1 The Stern-Gerlach Experiment and Spin

In atomic and nuclear physics, magnetism is intimately coupled to angular momentum. This is because a charge moving in a circular orbit produces a magnetic moment. Recall from freshman physics that a current loop of area A and strength I has a magnetic moment $\mu = AI$. The magnetic moment is actually a vector pointing normal to the area, as is shown in figure 1.

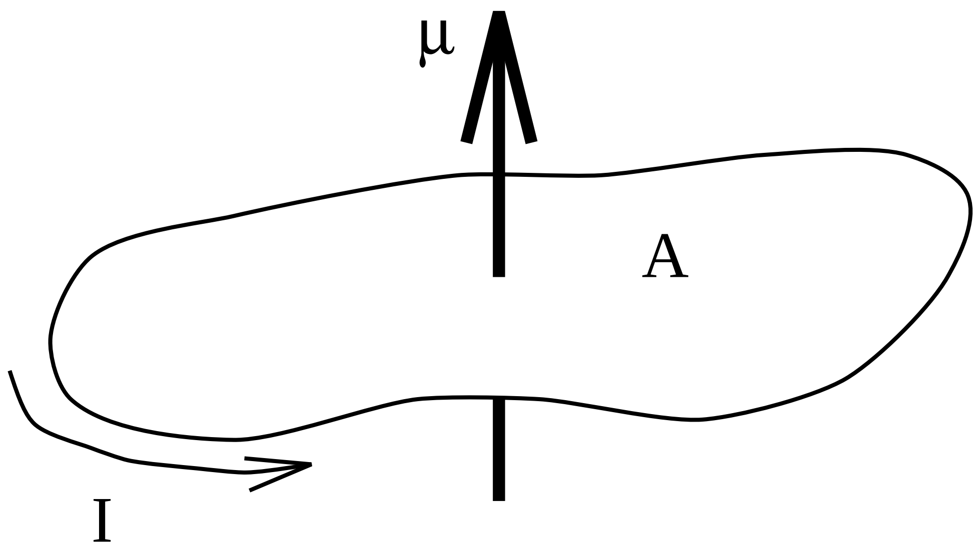

Figure 1: Sketch of magnetic moment due to a circulating charge.

A charge q moving in a circle of radius R at velocity v has a period of revolution of $T = 2\pi R/v$. It therefore produces a current $I = q/T = qv/(2\pi R)$. Since the area is $A = \pi R^2$, the magnetic moment is

$$\mu = \frac{q}{2M}\mathbf{L} \tag{1}$$

where M is the mass of the particle carrying the charge, and $\mathbf{L}$ is its angular momentum. This formula turns out to be valid for noncircular orbits and it also works in quantum mechanics. Note that the magnetic moment points opposite to the angular momentum for the electron, since the electron's charge is negative.

The component of the atomic magnetic moment in a particular direction is proportional to the component of charge in that direction. If we align the z-axis in the direction of interest, then equation (1) becomes

$$\mu_z = \frac{q}{2M} L_z = \frac{q\hbar m}{2M} \tag{2}$$

where m is the L_z quantum number.

For a constant magnetic field B in the positive z direction, the potential energy of a magnetic dipole with z-component μ_z is $U = -\mu_z B$. In addition, if the magnetic field is not constant, but exhibits a z-gradient $\partial B/\partial z$, then the dipole experiences a net force in the z direction, $F = \mu_z(\partial B/\partial z)$. This force is the basis of an important experiment first carried out by Stern and Gerlach in 1922.

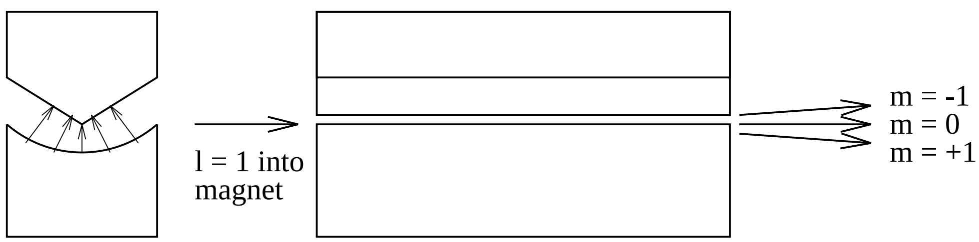

Figure 2: Sketch of the Stern-Gerlach apparatus. The magnet has a long narrow gap with asymmetric poles that create a vertical gradient of magnetic field, positive upward in this case. To the left is a cross-section of the magnet.

Figure 2 shows a sketch of the Stern-Gerlach apparatus. Particles with orbital angular momentum quantum number $l = 1$, but random orientation, are beamed into the left end of the magnet. Depending on the orientation of the angular momentum vector, and hence the magnetic moment, the particles are deflected upward (positive μ_z, $m = -1$), downward (negative μ_z, $m = +1$), or not at all (zero μ_z, $m = 0$.

The original experiment by Stern and Gerlach was carried out using silver atoms. However, in 1927 Phipps and Taylor redid this experiment using hydrogen atoms. In this experiment the beam separated into *two* sub-beams, indicating that the hydrogen atom was in a state exhibiting two possible m

values, which corresponds to $l = 1/2$. This would seem to be impossible from the theory of the hydrogen atom. First of all, the hydrogen atom is expected to be in the ground state, with $l = 0$, and thus no beam splitting. However, even if the hydrogen atoms were in the first excited state with $l = 1$, there should have been *three* beams, as discussed above. Thus, the results of Phipps and Taylor reveal an inconsistency between theory and experiment. This inconsistency is only resolved when it is realized that the electron carries an *intrinsic* angular momentum, or spin, with $l = 1/2$. The theory of spin is discussed in the next section dealing with the use of operator methods in angular momentum.