Physics 221 – Test 3 – Fall 2009

One-page reminder sheet allowed. *Show all work – no credit given if work not shown!*

1. Sally is swinging on a swing and George is standing next to the swing. (Don't consider the effects of general relativity here.)

   (a) Determine the direction of the acceleration experienced by Sally from George's point of view when Sally is at the bottom of the swing's arc.

   (b) Determine the direction of the inertial force experienced by Sally at the bottom of the swing's arc.

2. Quantum jumping beans. Assume that $\hbar = 1$ J s and $g = 10$ m s^{-2}.

   (a) A pinto bean of mass M is confined to a box of size a. Use the uncertainty principle to derive an approximate formula for its momentum, and hence its kinetic energy.

   (b) Íf the bean initially jumps off the surface, moving upward with the above kinetic energy, derive a formula for how high H it goes, using conservation of energy.

   (c) If the "box" is really the vertical region to which the bean is confined by gravity given the above initial kinetic energy, we can equate a to H. If the mass of the pinto bean is 10^{-4} kg, how high does it jump as a result of the uncertainty principle?

3. Quantum world with $\hbar = 100$ J s: Two of the front doors of MSEC are open as your wave packet enters enroute to physics 221 in MSEC 101. The door to this classroom is open as well. For some reason you cannot manage to enter MSEC 101 in spite of repeated tries.

   (a) Explain what quantum mechanical process is likely occurring here.

   (b) Devise at least two quantum mechanical solutions to your problem which increase your probability of entering the classroom.

4. A particle of mass M moves non-relativistically along the x axis with potential energy $U(x) = Ax^4 - Bx^2$, where A and B are positive constants.

   (a) Find the force on the mass as a function of x due to this potential energy.

   (b) Make a qualitative sketch of the potential energy function and show where the classically allowed regions and the turning points are if the total energy of the mass is slightly less than zero.

   (c) Derive an equation for the speed of motion of the mass, assuming that the total energy E is known. Use this to show how the wavelength of the matter wave associated with the particle varies with position.

5. A sky diver of mass 80 kg drops 500 m under the influence of his parachute, reaching a final speed of 10 m s^{-1} after exiting the airplane at 70 m s^{-1}. (The acceleration of gravity is 10 m s^{-2}. The sky diver hasn't reached the surface yet.)

   (a) Compute the work done by gravity on the sky diver (in the earth frame).

   (b) Compute the total work done on the sky diver.

   (c) Compute the work done by air friction.

6. Rotational symmetry: An object has rotational symmetry about the vertical axis, i. e., it looks the same no matter what angle θ it is rotated through.

   (a) What form does the wavefunction $\psi(\theta)$ take?

   (b) A rotation through an angle of 2π should leave the wavefunction completely unchanged. What condition does this impose on the wavefunction?

7. Compute the quantum mechanical energy levels of a mass M moving at relativistic speeds in a box of size a. Include the rest energy in your total energy.