Dog sled problem

8 dogs, $m_d = 16.5$ Kg on average

loaded sled + driver, $m_\ell = 208$ kg

The coefficients of static ($\mu_s$) and Kinetic friction ($\mu_K$) are

$\mu_s = 0.14$ and $\mu_K = 0.10$

If there is no sliding (slipping, skidding), then the frictional force between 2 surfaces is

$f_s \leq \mu_s N$

often, we want to know $f_s(\text{max}) = \mu_s N$

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But note: $\vec{f}_s \perp \vec{N}$ $(\vec{f} \neq \mu_s \vec{N})$

If there is sliding, then $f_K = \mu_K N$

$(\vec{f}_K \perp \vec{N})$

a) Starting from rest, each dog exerts a force $f_d$ on ground, so the ~~total~~ force on ground

$f_g = 8 f_d$

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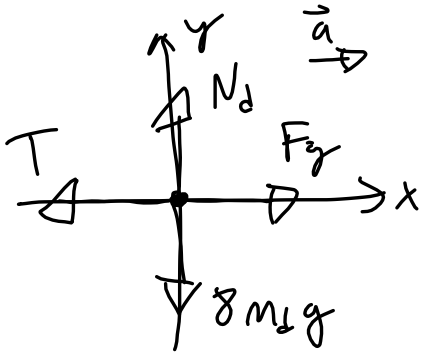
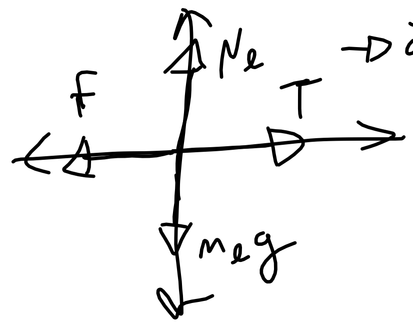
FBD for dogs (all 8)


FBD for loaded sled




If sled is not moving,

$a = 0, \quad f = f_s$

If sled starts from rest,

$a \neq 0, \quad f = f_s(max) = \mu_s N_\ell$

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eq's for load: x components: $T - f_s = m_\ell a$

y components: $N_\ell - m_\ell g = 0$

so $T = m_\ell(a + \mu_s g)$ where we used $f_s = \mu_s N$

eq's for dogs: x comp: $F_g - T = 8 m_d a$

y comp: $N_d - 8 m_d g = 0$ (not useful)

2 equations for T: $T = m_\ell(a + \mu_s g)$

$T = F_g - 8 m_d a$

eliminate T, $\boxed{a = \dfrac{F_g - \mu_s m_\ell g}{8 m_d + m_\ell}}$

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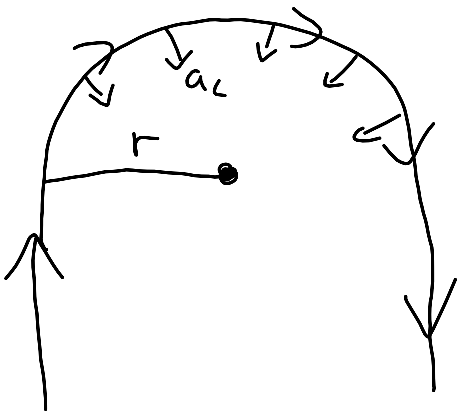
## Banked turns in NASCAR

If a car executes uniform circular motion (ucm) then for mass m and radius r, we know that




$$a_c = \frac{v^2}{r}$$

We can define a centripetal force $F_c = ma_c = \boxed{\frac{mv^2}{r}}$

also, note that since $V = r\underbrace{\omega}_{\frac{d\theta}{dt}}$, $\boxed{F_c = mr\omega^2}$

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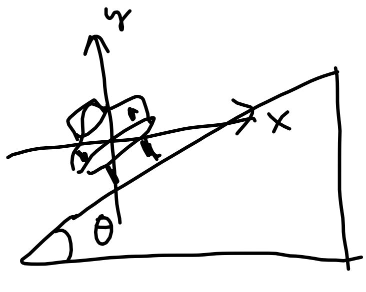
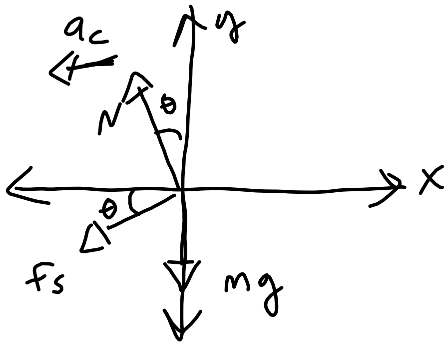
Suppose that the turns are banked




from rear of car (velocity is into page)

### FBD diagram for car




Note we're using $f_s$ here. Assume no skid, so

$$f = f_s$$

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If we want the max speed without skid, then

$f_s = \mu_s N$ (max static friction)

Newton's 2nd law:

$$\Sigma F_x = -m a_c = -\frac{mv^2}{r}$$

$$-N \sin\theta - \underbrace{f_s}_{\mu_s N} \cos\theta = -\frac{mv^2}{r}$$

$$N(\sin\theta + \mu_s \cos\theta) = \frac{mv^2}{r}$$

$$\Sigma F_y = 0$$

$$N \cos\theta - \mu_s N \sin\theta - mg = 0 \quad \Rightarrow N(\cos\theta - \mu_s \sin\theta) = mg$$

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Can this problem work if $\mu_s = 0$ ?

We have
$$N(\sin\theta + \mu_s \cos\theta) = \frac{mv^2}{r}$$
$$N(\cos\theta - \mu_s \sin\theta) = mg$$

Divide equations,
$$\tan\theta = \frac{v^2}{gr}$$

so $v = \sqrt{gr\tan\theta}$ (book, p. 360)

Now for $\mu_s \neq 0$, $v = \sqrt{rg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s \sin\theta}\right)}$

Indep of mass!

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Example Talladega, $\theta = 33^\circ$, $r \simeq 300\ m$

$\mu_s = 0.9$

$$v = \sqrt{(3\times10^3\ \tfrac{m^2}{s^2})\left(\tfrac{1.3}{.35}\right)}$$

$$= 105\ m/s\ (\sim 230\ mph)$$

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Another ucm example

ucm in a vertical plane. $r = 1\ m$

$m = ?$

$T \simeq 1\ s$ (maximum)

minimum $\omega$?

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FBD at top


$$\Sigma F_y = -m a_c$$

$$-T - mg = -m a_c$$

$$T = m(a_c - g)$$

but $a_c = r\omega^2$, $T = m\underbrace{(r\omega^2 - g)}_{\text{this must be zero}}$

For the minimum $\omega$,

$$\omega_{min} = \sqrt{\frac{g}{r}},$$

so for $r = 1\ m$,

$$\omega_{min} = \sqrt{10\ \tfrac{m^2}{s^2}} \approx 3.13\ s^{-1}$$

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So $T = \frac{2\pi}{\omega} = \frac{2\pi}{3.13} \sim 2$ s

We're only off by a factor of 1.5!

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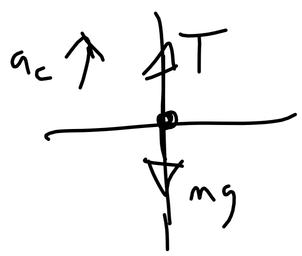
Note at the bottom, we would draw another FBD

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