Problem Set 4

1. If ϕ_i are the (column) eigenvectors of a Hermitian operator, A, show that the sum
$$\sum_i \phi_i \phi_i'$$
is the identity matrix. (Hint: Work in a reference frame in which A is diagonal.) Also, as an example of this, show *numerically* that this result holds for the eigenvectors of the harmonic oscillator. Any set of eigenvectors exhibiting this property is said to be *complete*.

2. Given the potential energy for a particle of mass $m = 1$,
$$U = -(6 + 5x^3)\exp(-x^2):$$
   (a) Compute all the bound state eigenvalues and list them.
   (b) Show that these vectors are orthonormal.
   (c) Compute the dipole moment matrix elements $\psi_i' x \psi_j$ for $i = 1$ and j for all the bound states. How do the results differ from those of *anho_demo.r*?

3. Consider the 2-D harmonic oscillator with Hamiltonian $H = (p_x^2 + p_y^2 + x^2 + y^2)/2$ and wave functions $\psi_{ij}(x, y)$. The indices $i = 1, 2, 3, ...,\ j = 1, 2, 3, ...$ indicate the state of the x and y harmonic oscillator components obtained by separating variables in Cartesian coordinates.
   (a) Compute (numerically or otherwise) the matrix elements
$$X_{ijkl} = \psi_{ij}' x \psi_{kl} = \int dx \int dy \psi_{ij}^* x \psi_{kl}$$
$$Y_{ijkl} = \psi_{ij}' y \psi_{kl} = \int dx \int dy \psi_{ij}^* y \psi_{kl}$$
   for the transitions from the ground state, $k = l = 1$, to the states $(i, j) = (1, 2), (2, 1), (2, 2)$. Explain your results, i. e., if one or more of these transitions is forbidden, explain why. (You may find *xydipole.r* useful.)

(b) Find the eigenvalues of the Hamiltonian $H = H_0 + 0.1x^2y^2$, where H_0 is the 2-D harmonic oscillator Hamiltonian described above. Use first order perturbation theory to find the energies of the states $(i, j) = (1, 1), (2, 1), (1, 2), (3, 1), (2, 2), (1, 3)$. (Hint: Try modifying *xydipole.r* to do the integrals.)