Physics 222 – Test 4 – Spring 2010

One-page reminder sheet allowed. *Show all work – no credit given if work not shown!* Note that $k_B = 1.38 \times 10^{-23}$ J K^{-1}, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}, and $c = 3 \times 10^8$ m s^{-1}.

1. Your insulated bottle contains 0.5 kg of water at a temperature of 20° C. What mass of ice (at 0° C) must be added to bring the water (including the melted ice) to 0° C? Hint: The specific heat of water is about 4200 J K^{-1} kg^{-1}. It takes about 3.3×10^5 J kg^{-1} to melt ice at 0° C, leaving water at 0° C.

2. A closed container contains an ideal gas at room temperature (300 K) and pressure (10^5 Pa). This is a very special container which can withstand very high temperatures and pressures.

   (a) Derive an equation for the pressure of the gas as a function of temperature. Hint: Ideal gas law.

   (b) Derive an equation for the pressure on the interior walls of the container at temperature T due to black body radiation.

   (c) Determine the temperature at which the gas pressure and the radiation pressure are equal.

3. Explain qualitatively why the entropy of a closed system never decreases using statistical mechanical arguments. In this explanation you will need to give the statistical mechanics definition of entropy.

4. Heat conduction between reservoirs:

   (a) Determine how much entropy is added to a thermal reservoir at temperature T if heat ΔQ is added to it.

   (b) From this, show that spontaneous heat conduction from a reservoir at a lower temperature T_1 to a reservoir at a higher temperature T_2 violates the second law of thermodynamics.

5. Consider a rapidly tumbling, nearly spherical asteroid of radius R subject to a solar flux of 900 W m^{-2}. If 85% of the sun's flux is absorbed (the rest is reflected) and then re-radiated as black body radiation (i. e., with emissivity $\epsilon = 1$), determine the steady-state temperature of the asteroid, i. e., energy in equals energy out. Hints: The solar radiation intercepted by the asteroid is equal to its cross-sectional area times the solar flux, whereas the surface available for re-radiation is the surface area of the asteroid. Because it is tumbling randomly, each patch of surface receives the same amount of solar radiation on the average. Does your result depend on R?

6. A hurricane can be thought of as a heat engine which receives heat from the ocean surface ($T = 300$ K) and ejects heat at the temperature of the tropopause ($T = 200$ K).

   (a) Determine the maximum fraction of the heat received from the ocean surface which could be converted into useful work, e. g., by creating winds.

   (b) If global warming both increases the surface temperature and decreases the tropopause temperature, what is the likely effect on the maximum possible intensity of hurricanes?