The moist static energy is widely used as a thermodynamic variable in diagnostic studies. Tellingly, it is rarely if ever used in dynamic models. There is a good reason for this, which we now discuss.

The moist static energy is defined

$$\mu = C_p T + L r_v + g z \tag{1}$$

where C_p is the specific heat of air at constant pressure, T is the absolute temperature, L is the (assumed constant) latent heat of vaporization, g is the acceleration of gravity, and z is the geopotential height. The supposed conservation property of moist static energy is derived as follows. Ignoring condensed water substance and the modifications of water substance to the specific heat, etc., the approximate differential of specific enthalpy is

$$dh = C_p dT + L dr_v = T ds + dp/\rho, \tag{2}$$

where s is the moist entropy and ρ is the density of air. We assume hydrostatic balance, $dp/\rho = -g dz$, from which it is easy to show that

$$d\mu = T ds. \tag{3}$$

In processes where the moist entropy is conserved, then the moist static energy is also conserved by this derivation. The assumption of hydrostatic balance is justified by the assertion that the atmosphere is never very far from such balance.

A deeper view of this assumption is revealed by examining its validity in steady, adiabatic, laminar flows in which Bernoulli's equation is valid. To the degree of approximation for moist processes used here, Bernoulli's equation can be written in differential form

$$d(v^2/2 + h + gz) = d(v^2/2 + \mu) = 0, \tag{4}$$

where v is the flow speed. The derivation of this equation assumes that the entropy is conserved. Hence, we have $d\mu = -d(v^2/2) \neq 0$ when $ds = 0$, which contradicts equation (3).

In the more general case of steady laminar flow in which heat transfer is allowed. e. g., by radiation, one can derive

$$d(\mu + v^2/2) = T ds, \tag{5}$$

which can be considered to be an extension of equation (3) to this special case.

Atmospheric flow is rarely steady and laminar. However, equation (5) is proof by counter-example that the hydrostatic approximation is not a valid assumption in the derivation of the conservation properties of moist static energy. Often the contribution of specific kinetic energy ($v^2/2$) to equation (5) is small, but there are extreme situations such as hurricanes and severe convective storms where its contribution is by no means negligible. The broad lesson here is that *moist static energy has no rigorously derivable conservation properties.*