# Physics 121 – September 7, 2017

## Assignments:

**This week:**

- Finish reading Chapter 3 of textbook (note: you will NOT be responsible in HW, quizzes, or exams for material in section 3.6)
- Start reading Chapter 4
- Complete ETA Problem Set #3 by Sept 11 at 4 PM
- Chapter 3 written problems 30, 31, 38, 48, 60, 72, and 96
- Quiz in recitation this week (simple vectors problem)

Office hours on Wed and Thur, 2-3 PM

## Key concepts for today:

- Using 1-D equations of motion with constant acceleration
- Free fall
- Extension of equations of motion to 3-D

# Summary of 1-D equations of motion

## Putting Equations Together

In the following examples, we continue to explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The note that follows is provided for easy reference to the equations needed. Be aware that these equations are not independent. In many situations we have two unknowns and need two equations from the set to solve for the unknowns. We need as many equations as there are unknowns to solve a given situation.

### Summary of Kinematic Equations (constant *a*)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

p. 134 of textbook

Examples 3.10 and 3.11 are pretty good for seeing how to apply the equations of motion to simple problems (both involve cars either slowing down or speeding up).

Also example 3.12 (rocket ship accelerating).

Note book says that there are 6 variables that describe motion in 1-D. I claim there are usually only 4 because we can usually set $t_o=0$ and $x_o=0$

There are also some example problems from ETA that we should go over now, involving graphs of velocity versus time.

For two-body problems (e.g. pursuit problems) involving constant acceleration: see example 3.13

I think this example has bad “symbology”. Let’s go over this one now.

### Cheetah Catching a Gazelle

A cheetah waits in hiding behind a bush. The cheetah spots a gazelle running past at 10 m/s. At the instant the gazelle passes the cheetah, the cheetah accelerates from rest at 4 m/s$^2$ to catch the gazelle. (a) How long does it take the cheetah to catch the gazelle? (b) What is the displacement of the gazelle and cheetah?

### Strategy

We use the set of equations for constant acceleration to solve this problem. Since there are two objects in motion, we have separate equations of motion describing each animal. But what links the equations is a common parameter that has the same value for each animal. If we look at the problem closely, it is clear the common parameter to each animal is their position $x$ at a later time $t$. Since they both start at $x_0 = 0$, their displacements are the same at a later time $t$, when the cheetah catches up with the gazelle. If we pick the equation of motion that solves for the displacement for each animal, we can then set the equations equal to each other and solve for the unknown, which is time.

### Solution

a. Equation for the gazelle: The gazelle has a constant velocity, which is its average velocity, since it is not accelerating. Therefore, we use Equation 3.10 with $x_0 = 0$:

$$x = x_0 + \bar{v}t = \bar{v}t.$$

Equation for the cheetah: The cheetah is accelerating from rest, so we use Equation 3.13 with $x_0 = 0$ and $v_0 = 0$:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2.$$

Now we have an equation of motion for each animal with a common parameter, which can be eliminated to find the solution. In this case, we solve for $t$:

$$x = \bar{v}t = \frac{1}{2}at^2$$
$$t = \frac{2\bar{v}}{a}.$$

The gazelle has a constant velocity of 10 m/s, which is its average velocity. The acceleration of the cheetah is 4 m/s$^2$. Evaluating $t$, the time for the cheetah to reach the gazelle, we have

$$t = \frac{2\bar{v}}{a} = \frac{2(10)}{4} = 5\text{ s}.$$

# Free fall

Does an object have to be moving toward the surface in order to be classified as being in "free fall"?

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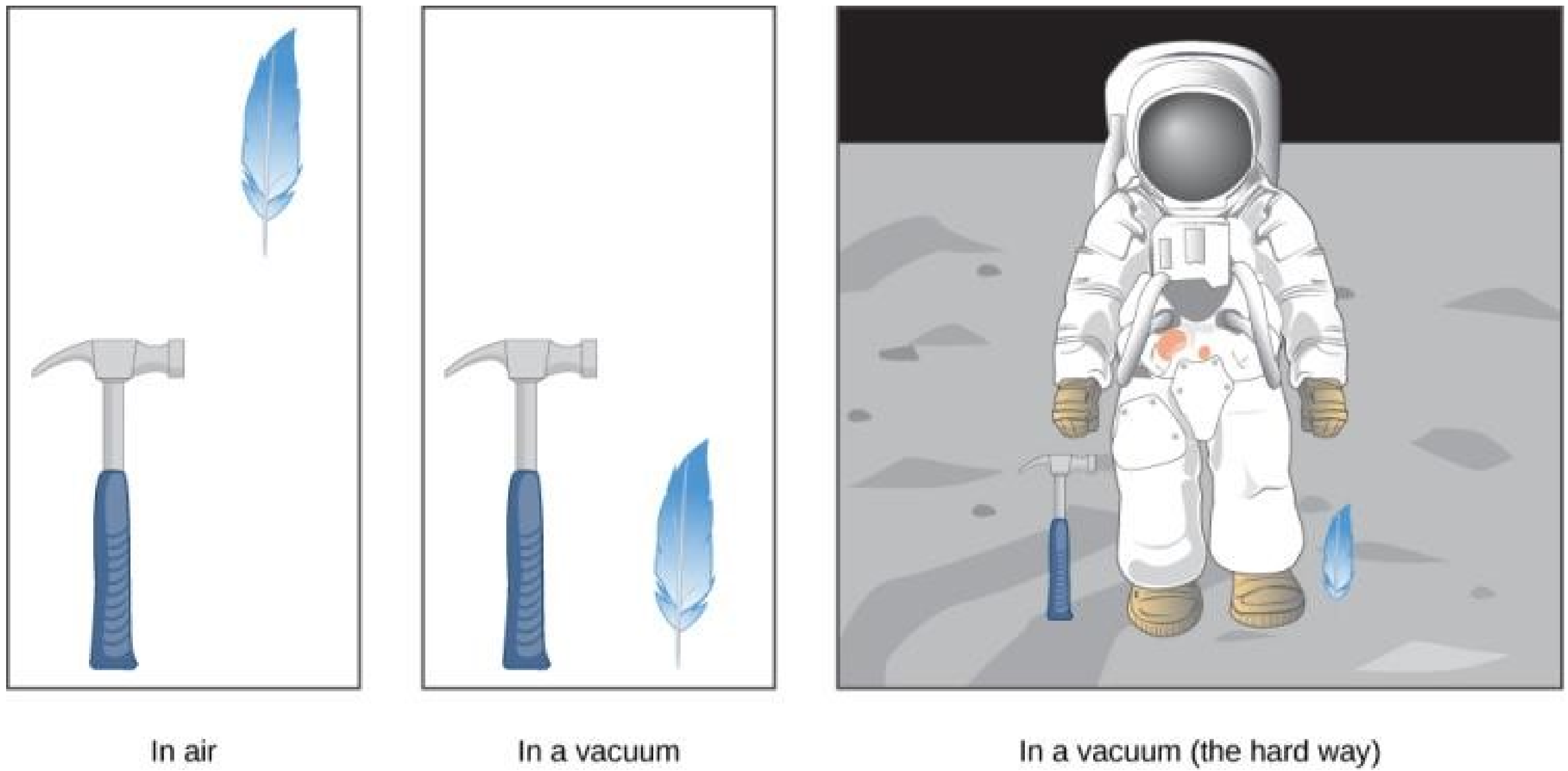
Does an object have to be moving toward the surface in order to be classified as being in “free fall”?

NO!  Free fall refers to an object that is moving under the influence of a gravitational force.

# Figure 3.26




A hammer and a feather fall with the same constant acceleration if air resistance is negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated in 1971 on the Moon, where the acceleration from gravity is only 1.67 $m/s^2$ and there is no atmosphere.

**Let’s do our own experiment!**

# This is pretty good advice on p. 142, but I have an issue with ex. 3.14

### Kinematic Equations for Objects in Free Fall

We assume here that acceleration equals $-g$ (with the positive direction upward).

$$v = v_0 - gt \tag{3.15}$$

$$y = y_0 + v_0 t - \frac{1}{2}gt^2 \tag{3.16}$$

$$v^2 = v_0^2 - 2g(y - y_0) \tag{3.17}$$

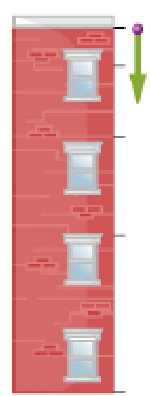
### Problem-Solving Strategy: Free Fall

1. Decide on the sign of the acceleration of gravity. In Equation 3.15 through Equation 3.17, acceleration $g$ is negative, which says the positive direction is upward and the negative direction is downward. In some problems, it may be useful to have acceleration $g$ as positive, indicating the positive direction is downward.
2. Draw a sketch of the problem. This helps visualize the physics involved.
3. Record the knowns and unknowns from the problem description. This helps devise a strategy for selecting the appropriate equations to solve the problem.
4. Decide which of Equation 3.15 through Equation 3.17 are to be used to solve for the unknowns.

## Example 3.14

### Free Fall of a Ball

Figure 3.27 shows the positions of a ball, at 1-s intervals, with an initial velocity of 4.9 m/s downward, that is thrown from the top of a 98-m-high building. (a) How much time elapses before the ball reaches the ground? (b) What is the velocity when it arrives at the ground?



| $t$ (s) | $x$ (m) | $v$ (m/s) |
|---|---|---|
| 0 | 0 | −4.9 |
| 1 | −9.8 | −14.7 |
| 2 | −29.4 | −24.5 |
| 3 | −58.8 | −34.3 |
| 4 | −98.0 | −44.1 |

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$
$$-98.0\text{ m} = 0 - (4.9\text{ m/s})t - \frac{1}{2}(9.8\text{ m/s}^2)t^2.$$

This simplifies to

$$t^2 + t - 20 = 0.$$

This is a quadratic equation with roots $t = -5.0\text{s}$ and $t = 4.0\text{s}$. The positive root is the one we are interested in, since time $t = 0$ is the time when the ball is released at the top of the building. (The time $t = -5.0\text{s}$ represents the fact that a ball thrown upward from the ground would have been in the air for 5.0 s when it passed by the top of the building moving downward at 4.9 m/s.)

b. Using Equation 3.15, we have

$$v = v_0 - gt = -4.9\text{ m/s} - (9.8\text{m/s}^2)(4.0\text{ s}) = -44.1\text{ m/s}.$$

### Significance

For situations when two roots are obtained from a quadratic equation in the time variable, we must look at the physical significance of both roots to determine which is correct. Since $t = 0$ corresponds to the time when the ball was released, the negative root would correspond to a time before the ball was released, which is not physically meaningful. When the ball hits the ground, its velocity is not immediately zero, but as soon as the ball interacts with the ground, its acceleration is not $g$ and it accelerates with a different value over a short time to zero velocity. This problem shows how important it is to establish the correct coordinate system and to keep the signs of $g$ in the kinematic equations consistent.

Key point!

## Example 3.15

### Vertical Motion of a Baseball

A batter hits a baseball straight upward at home plate and the ball is caught 5.0 s after it is struck Figure 3.28. (a) What is the initial velocity of the ball? (b) What is the maximum height the ball reaches? (c) How long does it take to reach the maximum height? (d) What is the acceleration at the top of its path? (e) What is the velocity of the ball when it is caught? Assume the ball is hit and caught at the same location.

Solution

a. Equation 3.16 gives

$$y = y_0 + v_0 t - \frac{1}{2}gt^2$$

$$0 = 0 + v_0(5.0\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(5.0\text{ s})^2,$$

Yuck!

which gives $v_0 = 24.5\text{ m/sec}$.

b. At the maximum height, $v = 0$. With $v_0 = 24.5\text{ m/s}$, Equation 3.17 gives

$$v^2 = v_0^2 - 2g(y - y_0)$$

$$0 = (24.5\text{ m/s})^2 - 2(9.8\text{m/s}^2)(y - 0)$$

or

$$y = 30.6\text{ m}.$$

c. To find the time when $v = 0$, we use Equation 3.15:

$$v = v_0 - gt$$

$$0 = 24.5\text{ m/s} - (9.8\text{m/s}^2)t.$$

This gives $t = 2.5\text{ s}$. Since the ball rises for 2.5 s, the time to fall is 2.5 s.

d. The acceleration is 9.8 $\text{m/s}^2$ everywhere, even when the velocity is zero at the top of the path. Although the velocity is zero at the top, it is changing at the rate of 9.8 $\text{m/s}^2$ downward.

e. The velocity at $t = 5.0\text{s}$ can be determined with Equation 3.15:

$$\begin{aligned} v &= v_0 - gt \\ &= 24.5\text{ m/s} - 9.8\text{m/s}^2(5.0\text{ s}) \\ &= -24.5\text{ m/s}. \end{aligned}$$

Significance

The ball returns with the speed it had when it left. This is a general property of free fall for any initial velocity. We used a single equation to go from throw to catch, and did not have to break the motion into two segments, upward and downward. We are used to thinking of the effect of gravity is to create free fall downward toward Earth. It is important to understand, as illustrated in this example, that objects moving upward away from Earth are also in a state of free fall.

Another key point!