Ph 340 – Problem Set 9 – Spring 1997

1. Analytic solution to the harmonic oscillator problem.

   (a) Try to find a solution to the scaled harmonic oscillator differential equation of the form $\psi = \exp(-\alpha x^2)$ where α is a constant. Determine α and the energy.

   (b) The above solution is the ground state solution to the harmonic oscillator problem. Find excited state solutions of the form $\psi = P(x)\exp(-\alpha x^2)$ where α is the same as above and $P(x)$ is a polynomial in x. Look for two additional solutions with the highest power in the polynomal being respectively 5 and 6. (Solutions for 2, 3, and 4 are in the notes.) Find $\psi(x)$ and the energy for each solution. Hint: The problem simplifies because in order to satisfy parity conservation, $P(x)$ must be either even or odd. This causes all the terms of the polynomial for each case to be either even or odd powers of x.

2. Delta function potential well.

   (a) Solve the time-independent Schrödinger equation for bound state solutions for the potential $U(x) = -\Delta\delta(x)$. Hint: Outside the potential the solutions will be exponentials. To obtain a relationship between the exponential solutions on each side of the delta function, integrate the Schrödinger equation from $x = -\epsilon$ to $x = +\epsilon$, where ϵ is a very small positive number. In this integration we find

   $$\int_{-\epsilon}^{\epsilon} \frac{d^2\psi}{dx^2} dx = \left.\frac{d\psi}{dx}\right|_{\epsilon} - \left.\frac{d\psi}{dx}\right|_{-\epsilon}.$$

   Also,

   $$\int_{-\epsilon}^{\epsilon} [a(x)\delta(x) + b(x)]dx \approx a(0) + 2\epsilon b(0) \approx a(0),$$

   since ϵ is assumed to be small. Use the above to obtain a condition on the jump in $d\psi/dx$ across the potential well. From this you can obtain the full solution and the associated energy.

   (b) Use the above ideas to obtain the two bound state solutions and their energies to the double potential well $U(x) = -\Delta_1\delta(x+a) -$

$\Delta_2\delta(x-a)$ where a is a constant. Hint: Try $\psi = A\exp(\kappa x) + B\exp(-\kappa x)$ between the wells and simple exponentials elsewhere.