Physics 340 — Final Exam — Spring 1996

Reminder sheet allowed. **Show all work!**

1. A particular kind of wave has a dispersion relation

$$\omega = \alpha k + \beta / k$$

   where α and β are constants, k is the wavenumber, and ω is the frequency.

   (a) Compute the phase speed.

   (b) Compute the group velocity.

   (c) Sketch how a wave packet with this dispersion relation evolves with time for a central wavenumber $k = (\beta/\alpha)^{1/2}$, paying particular attention to the movement of the phase waves through the wave packet.

2. Compute the following commutation relations, showing your work:

   (a) $[p_x, x^3]$.

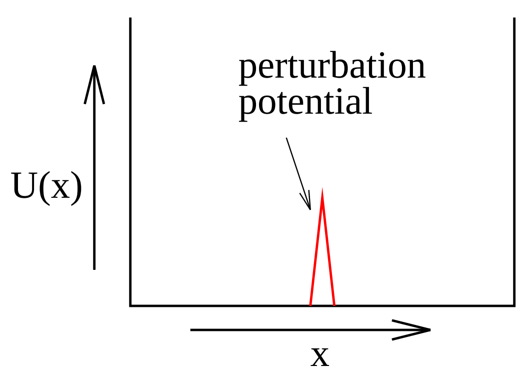
   (b) $[p_x, y^3]$.

   (c) $[L_z, L_y]$, where $L_z = xp_y - yp_x$ and $L_y = zp_x - xp_z$.

3. An infinite square well, as seen below, with dimensions $0 < x < 1$ contains a particle of mass $M = 1$. (Assume $\hbar = 1$.)

   (a) Compute the energies and eigenfunctions for the first three states, assuming a pure square well with no perturbation potential.

   (b) A perturbation potential of the form $U(x) = 0.1\delta(x - 1/2)$ is added to the system, where δ is the Dirac delta function ($\delta(x) = 0$ if $x \neq 0$, $\int \delta(x)dx = 1$). Compute the resulting perturbed energies of the first three states according to first order perturbation theory.



4. Weird atomic structure:

   (a) Suppose the electron had spin 3/2. What would the atomic number of the first noble gas (the helium analog) be in this case? Explain.

   (b) Suppose the electron had spin 1. What would atomic structure be like in this world?

5. The neutral kaon is not its own antiparticle like the photon, but it can be transformed into its antiparticle by the so-called strange force. If A_p is the amplitude to be a particle and A_a is the amplitude to be an antiparticle, then the governing equation for the particle/antiparticle system at rest is

$$i\frac{\partial}{\partial t}\begin{pmatrix} A_p \\ A_a \end{pmatrix} = \begin{pmatrix} M & \epsilon \\ \epsilon & M \end{pmatrix}\begin{pmatrix} A_p \\ A_a \end{pmatrix},$$

   where M is the kaon mass and ϵ is a constant having to do with the strange force. (Note: $\hbar = c = 1$ here.)

   (a) Find the two energy eigenvalues and the normalized eigenvectors of this system.

   (b) What is the mass difference, ΔM, between these two eigenstates?

   (c) Solve for A_p and A_a as a function of time, assuming that $A_p = 1$ and $A_a = 0$ at $t = 0$. How long does it take before the particle turns into the antiparticle?

6. The figure below shows the joint probability function for two particles in an infinite square well, one in the first excited state, the other in the second excited state. One of the pictures shows the case of two non-identical particles, one shows two identical Fermions, and one shows two identical Bosons. Which is which and why? (Vertical hatching shws where the probability is large. x_1 and x_2 are the positions of particle 1 and particle 2.)