1 Big Ideas of Quantum Mechanics

Quantum mechanics represents the state of a system by an amplitude, which is a complex number, associated with each possible set of values of the physical variables associated with the system, such as position, momentum, angular momentum, energy, etc. The probability of the system having a particular set of values of the variables is the absolute square of the associated amplitude.

If multiple amplitudes are nonzero, the actual values of the variables are uncertain. The *expectation value* of each variable is the probability-weighted average of the possible values of that variable. The set of all possible amplitudes, representing all possible combinations of values of the set of variables associated with the system, forms the components of a *vector*, which we shall call ψ. This vector is called the *state vector*, and represents everything we can possibly know about the system.

The number of variables needed to describe the state of a system is equal to the *number of degrees of freedom* of the system. For instance, a single particle moving in three dimensions has three translational degrees of freedom, and thus requires three variables for the construction of its state vector, for example, the x, y, and z components of its position.

There are multiple ways of representing the state vector of a system, which correspond to different sets of variables used to describe the system. For instance, a particle moving in one dimension has one degree of freedom, and therefore can be described by a single variable. This variable may be the position, x, the momentum, p, or perhaps the energy, E. Each variable (or set of variables in the case with more than one degree of freedom) has its own set of amplitudes associated it. However, these sets of amplitudes are not independent; rather, they are related by *unitary transformations*. In essence, the different sets of amplitudes are components of the state vector resolved in different coordinate systems. The coordinate systems are simply "rotated" with respect to each other.

In this picture, variables become Hermitian operators, which may be represented by self-adjoint matrices. In the principal axis coordinate system of the operator, the associated matrix is diagonal and the values along the diagonal represent the possible values of the associated variable. Resolved in some other coordinate system, the matrix associated with the variable generally isn't diagonal.

The expectation value of a variable, A, can be obtained in *any* coordinate system as $< A >= \psi' A \psi$, where A also represents the operator associated with the variable A.

If two operators A and B commute (i. e., if $AB = BA$), then these operators represent *consistent* variables whose values can simultaneously be assumed by a system. If A and B don't commute, then these variables are *inconsistent*, and a definite value of A is always associated with an indefinite value of B, and vice versa. This is the basis of the *uncertainty principle*.