# Physics 121 – September 12, 2017

## Assignments:

**This week:**

- Finish reading chapter 4 of textbook (note: we will NOT cover the section on nonuniform circular motion, pp. 191-193 of textbook)

- Make sure that your clicker or phone app is registered for this class: "PHYS121_Minschwaner_F2017" The course ID is "NMTphys121_Minschwaner"

- Complete ETA Problem Set #4 and chapter 4 written problems 30, 36, 59, 62, 76, due by Sept 18 at 4 PM

- Do practice problems in recitation this week

- Start reading Chapter 5

## Key concepts for today:

- Velocity and acceleration in 2- or 3-D
- Independence of perpendicular motions
- Equations of motion in 2- or 3-D
- Projectile Motion

Consider a projectile moving only under the influence of gravity where the air drag is negligible (such as a javelin). Can it be thrown in such a way that its velocity and acceleration vectors are perpendicular to one another at some point along its path?

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A. Yes
B. No
C. Maybe, but the thrower would have to be strong enough to make it supersonic.


# Recall the 1-D equations of motion from last week

## Putting Equations Together

In the following examples, we continue to explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The note that follows is provided for easy reference to the equations needed. Be aware that these equations are not independent. In many situations we have two unknowns and need two equations from the set to solve for the unknowns. We need as many equations as there are unknowns to solve a given situation.

### Summary of Kinematic Equations (constant *a*)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

p. 134 of textbook

Now for 2- or 3-D, instead of just "x" we have to measure displacement using a position vector "**r**"

$$\vec{\mathbf{r}}(t) = x(t)\,\hat{\mathbf{i}} + y(t)\,\hat{\mathbf{j}} + z(t)\,\hat{\mathbf{k}}. \quad (4.2)$$

**Figure 4.2** shows the coordinate system and the vector to point *P*, where a particle could be located at a particular time *t*. Note the orientation of the *x*, *y*, and *z* axes. This orientation is called a right-handed coordinate system (**Coordinate Systems and Components of a Vector**) and it is used throughout the chapter.

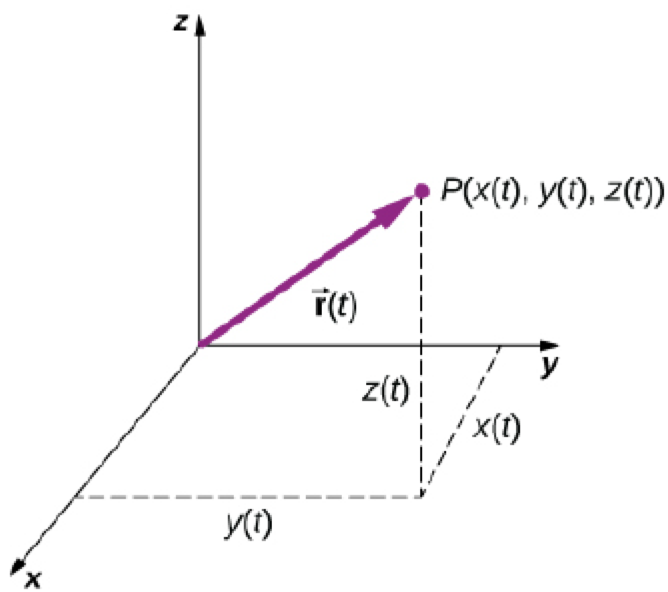



**Figure 4.2** A three-dimensional coordinate system with a particle at position $P(x(t), y(t), z(t))$.

Similarly, for 2- or 3-D we have to consider the instantaneous velocity vector "**v**" as the rate of change in the displacement vector **r** (note this could be a change in the magnitude of **r**, or just in the direction of **r**, or both).

## Velocity Vector

In the previous chapter we found the instantaneous velocity by calculating the derivative of the position function with respect to time. We can do the same operation in two and three dimensions, but we use vectors. The instantaneous **velocity vector** is now

$$\vec{\mathbf{v}}(t) = \lim_{\Delta t \to 0} \frac{\vec{\mathbf{r}}(t + \Delta t) - \vec{\mathbf{r}}(t)}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}. \tag{4.4}$$

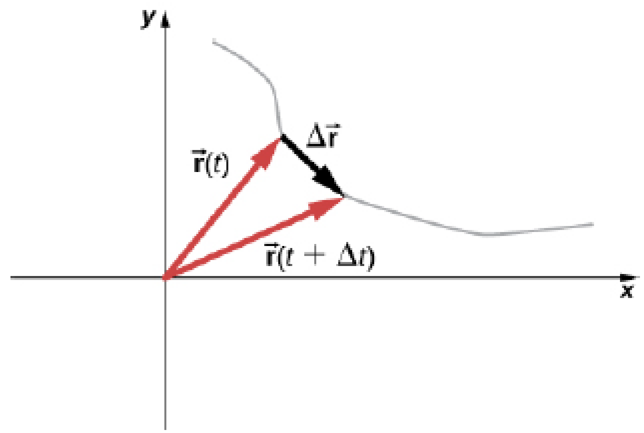


Figure 4.7 A particle moves along a path given by the gray line. In the limit as $\Delta t$ approaches zero, the velocity vector becomes tangent to the path of the particle.

Equation 4.4 can also be written in terms of the components of $\vec{v}(t)$. Since

$$\vec{r}(t) = x(t)\,\hat{\mathbf{i}} + y(t)\,\hat{\mathbf{j}} + z(t)\,\hat{\mathbf{k}},$$

we can write

$$\vec{v}(t) = v_x(t)\,\hat{\mathbf{i}} + v_y(t)\,\hat{\mathbf{j}} + v_z(t)\,\hat{\mathbf{k}} \qquad (4.5)$$

where

$$v_x(t) = \frac{dx(t)}{dt}, \quad v_y(t) = \frac{dy(t)}{dt}, \quad v_z(t) = \frac{dz(t)}{dt}. \qquad (4.6)$$

$v_x$ , $v_y$ , $v_z$ are called the **components** of the velocity

For vector acceleration “**a**” we use the rate of change in **v**

$$\vec{\mathbf{a}}(t) = \lim_{t \to 0} \frac{\vec{\mathbf{v}}(t+\Delta t) - \vec{\mathbf{v}}(t)}{\Delta t} = \frac{d\vec{\mathbf{v}}(t)}{dt}. \quad (4.8)$$

The acceleration in terms of components is

$$\vec{\mathbf{a}}(t) = \frac{dv_x(t)}{dt}\hat{\mathbf{i}} + \frac{dv_y(t)}{dt}\hat{\mathbf{j}} + \frac{dv_z(t)}{dt}\hat{\mathbf{k}}. \quad (4.9)$$

Also, since the velocity is the derivative of the position function, we can write the acceleration in terms of the second derivative of the position function:

$$\vec{\mathbf{a}}(t) = \frac{d^2x(t)}{dt^2}\hat{\mathbf{i}} + \frac{d^2y(t)}{dt^2}\hat{\mathbf{j}} + \frac{d^2z(t)}{dt^2}\hat{\mathbf{k}}. \quad (4.10)$$

where $\frac{dv_x}{dt} = \frac{d^2x}{dt^2} = a_x$ and similarly for $a_y$ and $a_z$

These are called the **components** of acceleration.

We will be working (mostly) with constant acceleration.

The only exceptions will be uniform circular motion (when the magnitude of “**a**” remains constant, but the direction of “**a**” is changing), and elliptical orbits in gravitational motion (when both the magnitude and direction of “**a**” can change).

# I cannot emphasize this point enough:

## The Independence of Perpendicular Motions

When we look at the three-dimensional equations for position and velocity written in unit vector notation, **Equation 4.2** and **Equation 4.5**, we see the components of these equations are separate and unique functions of time that do not depend on one another. Motion along the *x* direction has no part of its motion along the *y* and *z* directions, and similarly for the other two coordinate axes. Thus, the motion of an object in two or three dimensions can be divided into separate, independent motions along the perpendicular axes of the coordinate system in which the motion takes place.

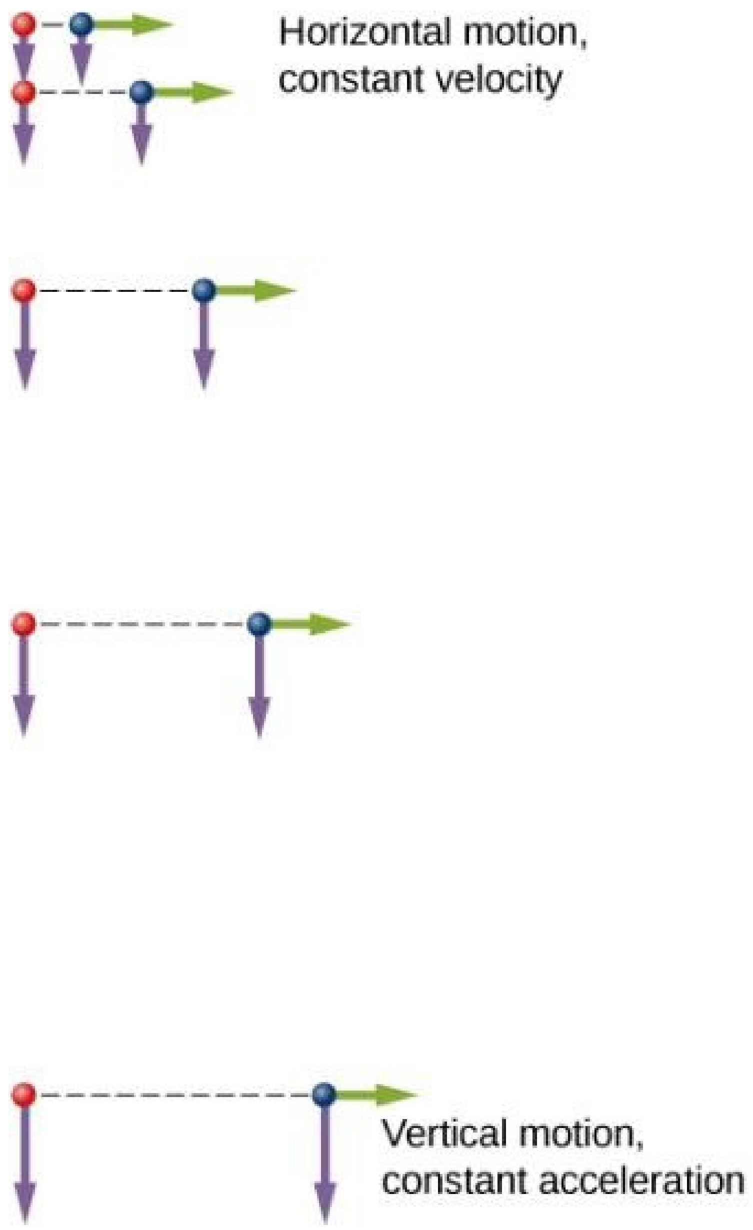
To illustrate this concept with respect to displacement, consider a woman walking from point *A* to point *B* in a city with square blocks. The woman taking the path from *A* to *B* may walk east for so many blocks and then north (two perpendicular directions) for another set of blocks to arrive at *B*. How far she walks east is affected only by her motion eastward. Similarly, how far she walks north is affected only by her motion northward.

### Independence of Motion

In the kinematic description of motion, we are able to treat the horizontal and vertical components of motion separately. In many cases, motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

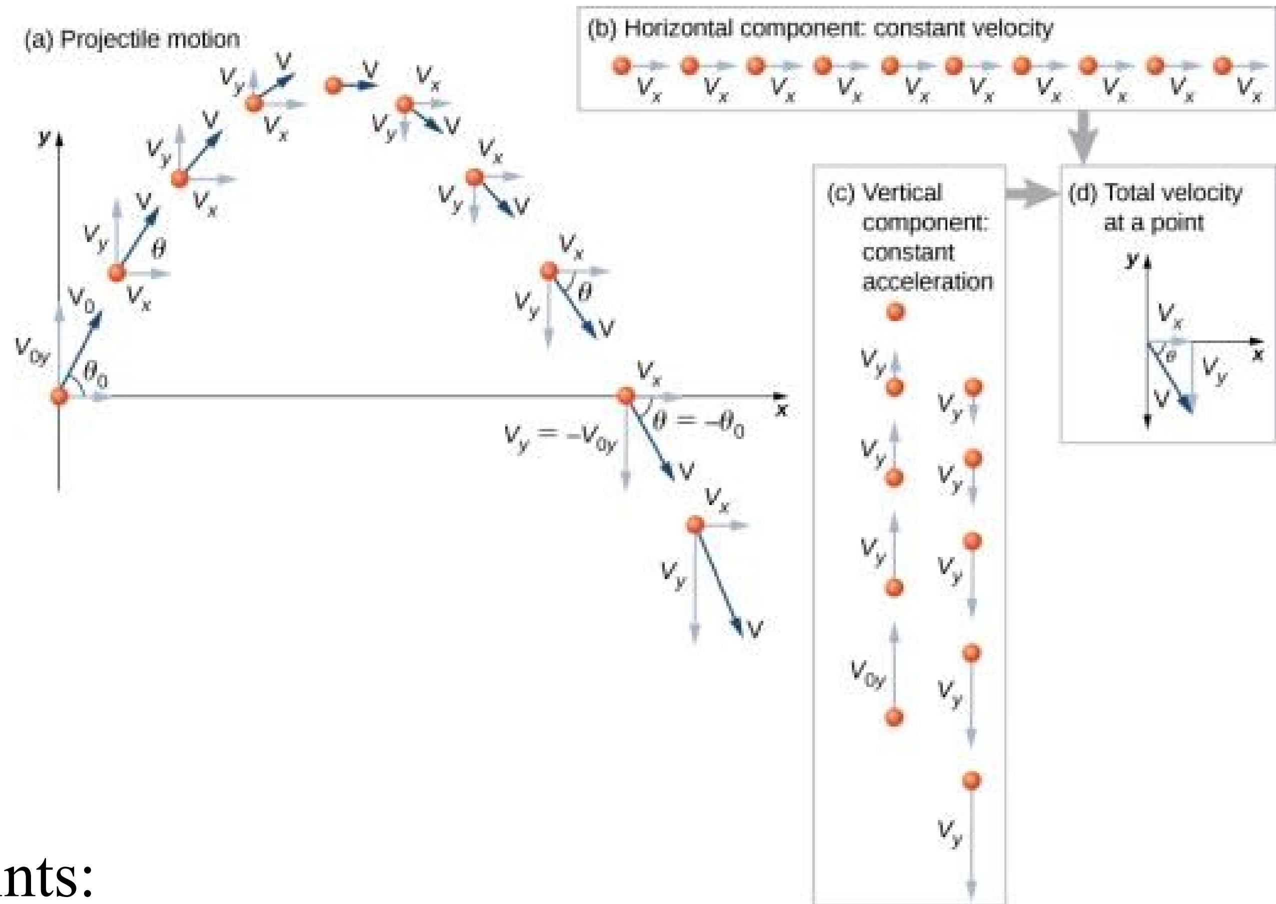
An example illustrating the independence of vertical and horizontal motions is given by two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and it follows a curved path. A stroboscope captures the positions of the balls at fixed time intervals as they fall (**Figure 4.8**).

# Figure 4.8




A diagram of the motions of two identical balls: one falls from rest and the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent the horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity whereas the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls, which shows the vertical and horizontal motions are independent.

This independence holds for all projectiles, even those with a vertical component to the initial velocity.




Key points:

1. $V_x$ = constant during the flight
2. $V_y$ varies during the flight, with acceleration = constant = -g
3. Vector velocity at any time is determined by components $V_x$ and $V_y$

Let's look at the equations of motion now.

This is pretty good advice from the textbook on p. 177

### Problem-Solving Strategy: Projectile Motion

1. Resolve the motion into horizontal and vertical components along the *x*- and *y*-axes. The magnitudes of the components of displacement $\vec{\mathbf{s}}$ along these axes are *x* and *y*. The magnitudes of the components of velocity $\vec{\mathbf{v}}$ are $v_x = v\cos\theta$ and $v_y = v\sin\theta,$ where *v* is the magnitude of the velocity and *θ* is its direction relative to the horizontal, as shown in **Figure 4.12**.
2. Treat the motion as two independent one-dimensional motions: one horizontal and the other vertical. Use the kinematic equations for horizontal and vertical motion presented earlier.
3. Solve for the unknowns in the two separate motions: one horizontal and one vertical. Note that the only common variable between the motions is time *t*. The problem-solving procedures here are the same as those for one-dimensional kinematics and are illustrated in the following solved examples.
4. Recombine quantities in the horizontal and vertical directions to find the total displacement $\vec{\mathbf{s}}$ and velocity $\vec{\mathbf{v}}$ . Solve for the magnitude and direction of the displacement and velocity using

$$s = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}(y/x), \quad v = \sqrt{v_x^2 + v_y^2},$$

where *θ* is the direction of the displacement $\vec{\mathbf{s}}$ .

# Let's try this out on example 4.7 from the textbook

## Example 4.7

### A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of $75.0°$ above the horizontal, as illustrated in Figure 4.13. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passes between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes? (d) What is the total displacement from the point of launch to the highest point?

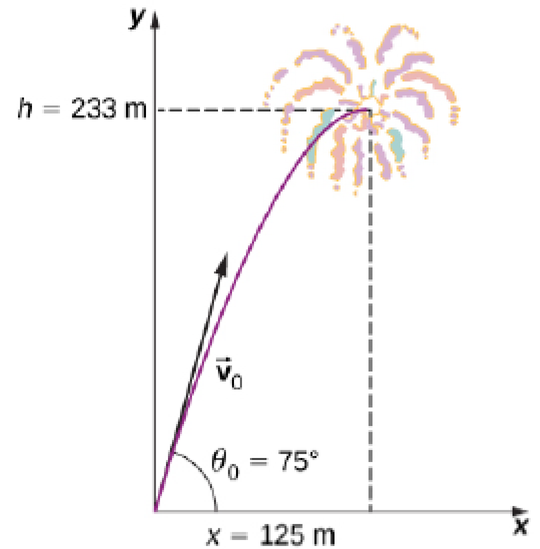



Figure 4.13 The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

This is a good example for problems where the final position is not level with the initial position (y is not equal to zero when the ball lands)

## Example 4.8

### Calculating Projectile Motion: Tennis Player

A tennis player wins a match at Arthur Ashe stadium and hits a ball into the stands at 30 m/s and at an angle $45°$ above the horizontal (Figure 4.14). On its way down, the ball is caught by a spectator 10 m above the point where the ball was hit. (a) Calculate the time it takes the tennis ball to reach the spectator. (b) What are the magnitude and direction of the ball's velocity at impact?

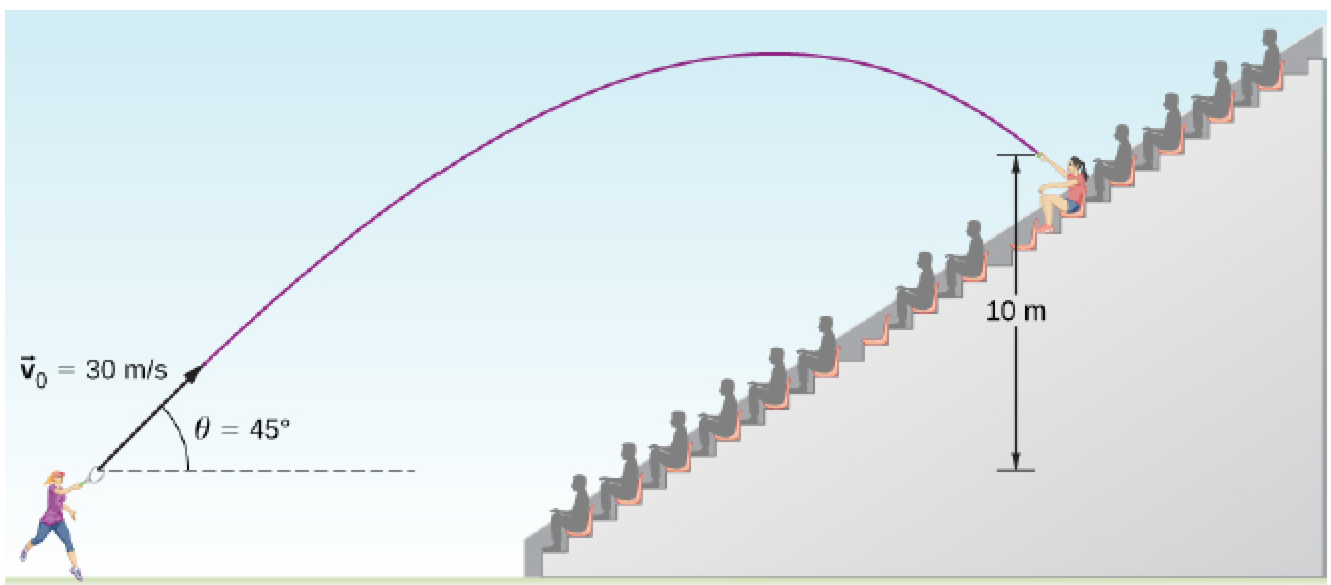



Figure 4.14 The trajectory of a tennis ball hit into the stands.

One of most convincing displays of the independence of vertical And horizontal motion is the classic “Shoot the monkey” problem:

**Shoot the monkey link**

Let’s analyze this problem is some detail, then we’ll try a live demo in class.

These equations may be useful at times, but be careful because they can only be applied in specific instances!

$$T_{\text{tof}} = \frac{2(v_0 \sin\theta_0)}{g}. \quad (4.24)$$

This is the **time of flight** for a projectile both launched and impacting on a flat horizontal surface. Equation 4.24 does not apply when the projectile lands at a different elevation than it was launched, as we saw in Example 4.8 of the tennis player hitting the ball into the stands. The other solution, $t = 0$, corresponds to the time at launch. The time of flight is linearly proportional to the initial velocity in the $y$ direction and inversely proportional to $g$. Thus, on the Moon, where gravity is one-sixth that of Earth, a projectile launched with the same velocity as on Earth would be airborne six times as long.

### Trajectory

The trajectory of a projectile can be found by eliminating the time variable $t$ from the kinematic equations for arbitrary $t$ and solving for $y(x)$. We take $x_0 = y_0 = 0$ so the projectile is launched from the origin. The kinematic equation for $x$ gives

$$x = v_{0x}t \Rightarrow t = \frac{x}{v_{0x}} = \frac{x}{v_0 \cos\theta_0}.$$

Substituting the expression for $t$ into the equation for the position $y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$ gives

$$y = (v_0 \sin\theta_0)\left(\frac{x}{v_0 \cos\theta_0}\right) - \frac{1}{2}g\left(\frac{x}{v_0 \cos\theta_0}\right)^2.$$

Rearranging terms, we have

$$y = (\tan\theta_0)x - \left[\frac{g}{2(v_0 \cos\theta_0)^2}\right]x^2. \quad (4.25)$$

Not so useful, but

This trajectory equation is of the form $y = ax + bx^2$, which is an equation of a parabola with coefficients

$$R = \frac{v_0^2 \sin 2\theta_0}{g}. \tag{4.26}$$

Note particularly that Equation 4.26 is valid only for launch and impact on a horizontal surface. We see the range is directly proportional to the square of the initial speed $v_0$ and $\sin 2\theta_0$, and it is inversely proportional to the acceleration of gravity. Thus, on the Moon, the range would be six times greater than on Earth for the same initial velocity. Furthermore, we see from the factor $\sin 2\theta_0$ that the range is maximum at $45°$. These results are shown in Figure 4.15. In (a) we see that the greater the initial velocity, the greater the range. In (b), we see that the range is maximum at $45°$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is somewhat smaller. It is interesting that the same range is found for two initial launch angles that sum to $90°$. The projectile launched with the smaller angle has a lower apex than the higher angle, but they both have the same range.