Physics 340 — Final Exam — Spring 1995

Conditions: This is a take home test. It is due Wednesday 10 May 1995 by 5:00 PM in my physics office box. You can use your notes, the book, or any resource in the library or on the Internet. However, you may not communicate with anybody except the instructor of the course in conjunction with this test.

1. Explain the role of the following effects or experiments in the development of quantum mechanics.
   (a) Photoelectric effect.
   (b) Atomic spectra.
   (c) Stern-Gerlach experiment.
   (d) Black body radiation.
   (e) Franck-Hertz experiment.
   (f) Davisson-Germer experiment.

2. Suppose that e_i, $i = 1, n$ form orthonormal column vectors in an n-dimensional space.
   (a) Show that $X = \sum_{i=1}^{n} e_i e_i'$ is the identity matrix, where e_i' is the complex conjugate transpose of e_i.
   (b) Show that XA resolves an arbitrary column vector A into its components with respect to the coordinate system defined by the e_i.

3. Consider the scaled one-dimensional Hamiltonian
$$H = -\frac{1}{2}\frac{\partial^2}{\partial x^2} - 2\left\{\exp[-(x-\sigma)^2] + \exp[-(x+\sigma)^2]\right\}.$$
   (a) Find (using rlab) the energies of the lowest and second lowest states as a function of σ for $0 \le \sigma \le 3$. Plot the energies of these two states vs σ on the same graph.
   (b) Explain the dependence of these energies on σ.

4. Construct the matrices representing L^+, L^-, L_x, L_y, L_z, and L^2, for $l = 3/2$. Demonstrate explicitly that $[L_x, L_y]$, $[L_y, L_z]$, and $[L_z, L_x]$ obey the general rules for these commutators. (You may find rlab useful. Also, for simplicity, set $\hbar = 1$.)

5. Imagine a one-dimensional harmonic oscillator potential with classical frequency ω. Compute the total energy of this system as a function of the number of identical particles (up to 5) in the potential well, assuming that they have
   (a) spin 1/2;

(b) spin 0.

Assume the particles are in the lowest possible energy states and don't have any potential energy of interaction with each other.