Physics 221 – Final Exam – Fall 2009

One-page reminder sheet allowed. *Show all work – no credit given if work not shown!*

1. The dispersion relation for a particular type of wave is $\omega = a\sin(bk)$ for $0 < k \leq \pi/b$, where ω and k are the frequency and the wavenumber and a and b are positive constants.
   (a) Determine for what value of k the phase speed of the wave is zero.
   (b) Determine for what value of k the group velocity of the wave is zero.

2. Light of wavelength λ is normally (perpendicularly) incident on a flat soap bubble film with film thickness d and index of refraction n.
   (a) Determine for what values of d constructive interference occurs between reflections off of the front and back surfaces.
   (b) Explain why destructive interference occurs between these reflections when $d \ll \lambda$.

3. Sirius is 8 ly distant from the sun. A spaceship leaves the solar system enroute to Sirius at speed 80% of the speed of light.
   (a) Draw a spacetime diagram in which the Sun and Sirius are stationary, showing the two stars' world lines, the world line of the spaceship, and the spaceship's line of simultaneity.
   (b) Determine the slopes of the world line and line of simultaneity of the spaceship in this diagram.
   (c) Compute the time experienced by the spaceship enroute to Sirius from the solar system in (1) the Sun's (and Sirius's) frame, and (2) the spaceship's frame.

4. The dispersion relation of relativistic matter waves is $\omega^2 = k^2c^2 + \mu^2$ where μ is a positive constant proportional to the associated particle's mass.
   (a) Show that the phase speed of these waves is greater than the speed of light.
   (b) Show that the group velocity of these waves is less than the speed of light.

5. Think of an atomic nucleus as a box with dimension $a = 10^{-15}$ m. A proton has mass 1.7×10^{-27} kg. Recall that $\hbar = 1.06 \times 10^{-34}$ J s. Hint: You may use either the uncertainty principle or particle-in-box theory.
   (a) Approximately what momentum will the proton have?
   (b) Compute the approximate kinetic energy of the proton.
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6. Assume that a particle of mass M has potential energy $U(x) = -U_0 \cos(kx)$ where U_0 and k are positive constants.

    (a) Compute the force on the mass due to this potential energy.

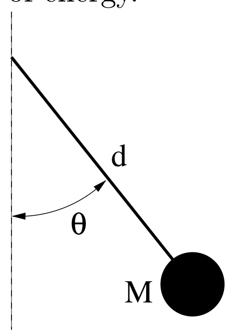
    (b) If the total energy of the mass is zero, find the turning points closest to the origin.

    (c) If the total energy of the mass is $+U_0$, find the particle speed at the origin.

7. Pendulum of length d with mass M:

    (a) Compute the torque about the pendulum pivot point on the mass M as a function of θ as shown below.

    (b) Compute the minimum angular momentum of the mass about the pivot point needed at $\theta = 0$ for the pendulum to swing up to $\theta = \pi/2$. Hint: Use conservation of energy.



8. Imagine a mass-spring system as shown below with an additional "shock absorber" force applied to the mass of the form $F_d = -bv$ where b is a positive constant and $v = dx/dt$ is the velocity of the mass.

    (a) Show that the differential equation for this oscillator is

$$\frac{d^2x}{dt^2} + \frac{b}{M}\frac{dx}{dt} + kx = 0.$$

    (b) Try a solution of the form $x = x_0 \exp(-i\omega t)$ and determine ω.

    (c) The real part of x is the physical solution. Take the real part and show that Real$[x(t)]$ takes the form of an oscillation which decays in amplitude with time.