Ph 340 – Problem Set 8 – Spring 1997

1. Solve the 1-D infinite square well problem for a relativistic particle. Hint: Use the Klein-Gordon equation:

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2} - \frac{m^2 c^4}{\hbar^2} \psi.$$

2. Show that

$$\frac{d}{dt} \int_{-\infty}^{\infty} \left(\psi^* \frac{\partial \psi}{\partial t} - \frac{\partial \psi^*}{\partial t} \psi \right) dx = 0$$

for any solution ψ to the 1-D Klein-Gordon equation which goes to zero at $\pm\infty$.

3. Assume that the 1-D time-independent Schrödinger equation

$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0$$

has a solution of the form

$$\psi(x) = A(x) \exp(\pm i \int k dx),$$

where $k^2 = 2m[E - U(x)]/\hbar^2$.

   (a) Find a differential equation for $A(x)$.

   (b) For slowly varying potentials compared to the wavelength, we can ignore the term $d^2 A/dx^2$. Find the resulting equation for A with this simplification. (This is called the WKB approximation.)

   (c) Making the WKB approximation, seek solutions of the form $A = k^n$, where n is a constant which must be determined.

   (d) Compute the probability flux for your solution.

   (e) Qualitatively compare your WKB results with the numerical solutions for positive and negative Gaussian potentials produced by `scatter1_demo.r`.

4. In elementary particle physics the neutral kaon particle has the peculiar property that it can turn into its own antiparticle under the effects of

the weak interactions. The state of this particle-antiparticle system can be written as

$$\psi = \begin{pmatrix} A \\ B \end{pmatrix}$$

where A is the amplitude for it to be a particle and B is the amplitude for it to be an antiparticle. The strong interaction Hamiltonian is

$$H_S = \begin{pmatrix} Mc^2 & 0 \\ 0 & Mc^2 \end{pmatrix}$$

where M is the kaon-antikaon mass and c is the speed of light. The weak interaction Hamiltonian is

$$H_W = \begin{pmatrix} 0 & W \\ W & 0 \end{pmatrix}$$

where W is a constant.

(a) Show that the eigenvalues of the strong Hamiltonian by itself are degenerate.

(b) Find the eigenvalues and eigenvectors of the combined strong-weak Hamiltonian $H = H_S + H_W$.

(c) What are the masses of these two states?

(d) The *charge conjugation* operator C turns particles into antiparticles and antiparticles into particles:

$$C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Show that the eigenstates of the combined Hamiltonian are also eigenstates of the charge conjugation operator. What are the eigenvalues of C for the eigenstates of the combined Hamiltonian?