Special problem 1 for chapter 21.

1. Isotope A decays into isotope B with decay rate λ_A, which decays into isotope C with decay rate λ_B.

   (a) If A, B, and C are the respective numbers of nuclei of these three isotopes, show that the governing equations for these quantities are

   $$\frac{dA}{dt} = -\lambda_A A \qquad \frac{dB}{dt} = \lambda_A A - \lambda_B B \qquad \frac{dC}{dt} = \lambda_B B.$$

   (b) If A_0 is A at zero time, derive an equation for $A(t)$. Hint: Try a solution of the form $A \propto \exp(ct)$ where c is a constant to be determined.

   (c) If $B = 0$ at zero time, derive an equation for $B(t)$. Hint: Try a solution of the form $B = a\exp(-\lambda_A t) + b\exp(-\lambda_B t)$ and determine the constants a and b in terms of A_0, B_0, λ_A, and λ_B.

   (d) If $\lambda_B \gg \lambda_A$, determine the ratio B/A for large times ($\lambda_B t \gg 1$).

   (e) Show that $A + B + C$ doesn't change with time.

   (f) If $C = 0$ at zero time, determine $C(t)$ using the above results.

2. An isotope has a nuclear gamma decay mode with half-life of 10^{-10} sec in which the emitted photon has energy 1 MeV.

   (a) From this half-life, compute the uncertainty in the energy of the emitted photon.

   (b) From the above result, compute the uncertainty in the frequency.

   (c) Compute the Doppler shift velocity that results in a frequency shift equal to the above uncertainty in the frequency. Hint: This velocity is small, justifying the approximation that $\omega' = \omega(1 + v/c)$.