Ph 340 – Problem Set 11 – Spring 1997

1. Time independent perturbation theory: Assume that the Hamiltonian splits into two parts, $H = H_0 + \epsilon J$, where

$$H_0 = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + U_0(x),$$

   $U_0 = 0$ for $0 < x < 1$ and $U_0 = \infty$ otherwise.

   (a) Find the eigenfunctions and energies for H_0. (This should be easy by now, as this is just the scaled infinite square well.)

   (b) If $J = 1$ for $0 < x < 1/2$ and $J = 0$ otherwise, find the energy shifts due to this perturbation potential.

2. Time dependent perturbation theory:

   (a) Write out equation (21) in the section of the web notes on perturbation theory for the case of a system with two states with amplitudes A_1 and A_2 and energies E_1 and E_2.

   (b) Suppose that $\omega = E_2 - E_1$. Drop the exponential terms which oscillate rapidly with time since their net effect on the solution is small. Then solve the pair of equations for $A_1(t)$ and $A_2(t)$.

   (c) A real-world example of this type of solution is nuclear magnetic resonance. Look up the magnetic moment of the proton and compute E_1 and E_2 for up and down spin orientations for a magnetic field of 0.1 Tesla. Determine the frequency for which $\omega = (E_2 - E_1)/\hbar$. (Note that we set $\hbar = 1$ in the first two parts of the problem.)

3. Show either numerically or analytically that $(\psi_m, x\psi_n)$ is only non-zero for $m = n \pm 1$ for the harmonic oscillator eigenfunctions. (Hint: First show that $A^+ + A^- = 2^{1/2}x$ to do it analytically.)