Physics 222 – Test 4 – Spring 2012

One-page reminder sheet allowed. Constants: Boltzmann's constant: $k_B = 1.38 \times 10^{-23}$ J K^{-1}; Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}; thermal frequency constant: $K = 3.67 \times 10^{11}$ s^{-1} K^{-1}. *Show all work – no credit given if work not shown!*

1. The earth's interior increases in temperature downward from the surface, with the rate of increase being 22 K km^{-1}.

   (a) Assuming that the thermal conductivity of the earth is approximately the same as brick, or $\kappa \approx 0.5$ W m^{-1} K^{-1}, compute the energy per unit area per unit time, F_e, being conducted upward out of the earth.

   (b) Assuming no internal heat source in the earth (not really a good assumption, as radioactive decay supplies significant heat), use the above result for F_e to compute the average cooling rate of the earth in units of degrees Kelvin per million years. The specific heat of the earth is about 400 J kg^{-1} K^{-1}, its average density is about 6×10^3 kg m^{-3}, and its radius is about 6300 km.

2. The solar energy per unit area per unit time reaching the earth's orbit is about 1360 W m^{-2}. Assuming that the earth absorbs all of the solar radiation incident on it and that this energy is re-radiated to space as black body radiation, compute the earth's surface temperature. Hint: The effective area of absorption of solar radiation is 1/4 the area over which re-radiation occurs. Explain why.

3. A system with N degrees of freedom has energy E and $\Delta\mathcal{N} = AE^N$ available states, where A is a constant.

   (a) Compute the entropy of the system. From this compute the temperature T in terms of E and N.

   (b) Rewrite $\Delta\mathcal{N}$ in terms of T and N.

   (c) If the temperature of the system increases from T_1 to T_2, derive an equation for the ratio of available states $\Delta\mathcal{N}_2/\Delta\mathcal{N}_1$ at the two temperatures.

   (d) If $T_1 = 300$ K, $T_2 = T_1 + \delta T$ where $\delta T = 10^{-6}$ K, and $N = 10^{23}$, compute the natural log of $\Delta\mathcal{N}_2/\Delta\mathcal{N}_1$. Hint: You may find the approximation $\ln(1+\epsilon) \approx \epsilon$ for $|\epsilon| \ll 1$ to be useful. (TEST CONTINUED ON OTHER SIDE.)

4. A tank of compressed helium in a vacuum has temperature T. Assume that helium behaves like an ideal gas.

   (a) If the valve on the tank is opened slightly, so that the helium escapes the tank, is the temperature of the helium coming out greater than, less than, or equal to T? Hint: Is the escaping gas doing any work on the valve? Explain.

   (b) If the escaping helium drives a turbine that is connected to an electric generator that in turn lights a light bulb, is the temperature of the helium as it leaves the turbine greater than, less than, or equal to T? Explain.

5. Heat source A produces Q joules of heat per unit time at temperature $T_A = 800$ K, while heat source B produces $2Q$ joules of heat per unit time at $T_B = 400$ K. The heat in each case is used to drive a Carnot engine, each with an output temperature of $T_C = 300$ K.

   (a) Determine what fraction of the heat input is converted to useful work by the Carnot engine in each case.

   (b) Compute the useful work produced per unit time in each case.