1 Problem Set 2

1. Show that if an operator is self-adjoint in one coordinate system, it is self-adjoint in all coordinate systems. (Hint: Transforming the operator A from one coordinate system to another is done by the operation qAq', where q' is a unitary matrix.)

2. Show that $(\psi, A\psi) = (A'\psi, \psi)$.

3. Show that if $A\psi = a\psi$ in one reference frame, where ψ is a vector, A is an operator, and a is a number, it holds in all reference frames. ψ is an *eigenvector* of A, and a is the associated *eigenvalue*.

4. Find the eigenvalues and eigenvectors of the equation

$$\begin{pmatrix} 1.5 & 0.5 & 0 \\ 0.5 & 1.5 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix}.$$

   Use the eigenvectors to construct the transformation matrix from the original coordinate system to the principal axis system. Verify that the original matrix transforms into principal axis form under this transformation. Do these operations both by hand and with RLaB. (The RLaB routine "eigs" finds eigenvalues and eigenvectors — do "help eig".)

5. Do the same thing for the matrix

$$\begin{pmatrix} 1.5 & 0.5 & 0 \\ 0.5 & 1.5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

   (Yes, there is a good reason to redo this with a different zz component!)