Chapter 10

Baroclinic Instability

Baroclinic instability is the most important mechanism for generating weather in the middle latitudes. It also plays a crucial role in the meridional transport of properties in the atmosphere. In this chapter we first discuss an example of baroclinic instability, that developed by Eady (1949). This instability mode arises from the interaction of upper and lower surface Rossby modes. Following our discussion of the Eady model, we develop an interpretation of surface potential temperature gradients as gradients in a thin sheet of potential vorticity at a fixed potential temperature level near the surface. This interpretation shows that surface Rossby waves are waves on a potential temperature gradient, just as internal Rossby waves. We then introduce two general theorems pertinent to small amplitude disturbances on a zonally symmetric flow in the atmosphere, the Charney-Stern necessary condition for baroclinic instability and the non-interaction theorem for low Rossby number disturbances.

10.1 Eady problem

We now discuss the classic example of baroclinic instability developed by Eady (1949). Eady analyzed instabilities in a flow on an f-plane with uniform zonal shear between upper and lower bounding surfaces, using the Boussinesq approximation to the full fluid equations. The upper lid has an effect similar to that of the tropopause.

We analyze the Eady problem using quasi-geostrophic theory in Boussinesq isentropic coordinates on an f-plane. Figure 10.1 shows the base state for the Eady problem in geometric and isentropic coordinates. In the former presentation the isentropic surfaces slope up to the north, which reflects the horizontal buoyancy gradient related to the wind shear shown in the right panel. In isentropic coordinates the isentropic surfaces are horizontal and the upper and lower lids slope down to the north.

The zonally symmetric part of the Montgomery potential is taken to be

$$M_Z = -f\Lambda(\theta - \theta_M)y \tag{10.1}$$

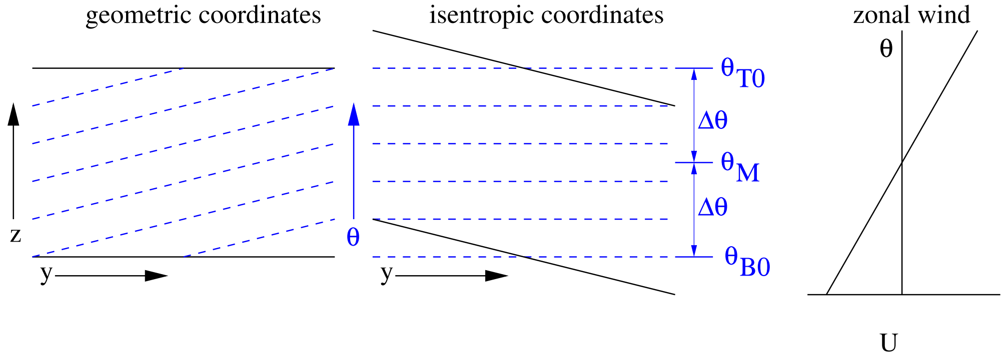

Figure 10.1: Base state for Eady problem. The left and center panels show the upper and lower lids (solid lines) and the isentropic surfaces (dashed lines) in the $y-z$ plane in geometric and isentropic coordinates respectively. The right panel shows the ambient zonal wind.

where θ_M is the mid-point potential temperature as illustrated in figure 10.1. The ambient zonal wind is

$$U(\theta) = -\frac{1}{f}\frac{\partial M_Z}{\partial y} = \Lambda(\theta - \theta_M). \tag{10.2}$$

Since we shall need the ambient winds at the upper and lower reference levels θ_{T0} and θ_{B0}, we define $U_T = \Lambda\Delta\theta$ and $U_B = -\Lambda\Delta\theta$ where $\Delta\theta = \theta_{T0} - \theta_M = \theta_M - \theta_{B0}$. The shear of the mean wind in geometric coordinates is $S = \partial U/\partial z = (\partial U/\partial\theta)(\partial\theta/\partial z) = \Lambda\Gamma_R$. The potential vorticity associated with the mean sheared flow is

$$q_Z = q_0\left(\frac{1}{f^2}\nabla_h^2 M_Z + \frac{\Gamma_R^2}{N^2}\frac{\partial^2 M_Z}{\partial\theta^2}\right) = 0. \tag{10.3}$$

Thus, the ambient potential vorticity in the interior of the flow is constant and equal to q_0, and no potential vorticity anomalies can be created by advection. The potential vorticity advection equation $dq/dt = 0$ is thus trivially satisfied, and the dynamics of the system is carried by the advection of potential temperature on the upper and lower lids. Surface Rossby waves associated with these lids interact to produce barotropic instability in analogy to baroclinic instability in the shallow water system.

The part of the Montgomery potential associated with any disturbance that develops is M'. Since no interior potential vorticity anomalies exist in this case, M' satisfies

$$\frac{1}{f^2}\nabla_h^2 M' + \frac{\Gamma_R^2}{N_R^2}\frac{\partial^2 M'}{\partial\theta^2} = 0. \tag{10.4}$$

Upper and lower boundary conditions on $M^* = M_Z + M'$ in the case of no terrain ($\Phi_T^* = \Phi_B^* = 0$)

$$\left(\frac{\partial M^*}{\partial\theta}\right)_{T0} = \frac{N_R^2}{\Gamma_R^2}\theta_T^* \qquad \left(\frac{\partial M^*}{\partial\theta}\right)_{B0} = \frac{N_R^2}{\Gamma_R^2}\theta_B^* \tag{10.5}$$

are evaluated on levels θ_{T0} and θ_{B0} respectively (see figure 10.1). We remind ourselves that $\theta_T^* = \theta_T - \theta_{T0}$ and $\theta_B^* = \theta_B - \theta_{B0}$.

The governing equations for θ_T^* and θ_B^* are

$$\frac{\partial \theta_T^*}{\partial t} + u_{gT0}\frac{\partial \theta_T^*}{\partial x} + v_{gT0}\frac{\partial \theta_T^*}{\partial y} = 0 \tag{10.6}$$

$$\frac{\partial \theta_B^*}{\partial t} + u_{gB0}\frac{\partial \theta_B^*}{\partial x} + v_{gB0}\frac{\partial \theta_B^*}{\partial y} = 0. \tag{10.7}$$

Inserting the geostrophic winds at the upper and lower reference levels, eliminating θ_T^* and θ_B^* using equation (10.5), splitting M^* into the zonal part M_Z and the disturbance part M', and finally linearizing in primed quantities yields

$$\frac{\partial}{\partial t}\left(\frac{\partial M'}{\partial \theta}\right)_{T0} + \Lambda\Delta\theta\frac{\partial}{\partial x}\left(\frac{\partial M'}{\partial \theta}\right)_{T0} - \Lambda\frac{\partial M'_{T0}}{\partial x} = 0 \tag{10.8}$$

and

$$\frac{\partial}{\partial t}\left(\frac{\partial M'}{\partial \theta}\right)_{B0} - \Lambda\Delta\theta\frac{\partial}{\partial x}\left(\frac{\partial M'}{\partial \theta}\right)_{B0} - \Lambda\frac{\partial M'_{B0}}{\partial x} = 0. \tag{10.9}$$

Between equations (10.4), (10.8), and (10.9), our governing equations for the Eady problem are complete.

Let us try a solution of the form

$$M' = A\exp\left[i(kx - \omega t) + m(\theta - \theta_M)\right] + B\exp\left[i(kx - \omega t) - m(\theta - \theta_M)\right]. \tag{10.10}$$

Substitution in equation (10.4) tells us that

$$m = \frac{kN_R}{f\Gamma_R}, \tag{10.11}$$

while equations (10.8) and (10.9) result in

$$\begin{pmatrix} (c + \Lambda\Delta\theta + \Lambda/m)e^{-m\Delta\theta} & -(c + \Lambda\Delta\theta - \Lambda/m)e^{m\Delta\theta} \\ (c - \Lambda\Delta\theta + \Lambda/m)e^{m\Delta\theta} & -(c - \Lambda\Delta\theta - \Lambda/m)e^{-m\Delta\theta} \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = 0. \tag{10.12}$$

Setting the determinant of the matrix to zero to get the dispersion relation yields

$$c = \pm(U_T - U_B)\left[\left(\frac{1}{\kappa} - \frac{1}{2}\right)^2 - \frac{\coth(\kappa) - 1}{\kappa}\right]^{1/2} \tag{10.13}$$

after some algebra, where we have recognized that $2\Lambda\Delta\theta = U_T - U_B$ and have defined the dimensionless horizontal wavenumber

$$\kappa = 2m\Delta\theta = Lk \tag{10.14}$$

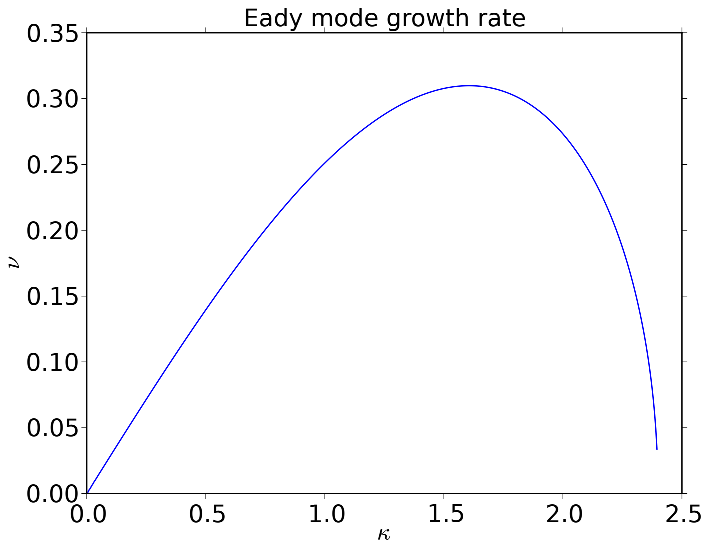

Figure 10.2: Plot of the non-dimensional growth rate ν vs. the non-dimensional wavenumber κ for the Eady mode.

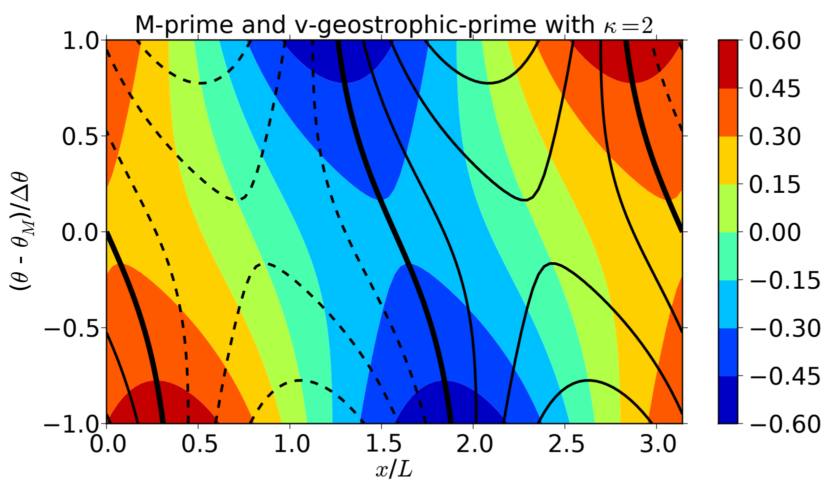

Figure 10.3: Filled color plot of η' and contour plot of v_g for the Eady model with $\kappa = 2$ and $E = 0.2$. Solid contours show positive values of v_g (toward the north) while dashed contours show negative values. Zero v_g is shown by the thick solid line.

where $L = 2N_R\Delta\theta/(f\Gamma_R)$. Taking $N_R = 10^{-2}$ s^{-1}, $\Delta\theta = 25$ K, $f = 10^{-4}$ s^{-1}, and $\Gamma_R = 5 \times 10^{-3}$ K m^{-1}, then $L = 10^3$ km.

Since $\coth(\kappa) > 1$ for positive κ, equation (10.13) is guaranteed to produce imaginary values of c for a range of κ values around $\kappa = 2$. Thus, we have an unstable disturbance which grows in amplitude with time. This is the simplest example of baroclinic instability. We define the dimensionless growth rate ν as a function of κ

$$\nu = \frac{\kappa c_I}{U_T - U_B} = \left\{-(1-\kappa/2)^2 + \kappa[\coth(\kappa) - 1]\right\}^{1/2} \tag{10.15}$$

and plot the results in figure 10.2. The growth rate has a short-wavelength cutoff at $\kappa \approx 2.4$ and peaks near $\kappa \approx 1.6$. There is no long-wavelength cutoff in the Eady mode.

As in the barotropic instability mode studied in the previous chapter, symmetry implies that $|A| = |B|$. Setting $A = E\exp(i\phi)$ and $B = E\exp(-i\phi)$ where E is real, equation (10.12) informs us that

$$\frac{A}{B} = \exp(2i\phi) = -\frac{\kappa - 2 + i\nu}{\kappa + 2 + i\nu}\exp(\kappa), \tag{10.16}$$

from which we conclude that

$$\phi = \frac{1}{2}\arctan\left(\frac{4\nu}{\kappa^2 + \nu^2 - 4}\right). \tag{10.17}$$

Figure 10.3 shows M' and v_g', the patterns of Montgomery potential and meridional geostrophic wind associated with a disturbance with wavenumber $\kappa = 2$ in the Eady model. The maximum amplitudes of these variables occur adjacent to the upper and lower boundaries, reflecting their character as a superposition of upper and lower surface waves. The upper wave is lagged to the west of the lower wave, so constant phase surfaces tilt to the west with height. As expected from geostrophic dynamics, southerly winds occur to the east of low values of M' and northerly winds occur to the west. The downward tilt to the north of constant geopotential surfaces in isentropic coordinates (or the upward tilt of isentropic surfaces; see figure 10.1) allows slantwise ascent of parcels with positive buoyancy, as occurs with symmetric instability. This in fact is one of the primary sources of energy for baroclinic instability.

10.2 Alternate interpretation of boundary conditions

We now show that a thin layer of potential vorticity just above the $\theta = \theta_{B0}$ surface can mimic the effect of a potential temperature perturbation at that surface. Imagine that we add such a layer to q^*, resulting in a total flow-associated potential vorticity

$$q_T^* = q_0 I_B(x, y, t)\delta(\theta - \theta_{B0} - \epsilon) + q^* \tag{10.18}$$

where ϵ is tiny, and set the potential temperature perturbation θ^*_{B0} to zero at θ_{B0}. The diagnostic equation for M^* in the Boussinesq case becomes

$$\frac{1}{f_0^2}\nabla_h^2 M^* + \frac{\Gamma_R^2}{N_R^2}\frac{\partial^2 M^*}{\partial\theta^2} = \frac{q_T^*}{q_0}. \quad (10.19)$$

Integrating equation (10.19) in θ over a thin layer surrounding $\theta_{B0} + \epsilon$ results in

$$\frac{\Gamma_R^2}{N_R^2}\left[\left(\frac{\partial M^*}{\partial\theta}\right)_+ - \left(\frac{\partial M^*}{\partial\theta}\right)_-\right] = I_B. \quad (10.20)$$

The "+" and "−" indicate the evaluation of $\partial M^*/\partial\theta$ just above and below the level $\theta = \theta_{B0}+\epsilon$. Since the potential temperature perturbation at the actual surface is zero, equation (10.5) tells us that $(\partial M^*/\partial\theta)_- = 0$. However, if we set

$$I_B = \theta_B^*, \quad (10.21)$$

then $(\partial M^*/\partial\theta)_+$ is equal to $(\partial M^*/\partial\theta)_{B0}$ and the new surface boundary condition is

$$\left(\frac{\partial M^*}{\partial\theta}\right)_{B0} = \frac{N_R^2}{\Gamma_R^2} I_B. \quad (10.22)$$

Note that a layer of potential vorticity at the surface is subject to the same governing equation as the surface potential temperature perturbation θ_B^*:

$$\frac{\partial I_B}{\partial t} + \boldsymbol{v}_{gB0}\cdot\boldsymbol{\nabla}_h I_B = 0. \quad (10.23)$$

Thus, the time evolution of the system can be considered to be contained totally within the potential vorticity conservation equation without recourse to a separate potential temperature equation at the boundary.

In the case of an upper lid, as occurs in the Eady problem, $(\partial M/\partial\theta)_+ = 0$ and $(\partial M/\partial\theta)_-$ is identified with $(\partial M^*/\partial\theta)_{T0}$ in equation (10.5). The relationship between the upper boundary potential temperature perturbation θ_T^* and the layer potential vorticity I_T therefore has a minus sign,

$$I_T = -\theta_T^*, \quad (10.24)$$

and the upper boundary condition is

$$\left(\frac{\partial M^*}{\partial\theta}\right)_{T0} = -\frac{N_R^2}{\Gamma_R^2} I_T. \quad (10.25)$$

In the Eady problem the potential temperature at both the upper and lower boundaries decreases with increasing y. Thus, the lower boundary potential vorticity decreases to the north while the potential vorticity at the upper boundary increases to the north.

10.3 Rossby waves and the mean flow

Jeffreys (1933) was perhaps the first to point out that midlatitude cyclones are likely to play an important role in determining the general circulation of the atmosphere. This is a complex problem of long-standing interest, and it is one which is not completely solved to this day. There are two aspects to this problem, determining the conditions under which dynamic instability (primarily barotropic and baroclinic instability) give rise to cyclone waves, and determining the effect of these waves on the mean zonal flow. Here we can only touch on this subject, outlining some of the most basic ideas.

We first develop the linearized quasi-geostrophic governing equations in Boussinesq isentropic coordinates for an arbitrary (but balanced) zonal-mean flow pattern on a beta-plane. We start by assuming that

$$\begin{aligned} M &= M_0(\theta) + M_Z(y,\theta) + M' = M_0(\theta) + M^* \\ \sigma &= \sigma_0 + \sigma_Z + \sigma' = \sigma_0 + \sigma^* \\ q_g &= q_0(1+\beta y/f_0) + q_Z(y,\theta) + q' = q_0(1+\beta y/f_0) + q^* \\ \Phi &= \Phi_0(\theta) + \Phi_Z(y,\theta) + \Phi' = \Phi_0(\theta) + \Phi^* \\ u_g &= U(y,\theta) + u_g' \\ v_g &= v_g' \end{aligned} \tag{10.26}$$

where a subscripted zero indicates the base state of a variable, the subscripted Z indicates the zonally symmetric part associated with the flow, and the prime indicates the non-zonally-symmetric disturbance. A superscripted asterisk indicates the zonal and perturbation parts taken together. The base state potential vorticity $q_0 = f_0/\sigma_0$ is constant, as is the base state density σ_0. Geostrophic balance tells us that

$$U = -\frac{1}{f_0}\frac{\partial M_Z}{\partial y} \tag{10.27}$$

while the relationship between Montgomery potential and geopotential yields

$$\Phi_Z = -\theta_R \frac{\partial M_Z}{\partial \theta}. \tag{10.28}$$

Cross-differentiation of equations (10.27) and (10.28) and elimination of M_Z gives us the thermal wind relationship for the zonal flow

$$\frac{1}{\theta_R}\frac{\partial \Phi_Z}{\partial y} = f_0 \frac{\partial U}{\partial \theta}. \tag{10.29}$$

while the definition of potential vorticity gives us

$$\frac{q_Z}{q_0} = -\frac{1}{f_0}\frac{\partial U}{\partial y} - \frac{\Gamma_R^2}{\theta_R N_R^2}\frac{\partial \Phi_Z}{\partial \theta} = \frac{1}{f_0^2}\frac{\partial^2 M_Z}{\partial y^2} + \frac{\Gamma_R^2}{N_R^2}\frac{\partial^2 M_Z}{\partial \theta^2}. \tag{10.30}$$

The disturbance Montgomery potential satisfies

$$\frac{1}{f_0^2}\nabla_h^2 M' + \frac{\Gamma_R^2}{N_R^2}\frac{\partial^2 M'}{\partial\theta^2} = \frac{q'}{q_0}. \tag{10.31}$$

The linearized potential vorticity advection equation is

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left(\nabla_h^2 M' + \frac{\partial^2 M'}{\partial\xi^2}\right) + \left(\frac{\partial Q}{\partial y}\right)\left(\frac{\partial M'}{\partial x}\right) = 0 \tag{10.32}$$

where

$$Q = f_0\left(\beta y/f_0 + q_Z/q_0\right) \tag{10.33}$$

is proportional to the part of the potential vorticity associated with the beta effect and the zonal mean flow. In equation (10.32), q'/q_0 has been eliminated using equation (10.31). As usual, the velocity has been replaced by the geostrophic velocity and the notation has been simplified by defining a scaled potential temperature ξ which has the units of length stretched by N_R/f_0:

$$d\theta = \frac{f_0\Gamma_R}{N_R}d\xi. \tag{10.34}$$

Unlike the shallow water case, there is no natural length scale analogous to the Rossby radius in the continuously stratified situation, so we do not non-dimensionalize the general governing equations. A Rossby radius appears only after the vertical scale of the problem is set.

We now assume a plane wave structure in the zonal direction $\exp\left[i(kx - \omega t)\right]$ and further define the trace speed in the x direction $c = \omega/k$. Substituting into equation (10.32) results in

$$\frac{\partial^2 M'}{\partial y^2} + \frac{\partial^2 M'}{\partial\xi^2} - \mu^2 M' = 0 \tag{10.35}$$

where

$$\mu^2 = \frac{1}{c - U}\frac{\partial Q}{\partial y} + k^2. \tag{10.36}$$

This is the analog to the Taylor-Goldstein equation for small, quasi-geostrophic perturbations on a zonally symmetric flow. Note that in terms of the scaled potential temperature ξ, the relationship between perturbation Montgomery potential M' and isentropic density σ' is

$$\frac{\partial^2 M'}{\partial\xi^2} = -f_0^2\frac{\sigma'}{\sigma_0} \tag{10.37}$$

while the relationship between M' and the perturbation geopotential Φ' is

$$\frac{\partial M'}{\partial\xi} = -\frac{f_0\Gamma_R}{N_R\theta_R}\Phi'. \tag{10.38}$$

These follow from the unscaled relations given in the chapter on quasi-geostrophic theory.

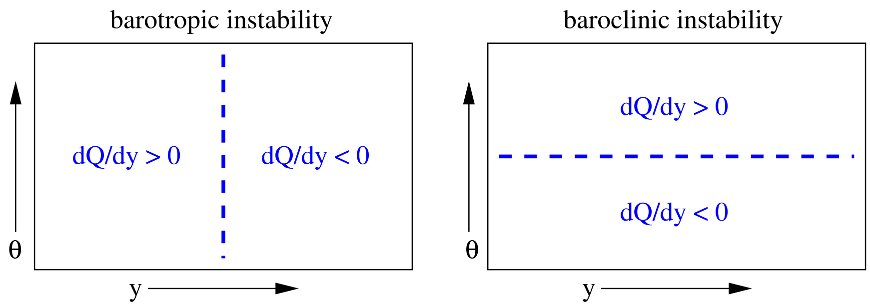

Figure 10.4: The orientation of the $\partial Q/\partial y = 0$ line determines whether the resulting instability is barotropic or baroclinic.

10.3.1 Conditions for instability

The quasi-geostrophic equivalent to the Miles-Howard theorem is the Charney-Stern theorem (Charney and Stern 1962). We multiply equation (10.35) by the complex conjugate of the Montgomery potential perturbation M'^* and integrate in y and ξ, noting first that

$$\int_{-\infty}^{\infty} M'^* \frac{\partial^2 M'}{\partial y^2} dy = \int_{-\infty}^{\infty} \left[\frac{\partial}{\partial y} \left(M'^* \frac{\partial M'}{\partial y} \right) - \left| \frac{\partial M'}{\partial y} \right|^2 \right] dy \tag{10.39}$$

and

$$\int_{B}^{T} M'^* \frac{\partial^2 M'}{\partial \xi^2} d\xi = \int_{B}^{T} \left[\frac{\partial}{\partial \xi} \left(M'^* \frac{\partial M'}{\partial \xi} \right) - \left| \frac{\partial M'}{\partial \xi} \right|^2 \right] d\xi \tag{10.40}$$

where B and T are the values of ξ corresponding to θ_{B0} and θ_{T0}. We chose the limits of integration for these two equations such that the first terms on the right sides vanish. In the y integration one must extend the limits into quiescent regions in which M' vanishes. We have chosen to carry the integration all the way to $\pm\infty$. For the ξ integration the artifice of replacing surface potential temperature anomalies by surface potential vorticity anomalies causes the first integral on the right to vanish as the actual surface boundary condition in this case is $\partial M'/\partial\theta \propto \partial M'/\partial\xi = 0$. If there is an upper lid, a similar condition exists there. Otherwise, the integral has to be carried to high enough altitudes that $M' = 0$.

The result is

$$\int_{-\infty}^{\infty} dy \int_{B}^{T} d\xi \left[\left| \frac{\partial M'}{\partial y} \right|^2 + \left| \frac{\partial M'}{\partial \xi} \right|^2 + \left(\frac{1}{c - U} \frac{\partial Q}{\partial y} + k^2 \right) |M'|^2 \right] = 0, \tag{10.41}$$

which has imaginary part

$$c_I \int_{-\infty}^{\infty} dy \int_{B}^{T} d\xi \left(\frac{1}{|c - U|^2} \frac{\partial Q}{\partial y} \right) |M'|^2 = 0. \tag{10.42}$$

In order for equation (10.42) to be satisfied with non-zero M', either $c_I = 0$, in which case there is no instability, or $\partial Q/\partial y$ must change sign within the domain of integration, making the integrand non-positive-definite. The latter condition is thus a necessary condition for instability to exist.

Figure 10.4 shows the difference between barotropic and baroclinic instability. In the former, the line of zero $\partial Q/\partial y$ is vertical whereas in the latter it is horizontal. Intermediate cases give rise to mixed barotropic-baroclinic instability. In the case of the Eady mode, $\partial Q/\partial y = 0$ throughout the interior of the domain. Positive $\partial Q/\partial y$ exists in the upper boundary sheet of potential vorticity, whereas negative $\partial Q/\partial y$ exists in the lower boundary sheet.

10.3.2 Non-interaction theorem (part 2)

We now develop the quasi-geostrophic equivalent of the non-interaction theorem discussed earlier in the context of non-rotating, stratified, shear flows. Since knowing the distribution of potential vorticity allows us to determine everything about the flow in quasi-geostrophic theory, it is useful to examine the zonally averaged meridional eddy flux of potential vorticity. Using the same techniques as applied to part 1 of the non-interaction theorem, we assume variables have the form of a single mode with x and t dependence $\exp[i(kx - \omega t)]$ and write the meridional flux of potential vorticity as

$$F_q = \frac{1}{L}\int_0^L \mathrm{Re}(v_g)\mathrm{Re}(q')dx = \frac{1}{4}\left[v_g q'^* + v_g^* q'\right] \tag{10.43}$$

where a superscripted asterisk indicates a complex conjugate and L is initially the zonal wavelength of the disturbance being analyzed, but is subject to reinterpretation below. Using equation (10.31) for the potential vorticity perturbation q' as well as the coordinate transformation implied by equation (10.34) we find that

$$q' = \frac{q_0}{f_0^2}\left(\frac{\partial^2 M'}{\partial y^2} + \frac{\partial^2 M'}{\partial \xi^2} - k^2 M'\right) = \frac{q_0 M'}{f_0^2(c - U)} \tag{10.44}$$

where the last step employs equations (10.35) and (10.36). Combining this with the definition of v_g

$$v_g = \frac{1}{f_0}\frac{\partial M'}{\partial x} \tag{10.45}$$

results in

$$F_q = \frac{ikq_0}{f_0^3}\left[\frac{1}{c^* - U} - \frac{1}{c - U}\right]|M'|^2 = -\frac{kq_0 c_I |M'|^2}{f_0^3|c - U|^2} \tag{10.46}$$

where as before c_I is the imaginary part of the phase speed c of the disturbance.

We conclude from equation (10.46) that the meridional flux of potential vorticity due to a linear disturbance which is neither growing nor decaying is zero. Thus, stable Rossby waves propagate meridionally without affecting the mean state of the atmosphere.

The above analysis is only valid for a single Fourier mode in the zonal direction as it stands. However, a superposition of two Fourier modes with wavenumbers k_1 and k_2 results in cross terms between the modes with x dependence of $\exp[i(k_1 - k_2)x]$ in equation (10.43). Increasing L so that it is an integer number of both wavelengths $\lambda_1 = 2\pi/k_1$ and $\lambda_2 = 2\pi/k_2$ causes these cross terms to disappear. By extension, multiple modes with wavenumbers k_j may be superimposed if the integration range is set to the circumference of the globe. In this case equation (10.46) becomes

$$F_q = -\frac{q_0}{4f_0^3} \sum_j \frac{k_j c_{jI} |M'_j|^2}{|c_j - U|^2} \tag{10.47}$$

where the subscript j labels the modes over which the sum is made. The conclusion is the same as for a single mode; if all modes have zero growth rate, the total meridional flux of potential vorticity due to eddies is zero.

10.4 References

Charney, J. G., and M. E. Stern, 1962: On the stability of internal baroclinic jets in a rotating atmosphere. *J. Atmos. Sci.*, **19**, 159-172. This is the original paper on the Charney-Stern theorem.

Eady, E. T., 1949: Long waves and cyclone waves. *Tellus*, **1**, 33-52. This is Eady's original paper on baroclinic instability.

Jeffreys, H., 1933: The function of cyclones in the general circulation. *Procès-Verbaux de l' Association de Météorologie*, UGGI (Lisbon), Part II (Mémoires), 219-230.

Vallis, G. K., 2006: *Atmospheric and oceanic fluid dynamics.* Cambridge University Press, 745 pp. The Eady problem is considered (in geometric coordinates) in chapter 6 as well as the Charney-Stern theorem. A different derivation of the non-interaction theorem is covered in chapter 7.

10.5 Questions and problems

1. Compute the meridional flux of potential vorticity for the Eady mode. Hint: Treat the upper and lower surface potential temperature distributions as potential vorticity distributions. Comment on the meridional heat transport of the Eady mode.

2. Using the theory developed in section 10.3, find the dispersion relation and the spatial form of u'_g, v_g, and M' for a Rossby wave with assumed structure $\exp[i(kx + ly + m\xi - \omega t)]$. Assume that $U = 0$ but that $\beta \neq 0$. Do not apply upper or lower boundary conditions.

3. Compute the meridional flux of potential vorticity for the above Rossby wave.

4. Redo the non-interaction theorem without substituting equations (10.31) and (10.34) into equation (10.44) for the potential vorticity perturbation.

   (a) Show that F_q can be written in the form

   $$F_q = -\frac{\partial E_y}{\partial y} - \frac{\partial E_\xi}{\partial \xi}$$

   and determine the components of the vector $\boldsymbol{E} = (E_y, E_\xi)$. This vector is called the *Eliassen-Palm* flux. The Eliassen-Palm flux will only be non-zero when there is wave growth or decay or nonlinear behavior.

   (b) Further show that E_y is proportional to the meridional flux of zonal momentum and that E_ξ is proportional to the meridional flux of geopotential.

   (c) Compute the Eliassen-Palm flux for the Eady mode.