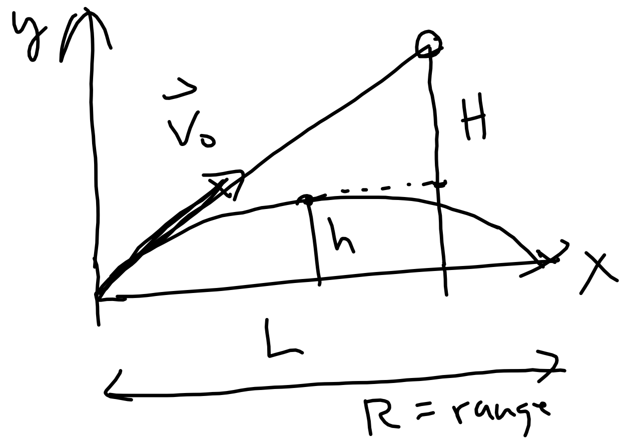
Shoot the monkey demo



for ball,

$$x = V_{ox} t \quad (1)$$

$$y = V_{oy} t - \frac{1}{2} g t^2 \quad (2)$$

$$V_y = V_{oy} - g t \quad (3)$$

$$V_y^2 = V_{oy}^2 - 2 g y \quad (4)$$

By (4), $y = 0 \Rightarrow V_y = \pm V_{oy}$

By (3), when $V_y = V_{oy}$, $t = 0$

$V_y = -V_{oy}$, $t = t_f$

time of flight

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So $t_f = \frac{2 V_{oy}}{g}$ time of flight

But from (1), $R = V_{ox} t_f$, $t_f = \frac{R}{V_{ox}}$

What is h? Take eq (4),

let $V_y = 0$ when $y = h$

$$0 = V_{oy}^2 - 2gh \Rightarrow h = \frac{V_{oy}^2}{2g}$$

For monkey, $y = H - \cancel{V_{oy}}^{\,0} t - \frac{1}{2} g t^2$

$$y = H - \frac{1}{2} g t^2$$

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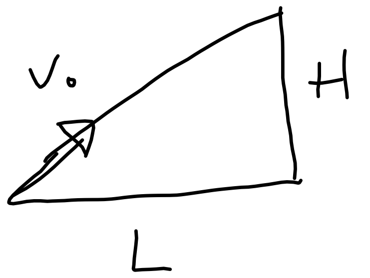
So we have 2 expression

$$y(\text{ball}) = V_{oy}t - \frac{1}{2}gt^2$$

$$y(\text{monkey}) = H - \frac{1}{2}gt^2$$



$V_{oy} = V_o \sin\theta$

$V_{ox} = V_o \cos\theta$

Since ball's velocity in x direction = constant
we can say that $L = V_{ox} t_c$ → time to collision

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Similarly, $\boxed{H = V_{oy} t_c}$

So, $y(\text{ball}, t = t_c) = V_{oy}t_c - \frac{1}{2}gt_c^2$

$\qquad\qquad\qquad\qquad\qquad\quad$ ||

$\qquad y(\text{monkey}, t = t_c) = H - \frac{1}{2}gt_c^2$

$$y(\text{ball}, t = t_c) = y(\text{monkey}, t = t_c)$$

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Range for level ground

$$R = V_{0x} t_f = V_{0x}\left(2\frac{V_{0y}}{g}\right)$$

$$V_{0x} = V_0 \cos\theta$$

$$V_{0y} = V_0 \sin\theta$$

$$R = V_0 \cos\theta \left(2\frac{V_0 \sin\theta}{g}\right)$$

$$= \frac{V_0^2\, 2\cos\theta \sin\theta}{g}$$

$$= \frac{V_0^2 \sin 2\theta}{g}$$

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$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{V_0^2}\right)$$

$$R = \frac{V_0^2 \sin 2\theta}{g}$$

for fixed V_0, max R is

when $2\theta = 90^\circ$

$$\theta = 45^\circ$$

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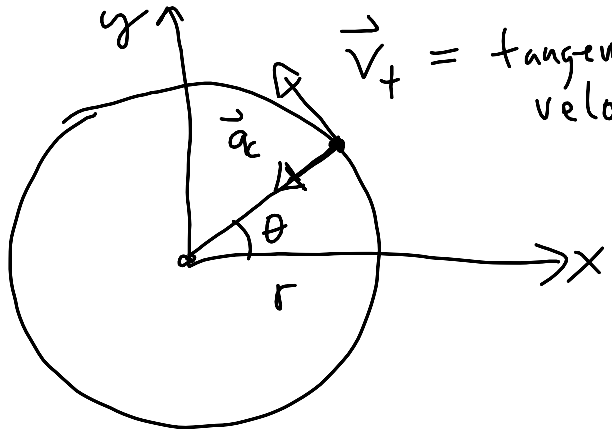

$\vec{v}_t$ = tangential velocity

$\vec{a}_c$ = centripetal acceleration
(sometimes called "radial")

Define $\omega = \frac{d\theta}{dt}$ = constant in uniform circ. motion

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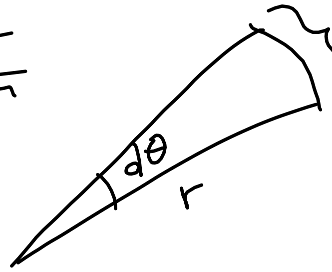
Define T = period
= time to complete 1 revolution

Then $\omega = \frac{2\pi}{T}$

$ds = r\,d\theta$



From geometry,

$$|\vec{v}_t| = \frac{ds}{dt} = \frac{d}{dt} r\,d\theta = r\frac{d\theta}{dt} = r\omega$$

In book, $|\vec{v}_t| = v$

They show that $\frac{\Delta r}{r} = \frac{\Delta v}{v}$

So $\Delta v = \frac{v}{r}\Delta r$

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Then $a_c = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{v}{r} \frac{\Delta r}{\Delta t}$

$$a_c = \frac{v}{r} \underbrace{\lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}}_{v} = \frac{v^2}{r}$$

↑

centripetal → towards the center

(centrifugal force → acts away from center

(we'll see later this is a phony force)

Note $a_c = \frac{v^2}{r}$, but $v = r\omega$

then $a_c = \frac{r^2\omega^2}{r} = r\omega^2$

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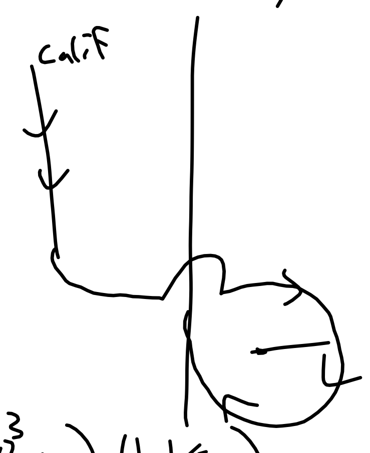
Examples:

1) I25 on ramp going north on I25, from south end of town



$v = 35$ mph

$r = 75$ m

what is a_c?

$$v = \left(35 \frac{\text{miles}}{\cancel{\text{hr}}}\right)\left(\frac{1\ \text{km}}{0.6\ \cancel{\text{mile}}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right)$$

$= 16.2$ m/s. $\quad a_c = \frac{v^2}{r} = 3.5\ \text{m/s}^2$

(over 1/3 g)

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3) Blu-ray disc is 12 cm in diameter

Typically, max $\omega = 10{,}000$ rpm

What is a_c at edge?

$$\omega = 10^4 \left(\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$

$$\simeq 10^3 \frac{\text{rad}}{\text{s}}$$

$$r = 0.06 \text{ m}$$

$$a_c = r\omega^2 = (0.06 \text{ m})\left(10^3 \frac{\text{rad}}{\text{s}}\right)^2$$

$$= 6 \times 10^4 \frac{\text{m}}{\text{s}^2} \quad (6{,}000 \text{ g's!})$$

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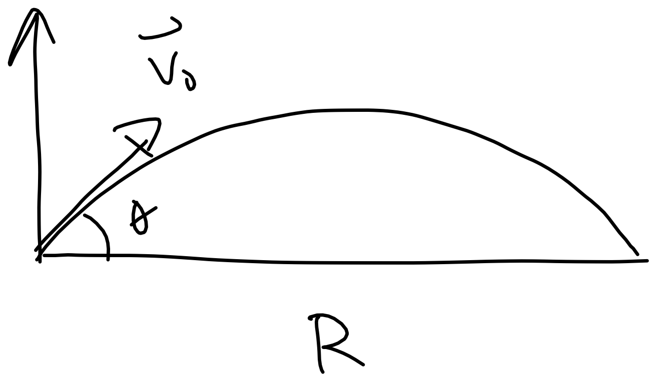

We know $|\vec{V}_0| = 13$ m/s

$R = 10$ m

what should θ be?

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