Ph 340 – Problem Set 2 – Spring 1997

1. If $\tilde{f}(k) = \exp(-\mu^2 k^2)$, where μ is a constant, we showed that the inverse Fourier transform yields $f(x) = (\pi^{1/2}/\mu)\exp(-x^2/4\mu^2)$. Show that the forward Fourier transform recovers the above form for $\tilde{f}(k)$.

2. If $f(x) = \exp[i\kappa x - (x-a)^2/b^2]$, where κ, a, and b are constants, find the Fourier transform of $f(x)$. Note that this function represents a wave packet of central wavenumber κ, width b, and location a. What role do each of these constants play in determining the form of $\tilde{f}(k)$?

3. Evaluate
$$f(x) = \int_{-\infty}^{\infty} \exp(-x'^2)\frac{\partial}{\partial x'}\delta(x - x')dx'. \tag{1}$$
Hint: Try integration by parts.

4. Using the default dispersion relation in `packet_demo.r`, describe what happens at large time to wave packets with different initial widths. (Hint: Use the default values of `dx`, `dt`, and `k0`, but increase `nx` and `nt`. Then vary `width`.