wtgsrcrt	WTG source of total cloud water	$kg/m^3/s$
wtgsrcent	WTG source of moist entropy	$W/K/m^3$
wtgsrcrtcum	Cumulative source of wtgsrcrt	kg/m^3
wtgsrcentcum	Cumulative source of wtgsrcent	$J/K/m^3$
rho	air density	kg/m^3
pres	air pressure	Pa
u(x,y,z)	air momentum density	$kg/m^2/s$
ddryent	dry entropy density	$J/K/m^3$
dent	moist entropy density	$J/K/m^3$
dsatent	saturated entropy density	$J/K/m^3$
drt	total cloud water density	kg/m^3
drr	precipitation density	kg/m^3
drsat	saturated water vapor density	kg/m^3

Table 3: Model variables. “Density” indicates the mixing ratio of a quantity times the air density. Parentheses in the first column indicate the vector components of a quantity.

greater computational efficiency, slowly changing source terms are calculated every nth time step where typically $n = 10$. Source terms are applied only to the full-step calculation.

The model domain is broken up into a horizontal rectangular grid of processor domains. The grid in a processor domain is illustrated in figure 1. Domains are three-dimensional with the same structure in the dimension normal to the page. Each processor domain is split into cells as shown. The data grid used to produce the output is cell-centered, as illustrated by the blue dots. The red dots represent a staggered grid that is used in intermediate calculations at each time step associated with the Lax-Wendroff scheme. No stretched grids are used; all grid cells are the same size.

Communication is needed with adjacent processor domains. The green dots represent data in the boundary cells of the adjacent domains. Boundary information is transferred between processor domains using MPI. These transfers wrap around the model domain for processor domains on the edge of the model domain. Thus, the full model domain is subject to periodic boundary conditions.

3 References

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