Physics 340 — Test 2 — Spring 1995

Conditions: This is a take home test. It is due Wednesday 29 March 1995 at the beginning of class. You can use your notes, the book, or any resource in the library or on the Internet. However, you may not communicate with anybody except the instructor of the course in conjunction with this test.

1. Get the harmonic oscillator RLaB demonstration ho_demo.r from

   `http://www.ee.nmt.edu/~raymond/ph340/demos.html`

   and modify it to use the scaled potential energy function $U(x) = -6\exp(-x^2)$, recalling that this program scales the Hamiltonian to to the form $H = p^2/2 + U(x)$. Answer the following.

   (a) Plot out the first 5 eigenstates and list their energies.

   (b) Describe how the solutions differ from the solutions to the harmonic oscillator problem.

   (c) Determine how many bound states there are, i. e., how many states have negative energy?

   (d) Demonstrate that the first 5 eigenfunctions are orthonormal.

2. Solve the one-dimensional scattering problem off of a square potential *analytically*. More specifically, solve the equation

$$\frac{\partial^2 \psi}{\partial x^2} + [1 - U(x)]\psi = 0 \tag{1}$$

   for

$$U(x) = \begin{cases} U_{max}, & |x| < W \\ 0, & |x| > W \end{cases} \tag{2}$$

   with radiation boundary conditions for $x > W$.

   (a) Compute the transmission probability as a function of U_{max} and W.

   (b) Demonstrate that the transmission probability has reasonable limits as U_{max} and W go to zero.

   (c) Determine the energy dependence of the transmission probability as we did in class by setting $U_{max} = U_0/\epsilon$ and $W = W_0\epsilon^{1/2}$. (Recall that ϵ is the relative energy as explained in class and in the hypertext course notes.)

   (d) Compare your energy dependence to the three cases computed numerically in the course notes.

   To do this one right you need to be *extremely careful* in your mathematical manipulations.