Physics 340 — Test 1 — Spring 1996

Reminder sheet allowed. **Show all work!**

1. A certain type of wave has a dispersion relation

$$\omega = ak^2 - b/k$$

where ω is the frequency, k is the wavenumber, and a and b are constants.

   (a) For what value of k is the phase speed zero?

   (b) Find the group velocity as a function of k.

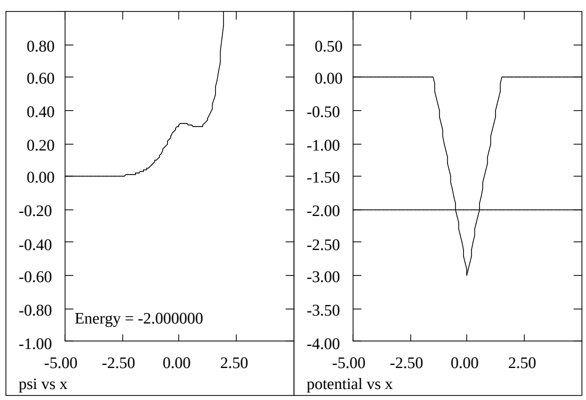
   (c) For value of k is the group velocity a minimum?

2. A wave function has the form $\psi(x) = \exp(ikx - \alpha x)$ where k is the wavenumber and α is a constant.

   (a) Compute the probability flux as a function of x.

   (b) Given the conservation of probability, qualitatively describe what will happen to $|\psi|^2$ with time.

3. You are trying to obtain the lowest energy level for the illustrated potential using the shooting method. For a given guess for the energy, the computed wave function is as illustrated below. Is your guess for the energy too high or too low? Explain.



4. Estimate the probability that an incident particle of mass m and energy $E < U$ will penetrate the barrier illustrated below. (Hint: An exact answer requires a full solution of the boundary value problem. Instead of doing this, simply assume that $\psi(x)$ is a purely decaying exponential in the barrier region.)