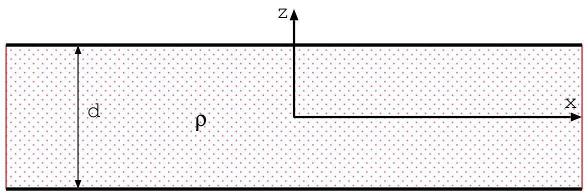
Physics 222 – Test 2 – Spring 2012

One-page reminder sheet allowed. Constants: $\epsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}; $\mu_0 = 4\pi \times 10^{-7}$ N s^2 C^{-2}; $c = 3 \times 10^8$ m s^{-1}; $\hbar = 1.06 \times 10^{-34}$ J s; $G = 6.67 \times 10^{-11}$ m^3 s^{-2} kg^{-1}. *Show all work – no credit given if work not shown!*

1. An infinite sheet of charge of finite thickness d has volume charge density $\rho > 0$ as shown below. Compute the direction and magnitude of the electric field inside the sheet as a function of the distance from the center of the sheet.



2. A lightning stroke lowers $q = -10$ C of (negative) charge vertically to the ground in $t = 10^{-3}$ s. Work out your answers in terms of symbols before inserting numbers!

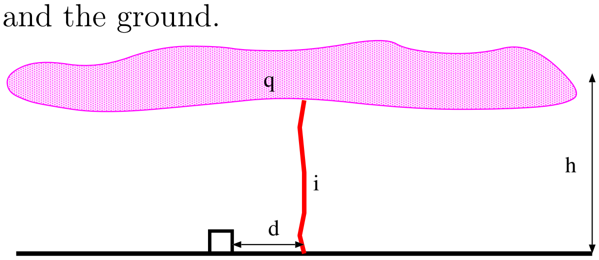
   (a) Compute the direction and approximate magnitude of the current i during the lightning stroke.

   (b) Compute the direction and magnitude of the magnetic field resulting from this current a distance $d = 100$ m from the stroke during the discharge.

   (c) If the charge in the cloud previous to the stroke is spread over an area of $S = 25$ km^2 at an elevation of $h = 1$ km above the surface, compute the direction and magnitude of the electric field between the cloud and the ground.

   (d) Compute the potential difference between the cloud and the ground before the lightning stroke.

   (e) Compute the energy stored in the electric field in the volume between the cloud and the ground.



3. An electromagnetic wave propagating in the $+z$ direction takes the form

$$\phi = 0; \qquad \boldsymbol{A} = (A_0 \cos(kz - \omega t), A_0 \sin(kz - \omega t), 0)$$

   where A_0 is a constant, ω is the angular frequency, and k is the wavenumber.

   (a) Compute the electric field $\boldsymbol{E}$ of the wave.

   (b) Compute the magnetic field $\boldsymbol{B}$ of the wave.

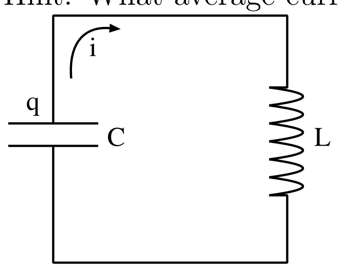
(c) Show that $\boldsymbol{E}$ and $\boldsymbol{B}$ are perpendicular to each other and to the direction of wave propagation.

(d) Sketch the change with time of the electric field vector in the x-y plane at $z = 0$ over one wave period.

4. The circuit shown below (called a tank circuit) starts out with charge q on the capacitor C and zero current through the inductor L.

   (a) Describe the qualitative behavior of the voltage across the capacitor, the charge on the capacitor, and the current through the inductor with time. You may want to sketch plots with these quantities on the vertical axis and time on the horizontal axis.

   (b) If the initial energy in the capacitor $U_E = q^2/(2C)$ is transferred totally to the inductor $U_B = Li^2/2$, estimate the time scale τ on which this transfer occurs. Hint: What average current i is required to discharge the capacitor in time τ?



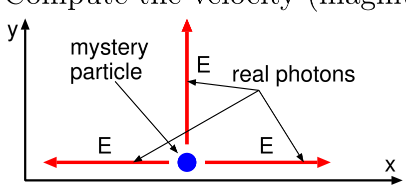
5. A mystery particle decays into three real photons, each with energy E, as shown below. One photon moves in the $+x$ direction, one in the $-x$ direction, and one in the $+y$ direction. You may work in units in which $c = 1$.

   (a) Compute the total energy of the mystery particle. (Remember, energy is a scalar!)

   (b) Compute the momentum of the mystery particle. (Remember, momentum is a vector and the individual momenta need to be added vectorially!)

   (c) Compute the mass of the mystery particle.

   (d) Compute the velocity (magnitude and direction) of the mystery particle.



6. Find α, β, and γ such that $\hbar^\alpha c^\beta G^\gamma$ has the units of mass. Compute the numerical value of this mass, which is called the *Planck mass*.