Physics 121 – September 14, 2017

Assignments:

This week:

- Finish reading chapter 4 of textbook (note: we will NOT cover the section on nonuniform circular motion, pp. 191-193 of textbook)

- Make sure that your clicker or phone app is registered for this class: "PHYS121_Minschwaner_F2017" The course ID is "NMTphys121_Minschwaner"

- Complete ETA Problem Set #4 and chapter 4 written problems 30, 36, 59, 62, 76, due by Sept 18 at 4 PM

- Do practice problems in recitation this week

- Start reading Chapter 5

Key concepts for today:

- Uniform circular motion
- Centripetal acceleration
- Relative motion (again, stressing the independence of the x and y components of position, velocity, and acceleration)

Recall the equations for projectile motion from Tuesday

To describe projectile motion completely, we must include velocity and acceleration, as well as displacement. We must find their components along the x- and y-axes. Let's assume all forces except gravity (such as air resistance and friction, for example) are negligible. Defining the positive direction to be upward, the components of acceleration are then very simple:

$$a_y = -g = -9.8\ \mathrm{m/s^2}\ \ (-32\ \mathrm{ft/s^2}).$$

Because gravity is vertical, $a_x = 0$. If $a_x = 0$, this means the initial velocity in the x direction is equal to the final velocity in the x direction, or $v_x = v_{0x}$. With these conditions on acceleration and velocity, we can write the kinematic **Equation 4.11** through **Equation 4.18** for motion in a uniform gravitational field, including the rest of the kinematic equations for a constant acceleration from **Motion with Constant Acceleration**. The kinematic equations for motion in a uniform gravitational field become kinematic equations with $a_y = -g,\quad a_x = 0$:

Horizontal Motion

(4.19)

Vertical Motion

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t \qquad (4.20)$$

(4.21)

(4.22)

(4.23)

These equations on p. 134 of the textbook should be shaded!

First, we need to finish analyzing the “shoot the monkey” demo.

Then we’ll look at some of the shortcut equations (good only for specific kinds of problems).

We’ll wrap this topic up with a clicker question for “shoot the instructor” exercise!

These equations may be useful at times, but be careful because they can only be applied in specific instances!

$$T_{\text{tof}} = \frac{2(v_0 \sin\theta_0)}{g}. \qquad (4.24)$$

This is the **time of flight** for a projectile both launched and impacting on a flat horizontal surface. Equation 4.24 does not apply when the projectile lands at a different elevation than it was launched, as we saw in Example 4.8 of the tennis player hitting the ball into the stands. The other solution, $t = 0$, corresponds to the time at launch. The time of flight is linearly proportional to the initial velocity in the y direction and inversely proportional to g. Thus, on the Moon, where gravity is one-sixth that of Earth, a projectile launched with the same velocity as on Earth would be airborne six times as long.

Trajectory

The trajectory of a projectile can be found by eliminating the time variable t from the kinematic equations for arbitrary t and solving for $y(x)$. We take $x_0 = y_0 = 0$ so the projectile is launched from the origin. The kinematic equation for x gives

$$x = v_{0x} t \Rightarrow t = \frac{x}{v_{0x}} = \frac{x}{v_0 \cos\theta_0}.$$

Substituting the expression for t into the equation for the position $y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$ gives

$$y = (v_0 \sin\theta_0)\left(\frac{x}{v_0 \cos\theta_0}\right) - \frac{1}{2}g\left(\frac{x}{v_0 \cos\theta_0}\right)^2.$$

Rearranging terms, we have

$$y = (\tan\theta_0)x - \left[\frac{g}{2(v_0 \cos\theta_0)^2}\right]x^2. \qquad (4.25)$$

Not so useful, but…

This trajectory equation is of the form $y = ax + bx^2$, which is an equation of a parabola with coefficients

$$R = \frac{v_0^2 \sin 2\theta_0}{g}. \tag{4.26}$$

Note particularly that **Equation 4.26** is valid only for launch and impact on a horizontal surface. We see the range is directly proportional to the square of the initial speed v_0 and $\sin 2\theta_0$, and it is inversely proportional to the acceleration of gravity. Thus, on the Moon, the range would be six times greater than on Earth for the same initial velocity. Furthermore, we see from the factor $\sin 2\theta_0$ that the range is maximum at $45°$. These results are shown in **Figure 4.15**. In (a) we see that the greater the initial velocity, the greater the range. In (b), we see that the range is maximum at $45°$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is somewhat smaller. It is interesting that the same range is found for two initial launch angles that sum to $90°$. The projectile launched with the smaller angle has a lower apex than the higher angle, but they both have the same range.

This figure is a great summary of how projectile motion depends on initial velocity and elevation angle

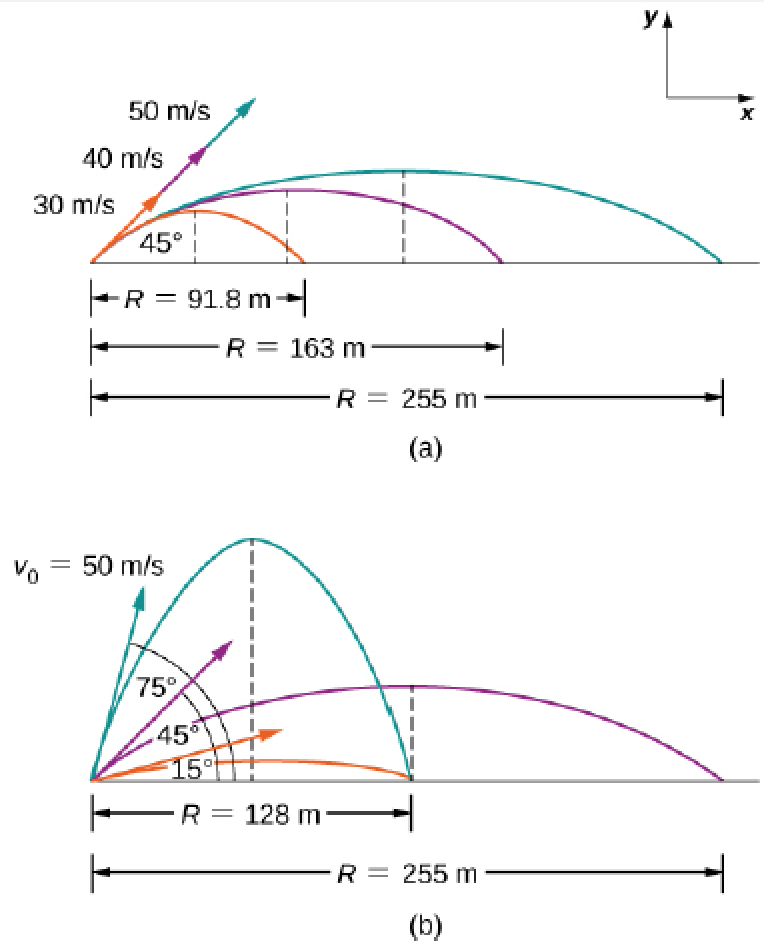


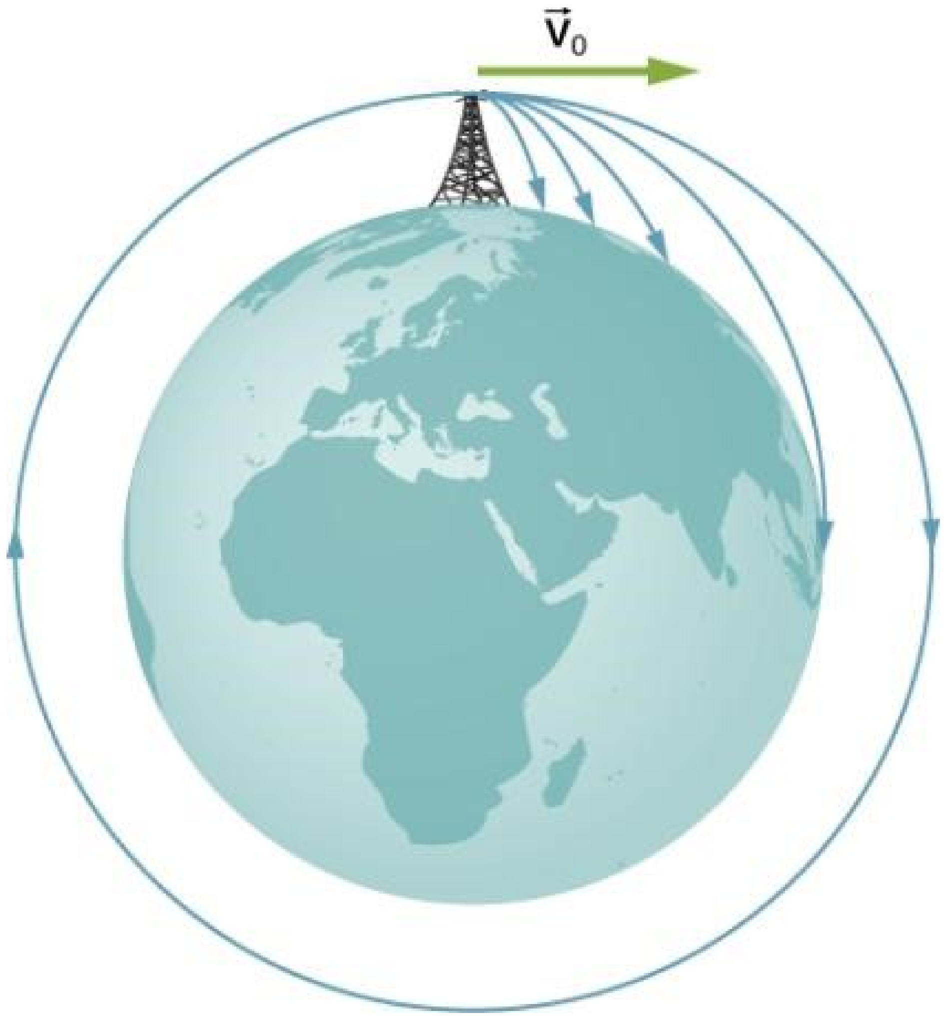
Figure 4.15 Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that the range is the same for initial angles of $15°$ and $75°$, although the maximum heights of those paths are different.

Clicker Question (3-minute response, you may discuss):

Consider a small projectile shot from a catapult at a target located a horizontal distance 10 m from the catapult. If the initial speed of the projectile is 13 m/s, at what elevation angle should the projectile leave the catapult?
(Note – answer with largest response will be tested on a live target….)

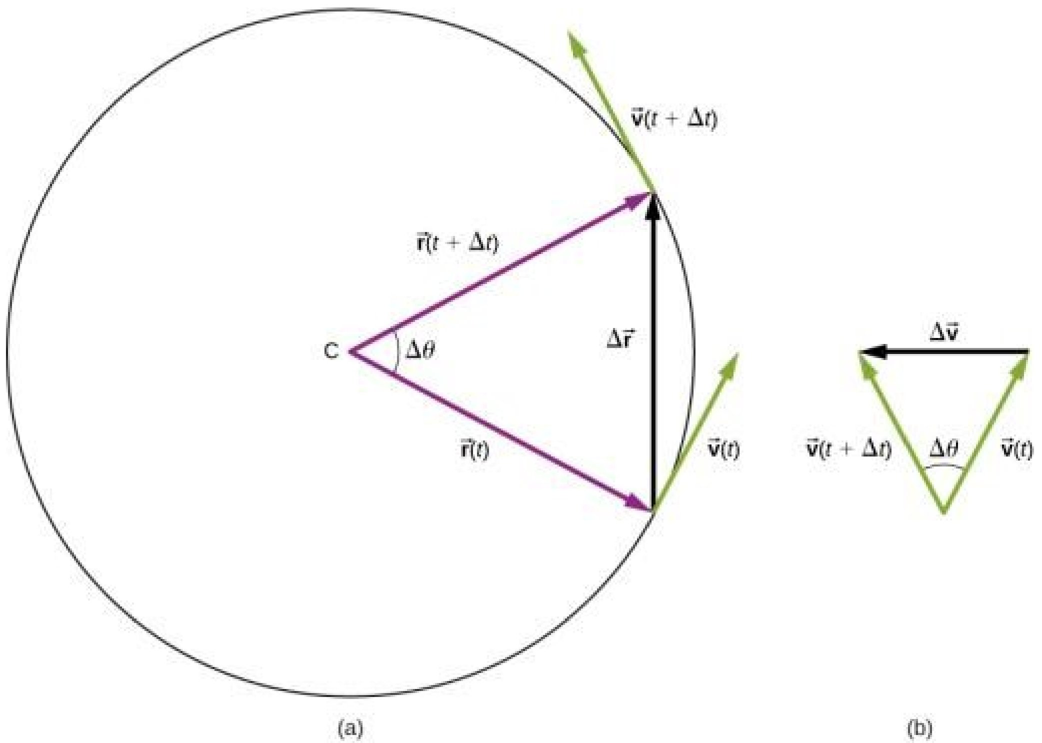
A. 12°
B. 18°
C. 36°
D. 45°

FIGURE 4.17



Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because Earth curves away beneath its path. With a speed of 8000 m/s, orbit is achieved.

Uniform circular motion



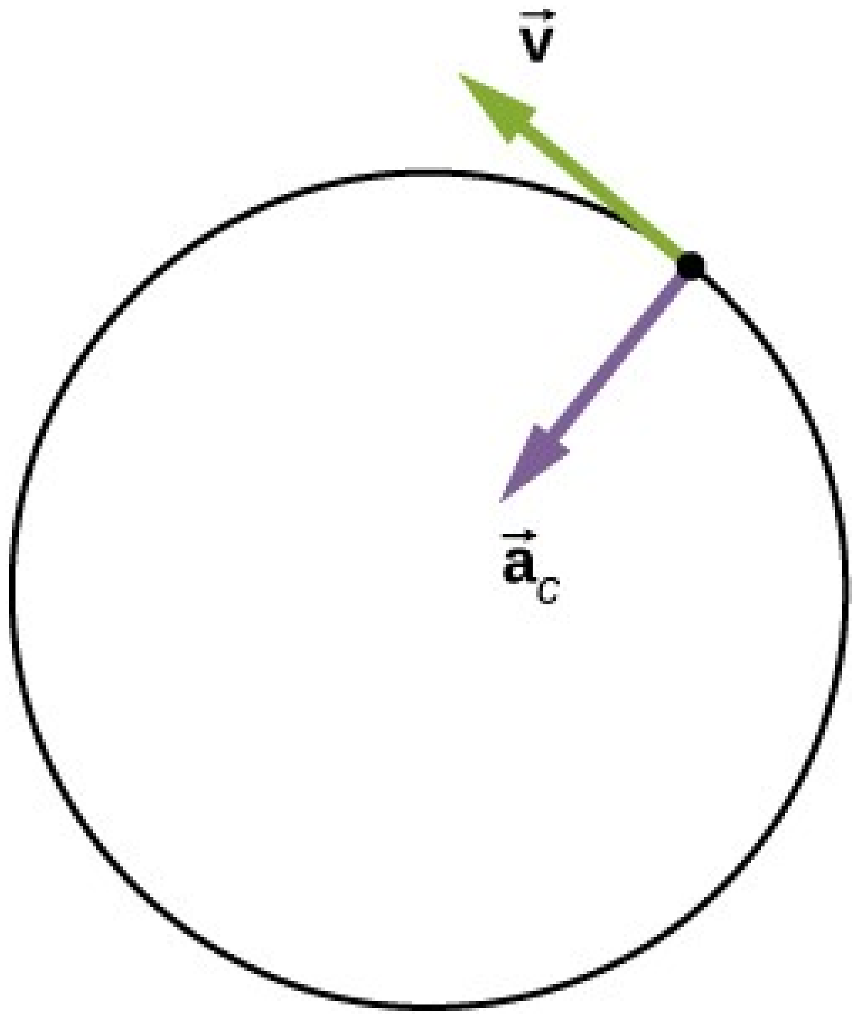
(a) A particle is moving in a circle at a constant speed, with position and velocity vectors at times t and $t + \Delta t$.

(b) Velocity vectors forming a triangle. The two triangles in the figure are similar. The vector $\Delta\vec{v}$ points toward the center of the circle in the limit $\Delta t \to 0$.

The book derives the acceleration in this case as

$$a_c = \frac{v^2}{r}$$

Centripetal acceleration ($\mathbf{a_c}$) in circular motion always points to the center of the circle.



In uniform circular motion, $|\mathbf{v}|$ is a constant, the only acceleration is $\mathbf{a_c}$, and $\mathbf{a_c}$ is always perpendicular to $\mathbf{v}$.

We'll see more of this later, but for now we should define some quantities for uniform circular motion.

A particle executing circular motion can be described by its position vector $\vec{\mathbf{r}}(t)$. Figure 4.20 shows a particle executing circular motion in a counterclockwise direction. As the particle moves on the circle, its position vector sweeps out the angle θ with the *x*-axis. Vector $\vec{\mathbf{r}}(t)$ making an angle θ with the *x*-axis is shown with its components along the *x*- and *y*-axes. The magnitude of the position vector is $A = |\vec{\mathbf{r}}(t)|$ and is also the radius of the circle, so that in terms of its components,

$$\vec{\mathbf{r}}(t) = A\cos\omega t\,\hat{\mathbf{i}} + A\sin\omega t\,\hat{\mathbf{j}}. \tag{4.28}$$

Here, ω is a constant called the **angular frequency** of the particle. The angular frequency has units of radians (rad) per second and is simply the number of radians of angular measure through which the particle passes per second. The angle θ that the position vector has at any particular time is ωt.

If *T* is the period of motion, or the time to complete one revolution (2π rad), then

Very useful relation, and sometimes you'll have to convert from rpm to rad/s

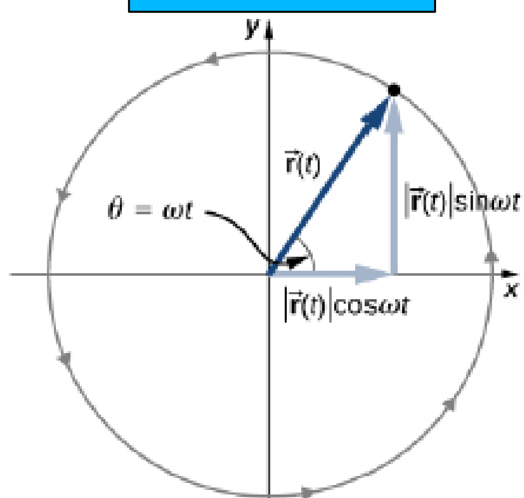
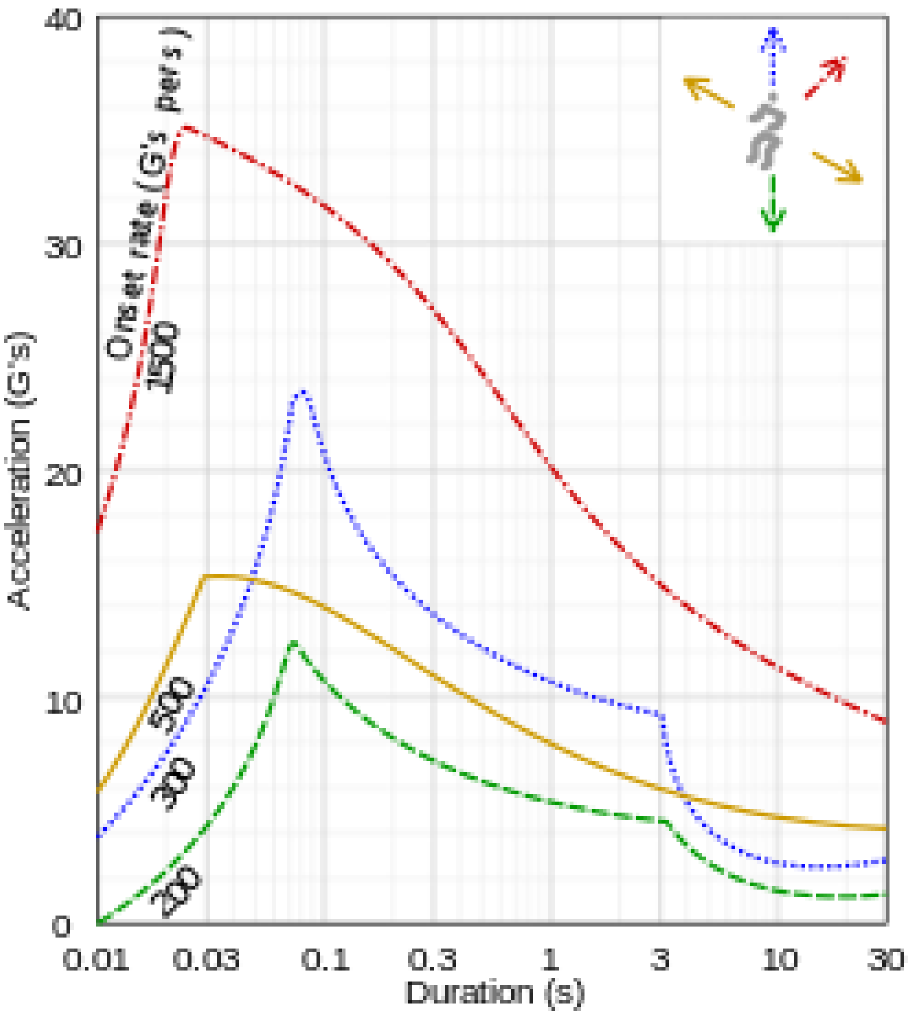


Figure 4.20 The position vector for a particle in circular motion with its components along the *x*- and *y*-axes. The particle moves counterclockwise. Angle θ is the angular frequency ω in radians per second multiplied by *t*.

Suppose you know ω instead of v. Can you still find a_c?

One reason that we might be interested in finding a_c :

The "g-force" (though it's not really a force!) exerted on us when we round curves or fly in airplanes.



Robert V. Brulle (2008)

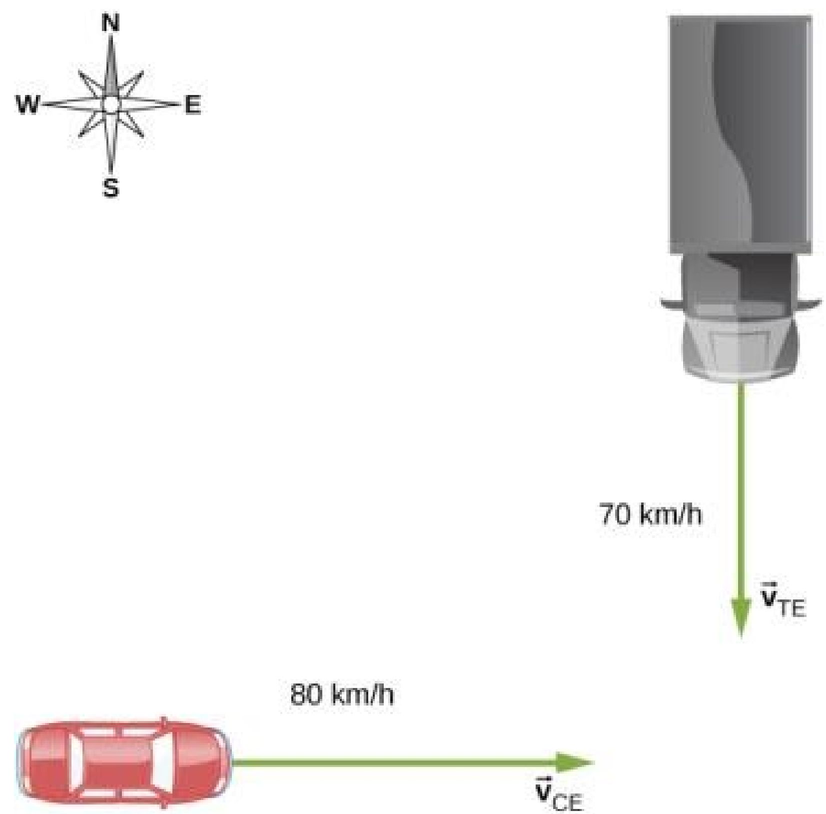
Relative Motion, key points

1. Define your coordinate system
2. Clearly define your velocities using subscripts
3. Simply add vector components of velocity

There are typically two kinds of problems we'll face:

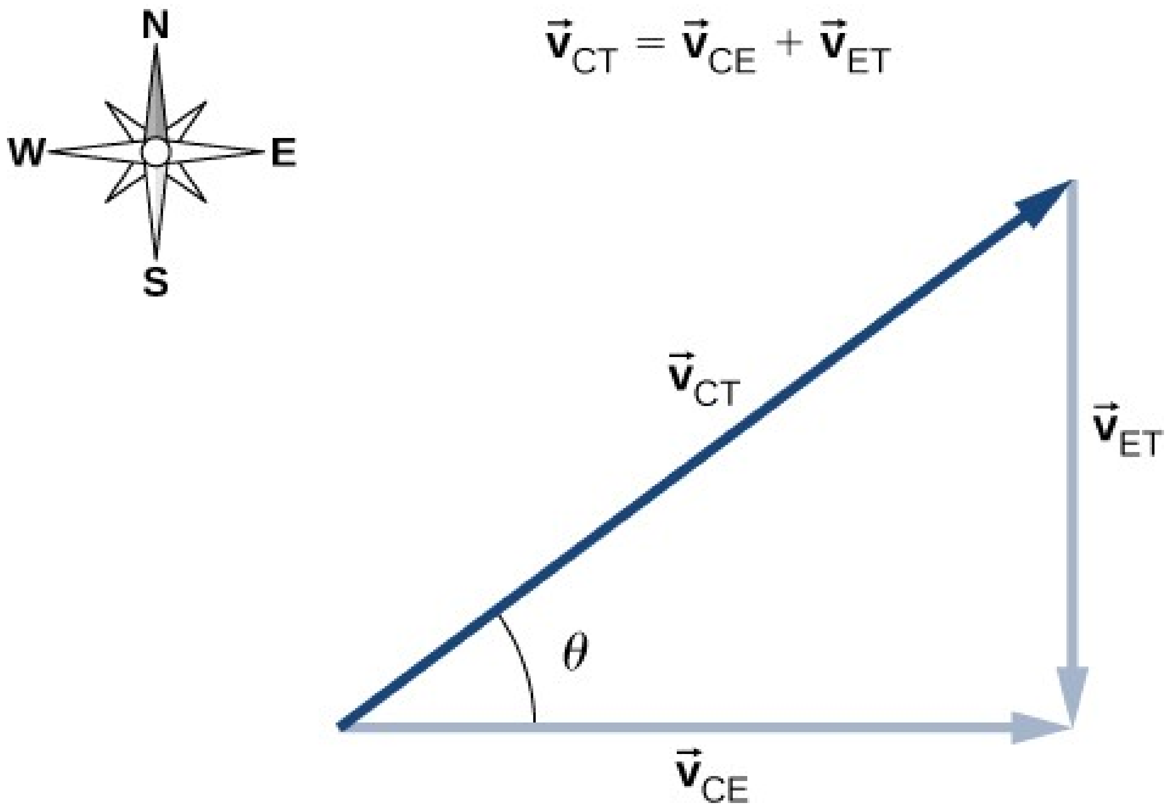
(i) Find the relative velocity between two objects
(ii) Find the velocity of an object that is moving within air or water that is also in motion (winds or currents)

Figure 4.27



A car travels east toward an intersection while a truck travels south toward the same intersection.

Figure 4.28



This arrow is pointing the wrong way!

Vector diagram of the vector equation $\vec{\mathbf{v}}_{\text{CT}} = \vec{\mathbf{v}}_{\text{CE}} + \vec{\mathbf{v}}_{\text{ET}}$.