The Spectral Weak Temperature Gradient Approximation*

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The weak temperature gradient (WTG) approximation is a mathematical simplification based on observations of the tropics.

WTG is a way to model the environment surrounding the domain of a numerical atmospheric model.
Motivation
Motivation

Science questions for the tropical atmosphere:

Q) What initiates convection?
Q) Why does it rain?
Q) Why is convection often clustered?
Q) Why does a tropical cyclone form?
Motivation

We want to know why it rains in the tropics.

1) Observe the tropical atmosphere

2) Take a theoretical approach

3) Model the atmosphere
We want to know why it rains in the tropics.

1) Observe the tropical atmosphere
   but the real atmosphere is very complicated
2) Take a theoretical approach
   but we may oversimplify our assumptions
3) Model the atmosphere
   but models lack many real phenomena

Observation ←→ Modeling ←→ Theory
Motivation: Choose modeling

We can model:

1) entire tropics

2) a limited area (simplified model)
Motivation: Choose modeling

We can model:

1) entire tropics
   expensive, time-consuming and complicated

2) a limited area (simplified model)
   lacking many real phenomena

Complex models ←→ Simple models
Motivation: Choose limited-area model

We can model 1, 2, or 3 dimensions
Motivation: Choose limited-area model

We can model 1, 2, or 3 dimensions

Limited-domain models are discrete

Missing large-scale flows and local thermodynamics
Motivation: Problems with limited-domains

energy in $\rightarrow$ energy out
Motivation: Problems with limited-domains

radiative cooling and surface energy fluxes
Motivation: Problems with limited-domains

unstable atmosphere $\rightarrow$ convection
Motivation: Problems with limited-domains

ascent $\rightarrow$ inflow and outflow
Motivation: Problems with limited-domains

create a limited domain
Motivation: Problems with limited-domains lose influence of environment
Motivation: Problems with limited-domains

A surface wind must be imposed within
Motivation: Problems with limited-domains

cyclic BCs $\rightarrow$ winds turn inward
Motivation: Problems with limited-domains

rigid top/bottom $\rightarrow$ compensating subsidence
Motivation: Problems with limited-domains

rain $\rightarrow$ latent heating
Motivation: Problems with limited-domains

temperature anomaly cannot escape
External winds can help!

1) Impose external winds from observation.

2) Let winds be parameterized within model.
Motivation: Problems with limited-domains

External winds can help!

1) Impose external winds from observation.
   * also specifies the rain rate
   * large temperature errors

2) Let winds be parameterized within model.
   * WTG is one method
Development of WTG
Equations describing the atmosphere

\[ \frac{D\vec{u}}{Dt} = \frac{\vec{F}}{m} \quad \text{(Newton’s 2nd law)} \]

\[ \frac{D\rho}{Dt} = -\rho \nabla \cdot \vec{v} \quad \text{(mass conservation)} \]

\[ \frac{D\theta}{Dt} = Q \quad \text{(thermodynamic equation)} \]

\[ \theta = T \left( \frac{p_0}{p} \right)^\kappa \quad \text{(potential temperature)} \]
The weak temperature gradient

Zonal gradients in W. Pacific, JJA 2013 (700 mb)

-45 -30 -15 0 15 30 45
latitude (degree)

- σ
+ σ

-45 -30 -15 0 15 30 45
latitude (degree)

- σ
+ σ

temperature gradient  precipitation gradient
The weak temperature gradient

Q) How does tropical $\Delta T$ remain small?
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$$\vec{F}_{atmos} = \vec{F}_{\Delta p} + \vec{F}_g + \vec{F}_{Coriolis} + \vec{F}_{small}$$
The weak temperature gradient

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\[ \vec{F}_{\text{Coriolis}} \propto \sin(\text{latitude}) \]
The weak temperature gradient

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\[
\vec{F}_{\text{atmos}} = \vec{F}_{\Delta p} + \vec{F}_g + \vec{F}_{\text{Coriolis}} + \vec{F}_{\text{small}}
\]

\[
\vec{F}_{\text{Coriolis}} \propto \sin(\text{latitude}) \implies
\]

\[
\frac{Du}{Dt} = \vec{F}_{\Delta p} \quad \frac{Dw}{Dt} = \vec{F}_{\Delta p} + \vec{F}_g
\]

\[
\frac{Dw}{Dt} \approx \text{small} \implies \vec{F}_{\Delta p} \approx -\vec{F}_g
\]
Q) How does tropical $\Delta T$ remain small?

Assume small density variations and linearize:
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$$w' \propto \exp(ikx - i\omega t) \implies \omega = kN/m$$
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$$w' \propto \exp(ikx - i\omega t) \implies \omega = kN/m$$

$$c = \omega/k = N/m$$
The weak temperature gradient
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The weak temperature gradient

\[ +p' \]

\[ -p' \]
The weak temperature gradient

\[ +p' - p' + p' - p' \]
The weak temperature gradient
The weak temperature gradient

\[ +p' - p' + p' - p' \]
The weak temperature gradient
Q) Why does it rain in the tropics?

- Hypothesis: rain $\propto F_s(\text{surface}) - F_s(\text{top})$
- $s = s(T, p, r)$: moist entropy
- 2D numerical model
- Employed WTG approximation
Wind field is decomposed:

\[ \frac{D\theta}{Dt} = \frac{\partial \theta}{\partial t} + \vec{v} \cdot \nabla \theta = Q_{\text{diabatic}} \]
Raymond and Zeng (2005) "relaxed" WTG

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\[ \frac{\partial \theta}{\partial t} + \vec{v}_{\text{int}} \cdot \nabla \theta = Q_{\text{diabatic}} - \mathcal{W}_{\text{ext}} \frac{\partial \bar{\theta}}{\partial z} \]
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\]

\[
\mathcal{W}_{\text{ext}} \frac{\partial \overline{\theta}}{\partial Z} = (\overline{\theta} - \theta_0)/\tau
\]
The environmental velocity:

$$w_{wtg} = \frac{(\bar{\theta} - \theta_0)}{\tau(\partial \bar{\theta}/\partial z)}$$
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\[ \tau = d / c \quad (d = 300 \text{ km}) \]
The environmental velocity:

\[ W_{\text{wtg}} = \frac{(\bar{\theta} - \theta_0)}{\tau (\partial \bar{\theta} / \partial z)} \]

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\[ c = Nh / \pi \approx 50 \text{ ms}^{-1} \]

\[ h = 15 \text{ km}, \text{ depth of troposphere} \]
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Note: \( \frac{(\bar{\theta} - \theta_0)}{(\partial \bar{\theta} / \partial z)} \approx (\bar{\theta} - \theta_0) \frac{\Delta z}{\Delta \theta} = \Delta \theta \frac{\Delta z}{\Delta \theta} = \Delta z \theta \)
Problem: $T'$ cannot escape and no outside effects can enter box.
How does WTG help?

WTG vertical velocity cancels $T'$ via adiabatic lifting ($\Delta z_\theta$)
Mass continuity requires horizontal advection of "exterior" quantities.
Generating a reference profile

- **radiation**
- **model domain**
- **surface fluxes**
- **reference**

\[ Z \]

**T**  \[ q \]
WTG experiment

Z

model domain

radiation

surface fluxes

reference

T q
WTG experiment

- Radiation
- Surface fluxes
- Model domain
- Reference: T, q
WTG experiment

Z

model domain

radiation

surface fluxes

reference

T

q
WTG experiment

\[ Z \]

model domain

radiation

surface fluxes

reference \[ T \] \[ q \]
Raymond and Zeng (2005) "relaxed" WTG

Results:

- Rain and entropy flux
- Vertical mass flux
Spectral WTG
Q) Can we improve the adjustment mechanism?

- Similar to relaxed WTG
- Project heating anomaly onto sine series
- Assign unique timescale ($\tau$) to each mode
In relaxed WTG, we have:

\[ \tau = \tau(\pi/h) \]
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\[ \tau = \tau(\pi/h) \]

But we’d rather have:

\[ \tau_j = \tau(m_j) \]

\[ m_j = j\pi/h \quad j = 1, 2, 3, \ldots \]
spectral WTG

Expand velocity in sine series:

$$\Delta z_\theta = \frac{\bar{\theta}(z) - \theta_0(z)}{(\partial \bar{\theta}/\partial z)} = \sum_j \Theta_j \sin(m_j z)$$
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\[ \Theta_j = \frac{2}{h} \int_0^h \Delta z_\theta \sin(m_j z) \, dz \]
spectral WTG

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$$\Theta_j = \frac{2}{h} \int_0^h \Delta z_\theta \sin(m_j z) \, dz$$

$$W_{swtg} = \sum_j \frac{\Theta_j \sin(m_j z)}{\tau_j} \quad \tau_j = \frac{j \pi d}{Nh}$$
Romps (2012)

Scaled T anomaly (depth = 7.5 km)

Scaled T anomaly (depth = 15.0 km)

Scaled T anomaly (depth = 3.0 km)
Raymond and Sessions (2007)

conventional WTG

spectral WTG

(after Raymond and Sessions, 2007)
Conclusions

WTG
- A tool for researching tropical atmosphere
- Environmental changes on convection can be explored

spectral WTG
- A modified form of WTG
- Improves transient response
- Smooth
- Less influenced by grid-scale numerical oddities