Vector velocity $\vec{v}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}\right)$$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

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Vector acceleration

$$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

a_x, a_y, a_z are components of acceleration

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For constant acceleration in either x or y,

$$x = x_0 + V_{0x}t + \frac{1}{2}a_x t^2$$

$$V_x = V_{0x} + a_x t$$

$$V_x^2 = V_{0x}^2 + 2a_x(x - x_0)$$

$$y = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$$

$$V_y = V_{0y} + a_y t$$

$$V_y^2 = V_{0y}^2 + 2a_y(y - y_0)$$

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For projectiles,

$a_x = 0$, $a_y = -g$

If projectile starts at origin

$x_0 = 0$, $y_0 = 0$

$x = V_{0x}t$, $V_x = V_{0x}$	where
$y = V_{0y}t - \frac{1}{2}gt^2$	$V_{0x} = V_0 \cos\theta$
$V_y^2 = V_{0y}^2 - 2gy$	$V_{0y} = V_0 \sin\theta$
$V_y = V_{0y} - gt$	where θ = launch elevation angle

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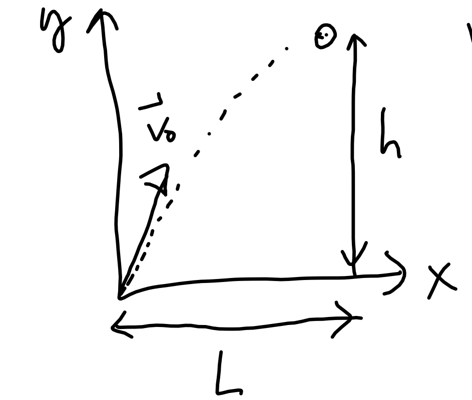
Example 4.7



We are given

$v_0 = 70\,\text{m/s}$

$\theta = 75^\circ$

Look at vertical motion first

(a) At highest pt, $y = h$, $v_y = 0$

$$v_y^2 = v_{oy}^2 - 2gy \Rightarrow 0 = v_{oy}^2 - 2gh$$

$$h = \frac{v_{oy}^2}{2g}$$

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But $v_{oy} = v_0 \sin\theta$

$$h = \frac{(70\,\text{m/s})^2 \sin^2(75^\circ)}{2\,(9.8\,\text{m/s}^2)} = 233\,\text{m}$$

(b) $v_y = v_{oy} - gt$

used often

again, at highest pt, $v_y = 0$, $t = t_h$

so $v_{oy} = g t_h \Rightarrow t_h = \frac{v_{oy}}{g} = 6.9\ \text{s}$

For horizontal motion $x = v_{ox} t$

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To find L, note that $x = L$ when $t = t_h$

$$L = V_{0x} t_h = \frac{V_{0x} V_{0y}}{g}$$

$$= \frac{(V_0 \cos\theta)(V_0 \sin\theta)}{g}$$

$$= \frac{V_0^2 \cos\theta \sin\theta}{g} = 125 \text{ m}$$

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(d) Displacement vector

$$\vec{D} = (125\text{ m})\hat{i} + (233\text{ m})\hat{j}$$

$$|\vec{D}| = \sqrt{(125)^2 + (233)^2} = 264 \text{ m}$$

Let ϕ = elevation angle of $\vec{D}$

$$= \tan^{-1}\left(\frac{233}{125}\right) = 62^\circ$$

(note - less than θ)

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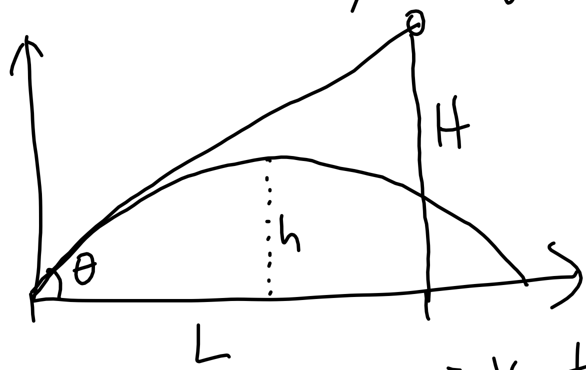
Shoot the monkey analysis



For the ball, $y = V_{oy}t - \frac{1}{2}gt^2$

$$V_y = V_{oy} - gt$$

$$V_y^2 = V_{oy}^2 - 2gy$$

From example 3.7, we know that $h = \frac{V_{oy}^2}{2g}$

For monkey, $y_0 = H$, $V_{oy} = 0$, so $y = H - \frac{1}{2}gt^2$

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$$V_y = -gt$$

$$V_y^2 = -2g(y - H) = 2g(H - y)$$

We could combine to show that

$y(\text{ball}) = y(\text{monkey})$

for all times $t > t_h$

For next time, we'll show that this is true.

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