The Spectral Weak Temperature Gradient Approximation*

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What is WTG?

The weak temperature gradient approximation is a mathematical simplification based on observations of the tropics.

WTG (founded on that observation) is a method of parameterizing the environment surrounding the domain of a numerical atmospheric model.
Motivation
Science questions for the tropical atmosphere:

Q) What initiates tropical convection?

Q) What conditions favor tropical precipitation?

Q) Why is tropical convection often clustered?

Q) What intensifies a tropical cyclone?
Motivation

We want to know why it rains in the tropics. We can:

1) Observe the tropical atmosphere
2) Model the tropical atmosphere
We want to know why it rains in the tropics. We can:

1) Observe the tropical atmosphere
   but the real atmosphere is very complicated

2) Model the tropical atmosphere
   but models lack many real phenomena

Observation $\leftrightarrow$ Modeling
Motivation: Choose modeling

We can model:

1) entire tropics

2) a limited area (simplified model)
Motivation: Choose modeling

We can model:

1) entire tropics
   
   expensive, time-consuming and complicated

2) a limited area (simplified model)
   
   lacking many real phenomena

Complex models $\leftrightarrow$ Simple models
Motivation: Choose limited-area model

We can model 1, 2, or 3 dimensions
Motivation: Choose limited-area model

We can model 1, 2, or 3 dimensions

Limited-domain models are discrete

Missing large-scale flows and local thermodynamics
Motivation: Problem

In a real limited-area, a domain-averaged vertical velocity, $w_e$, as well as a large-scale horizontal velocity, $\vec{u}_e$, may occur.

Mass continuity $\implies$ the model itself cannot generate this flow via its equations of motion.
Motivation: Problem

Again, we have (at least) a couple of options:

1) Impose fixed $w_e$, $\tilde{u}_e$ from observation.

2) Let $w_e$, $\tilde{u}_e$ be parameterized by model itself.
Motivation: Problem

Again, we have (at least) a couple of options:

1) Impose fixed $w_e$, $\tilde{u}_e$ from observation.
   * also specifies the rain rate
   * large temperature errors
   * causality issues

2) Let $w_e$, $\tilde{u}_e$ be parameterized by model itself.
   * WTG is one method
Q) Why does it rain in the tropics?

Use model (data is complicated)

Use limited-area model (large models are expensive and complicated)

But we need the influence of the surrounding environment!

Use WTG (specifying vertical velocity can be troublesome)
Literature
Primitive horizontal momentum equation:

\[ t \sim T \quad \nabla \sim 1/L \quad \vec{u} \sim U \]

\[ \frac{D\vec{u}}{Dt} + \vec{f} \times \vec{u} = -\frac{\nabla p}{\rho} \]

\[ T \geq L/U \quad H = \frac{p}{\rho g} \]
Charney (1963)

\[ H = 10 \text{ km} \quad L = 1000 \text{ km} \quad U = 10 \text{ ms}^{-1} \]

\[ \frac{\delta p}{\rho} \sim \frac{U^2}{gH} \approx 1 e^{-3} \]

\[ \delta p \approx 0.1\% \implies \]

temperature fluctuations are small in the tropics
The weak temperature gradient

Zonal gradients in W. Pacific, JJA 2013 (700 mb)

temperature gradient  precipitation gradient
Q) Why is tropical $\nabla T$ small, but $\nabla R$ isn’t?

- Simplified circulation model
- Parameterized environment using WTG
- Compared rain climatology with model
Sobel and Bretherton (2000) "strict" WTG

\[ \frac{d\theta}{dt} = Q_d \text{ (diabatic heating)} \]
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\[ \frac{\partial \theta}{\partial t} + \mathbf{\hat{u}}_h \cdot \nabla_h \theta + w \frac{\partial \theta}{\partial z} = Q_d \]
Sobel and Bretherton (2000) "strict" WTG

\[ \frac{d\theta}{dt} = Q_d \text{ (diabatic heating)} \]

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\frac{d\theta}{dt} = Q_d \quad \text{(diabatic heating)}
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\frac{\partial \theta}{\partial t} + \hat{u}_h \cdot \nabla_h \theta + w \frac{\partial \theta}{\partial z} = Q_d
\]
\[
W = Q_d / \frac{\partial \theta}{\partial z} \quad (w \rightarrow w_{\text{wtg}})
\]
Procedure:

- Set SST, radiation to climatology
- Ran model to equilibrium ($\frac{\partial \theta}{\partial t} = 0$)
- Fixed horizontal winds ($\overline{u} \rightarrow u_e$)
- Fixed $T$ to horizontal mean ($\nabla_h \cdot \theta = 0$)
- “Strict” WTG
Results: rainfall similar to climatology:

January Climatology

Model Output

(modified from Sobel and Bretherton, 2000)
Q) Why doesn’t deep moist convection homogenize tropospheric moisture?

- Hypothesis: gravity waves act faster than mixing
- 2D numerical model of convection
- Compared gravity wave theory w/ model
Linearized, hydrostatic, Boussinesq buoyancy:

\[ b = b_0 \sin(mz) \left[ \delta(x - ct) + \delta(x + ct) \right] \]

\[ c = N/m \]

\[ m = \pi/h \]

\[ b(x, t): \text{pulse at } x = |d| \text{ when } t = |d|/c \]
Q) Why does it rain in the tropics?

- Hypothesis: $rain = f(s_{surfaceflux} - s_{radiation})$
- 2D cloud resolving model (CRM)
Wind field is decomposed:

\[ \frac{\partial \theta}{\partial t} + \vec{v} \cdot \nabla \theta = Q_d \]
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\[
\frac{\partial \theta}{\partial t} + \vec{v}_{\text{grid}} \cdot \nabla \theta + \vec{v}_{\text{ext}} \cdot \nabla \bar{\theta} = Q_d
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\[ \frac{\partial \theta}{\partial t} + \vec{v}_{\text{grid}} \cdot \nabla \theta = Q_d - \mathcal{W}_{\text{ext}} \frac{\partial \bar{\theta}}{\partial z} \]
Wind field is decomposed:

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\[ \frac{\partial \theta}{\partial t} + \vec{v}_{\text{grid}} \cdot \nabla \theta = Q_d - w_{\text{ext}} \frac{\partial \bar{\theta}}{\partial z} \]

\[ w_{\text{ext}} \frac{\partial \bar{\theta}}{\partial z} = (\bar{\theta} - \theta_0) / \tau \]
Again, we derive $w_{\text{wtg}}$:

$$w_{\text{wtg}} = (\bar{\theta} - \theta_0)/(\tau \frac{\partial \bar{\theta}}{\partial z})$$
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$\tau \to 0$ : $w_{\text{wtg-relaxed}} \to w_{\text{wtg-strict}}$
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$$c = Nh/\pi \approx 50 \text{ m}\text{s}^{-1} \quad (h = 15\text{ km})$$
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$$w_{\text{wtg}} = (\bar{\theta} - \theta_0) / (\tau \frac{\partial \bar{\theta}}{\partial z})$$

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$$c = Nh/\pi \approx 50 \text{ ms}^{-1} \quad (h = 15 \text{ km})$$

$$\tau = d/c \quad (d = 300 \text{ km, arbitrary})$$
Obtaining a reference profile

- Radiation
- Surface fluxes
- Model domain
- Reference

Symbols:
- $Z$ (domain)
- $T$ (temperature)
- $q$ (humidity)
WTG experiment

radiation

model domain

surface fluxes

reference

T  q
WTG experiment

- Model domain
- Radiation
- Surface fluxes
- Reference

\( Z \)

\( T \quad q \)
WTG experiment

model domain

radiation

surface fluxes

reference

\( T \)

\( q \)
WTG experiment

Domain:
- Surface fluxes
- Radiation

Model:
- Domain

Reference:
- $T$ (Temperature)
- $q$ (Other parameter)
Results:

Raymond and Zeng (2005) “relaxed” WTG

Rain and entropy flux

Vertical mass flux
What is $d$?

The typical width of a convective cell
What is d?

The typical spacing of convective cells
What is d?

The width of the domain
Q) How does convection adjust to changes in its environment?

- Hypothesis: via gravity wave adjustment
- 3D numerical model of convection
- Applied step-functions in radiative cooling
- Diagnosed the adjustment time
Conclusions:

- Adjustment ($\tau_s$) depends on cloud spacing $s$
- Projected mass flux to find weights on $c_j$
  \[ \sum_j w_j c_j \propto s/\tau_s \]
- Gravity waves describe adjustment time

Perhaps $d \propto s$?
Spectral WTG
Q) Can we improve the adjustment mechanism?

- Similar to relaxed WTG (RZ05)
- Project heating anomaly onto sine series
- Assign unique time ($\tau$) to each mode
Relaxed WTG:

\[ w_{wtg}(z) = \frac{\bar{\theta}(z) - \theta_0(z)}{\tau(d\bar{\theta}/dz)} \]

\[ \tau = \tau(\pi/h) \]
Relaxed WTG:

\[ w_{w tg}(z) = \frac{\bar{\theta}(z) - \theta_0(z)}{\tau(d\bar{\theta}/dz)} \]

\[ \tau = \tau(\pi/h) \]

But we’d rather have:

\[ \tau_j = \tau(m_j) \]

\[ m_j = j\pi/h \quad j = 1, 2, 3, \ldots \]
Expand RHS in Fourier sine series:

\[
\frac{\bar{\theta}(z) - \theta_0(z)}{(d\bar{\theta}/dz)} = \sum_j \Theta_j \sin(m_j z)
\]
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\[
\Theta_j = \frac{2}{h} \int_0^h \frac{\bar{\theta} - \theta_0}{(d\bar{\theta}/dz)} \sin(m_j z) \, dz
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Expand RHS in Fourier sine series:

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\frac{\bar{\theta}(z)-\theta_0(z)}{(d\bar{\theta}/dz)} = \sum_j \Theta_j \sin(m_j z)
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\Theta_j = \frac{2}{h} \int_0^h \frac{(\bar{\theta}-\theta_0)}{(d\bar{\theta}/dz)} \sin(m_j z) \, dz
\]

\[
W_{\text{wtgs}} = \sum_j \frac{\Theta_j \sin(m_j z)}{\tau_j} \quad \tau_j = d/c_j = dm_j/N
\]
Spectral WTG: Fourier coefficients

\[ m_j = j\pi / h \]

WTG experiment (extra surface forcing)
Spectral WTG: PBL treatment

Conventional WTG:

\[ z_0 \approx 1 \text{ km} \quad \text{(PBL top)} \]

\[ w_{wtg_{base}} = w_{wtg}(z_0) \]

\[ w_{wtg}(z < z_0) = \left( \frac{z}{z_0} \right) w_{wtg_{base}} \]

Spectral WTG:

No special treatment
Multiple equilibria

$q(0)$ $\rightarrow$ $q'$

model
domain

reference
Multiple equilibria

WTG

SWTG
Linear, hydrostatic Boussinesq buoyancy:

\[ b(z, 0) = b_0 \sin(m_j z) \]
Q) How does increased stability affect cyclogenesis?

- Used RZ05’s WTG scheme
- Changed stability the external environment
- Observed vertical mass flux ($\overline{w\rho}$).
Raymond and Sessions (2007)
Raymond and Sessions (2007)

Conclusion:

more stability $\implies$ more low-level convergence

conventional WTG

spectral WTG

(after Raymond and Sessions, 2007)
Conclusions

- SWTG is a modified form of relaxed WTG
- No PBL artifice is needed
- Multiple equilibria results are more robust
- SWTG improves transient response
- SWTG is smooth
- Mass flux profiles are stronger and less top-heavy
- SWTG less influenced by grid-scale numerical oddities