Physics 340 — Test 1 — Spring 1995

Reminder sheet allowed. Show all work.

1. A free, nonrelativistic particle wavefunction is given by $\psi = \exp(ik_1x - i\omega_1 t) + \exp(ik_2x - i\omega_2 t)$, where $\hbar\omega_{1,2} = \hbar^2 k_{1,2}/2m$, and where the difference between k_1 and k_2 is small.

   (a) In this wavefunction the two terms interfere constructively in some locations and destructively in others. Determine how fast a region of constructive interference moves, and relate this to the group velocity of the wave.

   (b) Find the wavefunction transformed to wavenumber space.

2. The normalized ground state eigenfunction for the harmonic oscillator is $\psi = (\pi^{1/2}/B)\exp(-B^2x^2)$, where B is a constant.

   (a) Compute the expectation value for the momentum for this eigenfunction.

   (b) Compute the expectation value for the kinetic energy, $p^2/2m$.

   (If you can't evaluate the integrals, leave them in unevaluated form for full credit.)

3. A neutral K particle at rest can change spontaneously into its own antiparticle, the $\overline{K}$. We can represent the state of the system by the vector

$$\psi = \begin{pmatrix} P \\ A \end{pmatrix}$$

   where P is the amplitude to be a K and A is the amplitude to be a $\overline{K}$. The Hamiltonian for this system is

$$\hat{H} = \begin{pmatrix} M & \epsilon \\ \epsilon & M \end{pmatrix}$$

   where M is the rest energy of the K and the $\overline{K}$ and ϵ is a small, real energy.

   (a) Find the eigenvalues, M_1 and M_2, and eigenvectors, ψ_1 and ψ_2, of the Hamiltonian. Be sure to normalize the eigenvectors.

   (b) Find the time evolution of ψ assuming that $P = 1$ and $A = 0$ initially, i. e., the system is in a pure "particle" state.

   (c) The "particleness" operator, which has the value 1 if we have a particle and -1 if we have an antiparticle, is (in the original frame)

$$\hat{\Pi} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

   Determine whether $\hat{\Pi}$ commutes with the Hamiltonian.

   (d) Determine the expectation value of $\hat{\Pi}$ as a function of time.