Ph 340 – Problem Set 3 – Spring 1997

1. Given
$$\psi(x,t) = \int_{-\infty}^{\infty} \phi(k) \exp[i(kx - \omega t)] dk,$$
where $\omega = \omega(k)$ is real, evaluate
$$\int_{-\infty}^{\infty} \psi \psi^* dx,$$
showing that it equals
$$2\pi \int_{-\infty}^{\infty} \phi(k) \phi^*(k) dk.$$
Note that this result is independent of time. Hint: Use
$$\int_{-\infty}^{\infty} \exp[i(k - k')x] dx = 2\pi \delta(k - k').$$

2. Given
$$\psi(x,t) = \sum_m C_m f_m(x) \exp(-i\omega_m t)$$
over the interval $[-L/2, L/2]$, where ω_m is real and where
$$\int_{-L/2}^{L/2} f_m(x) f_n^*(x) dx = \delta_{mn},$$
Evaluate
$$\int_{-L/2}^{L/2} \psi \psi^* dx,$$
showing that the result is independent of time. Note that δ_{mn} is 1 if $m = n$ and 0 otherwise. This is called the Kronecker delta. The $f_m(x)$ are called a complete, orthonormal set of functions over the interval $[-L/2, L/2]$. Fourier series is a special case of this in which the fs are complex exponentials.