Displacement $\Delta x = x_1 - x_0$

Average velocity $\bar{v} = \frac{x_1 - x_0}{t_1 - t_0}$

Average speed $\bar{s} = \frac{\text{distance along path}}{t_1 - t_0}$

Example

distance = 1 mile

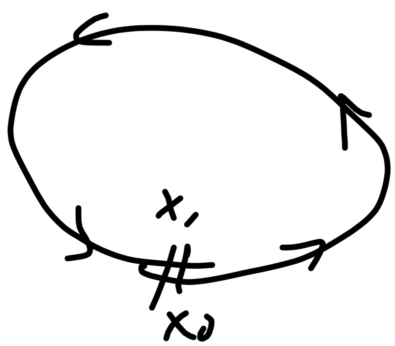
time = 30 seconds

$\bar{s} = \frac{1 \text{ mile}}{30 \text{ s}} \left(\frac{3600 \text{ s}}{1 \text{ hr}} \right) = 120 \text{ mph}$

Sep 5-9:34 AM

Average velocity?

We know $x_1 = x_0$, so $\bar{v} = 0$



Sep 5-9:47 AM

Instantaneous velocity $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

Distinction: $\bar{v}$ is defined between 2 points

v is defined at a point

Notes:

1) v is the slope of a line tangent to displacement vs. time curve

2) $v = \frac{dx}{dt}$ = derivative of x with respect to t

Sep 5-9:53 AM

Instantaneous speed $s = |\vec{v}(t)|$

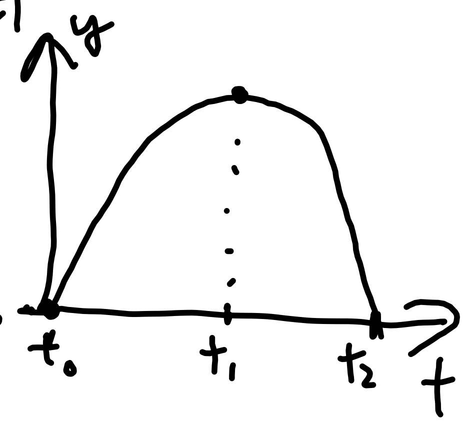
Example:

You fire a rocket straight up, and we know $y = bt - ct^2$

where $b = 82\ m/s$

$c = 4.9\ m/s^2$

When is $v = 0$?



Sep 5-9:58 AM

$$v = \frac{dy}{dt}$$

Use "polynomial rule" $x(t) = bt^n$

$$\frac{dx}{dt} = bnt^{n-1}$$

$$V = \frac{dy}{dt} = b - 2ct$$

Now $v = 0$ when $b = 2ct_1$

or $t_1 = \frac{b}{2c} = \frac{82\ m/s}{9.8\ m/s^2} = 8.4s$

Sep 5-10:04 AM

When does it land?

We know $y_0 = 0$, $t_0 = 0$

$$y = bt - ct^2$$

$$0 = bt_2 - ct_2^2$$

$$t_2 = \frac{b}{c} = \frac{82\ m/s}{4.9\ m/s^2} = 16.8\ s$$

Sep 5-10:10 AM

Instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Note:

1) $a = \frac{dv}{dt} = \frac{d}{dt} v = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2x}{dt^2}$

Second derivative

2) If a = constant (most problems)

then $a = \bar{a}$

Sep 5-10:12 AM

3) If v is positive, and a is negative, then "deceleration"

(same is true if v is negative, a is positive)

4) Units: $\frac{L}{T^2}$

Equations of motion

If $a = \bar{a}$ = constant, then

$$a = \frac{v - v_0}{t - t_0}$$

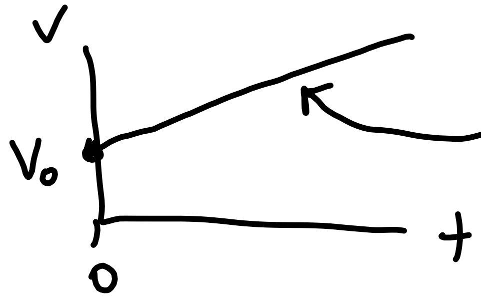
where v_0 = velocity at time t_0

Sep 5-10:18 AM

We can always let $t_0 = \emptyset$

then $a = \dfrac{v - v_0}{t}$

$$\boxed{v(t) = v_0 + at} \quad 3.12$$



slope = a

Sep 5-10:18 AM

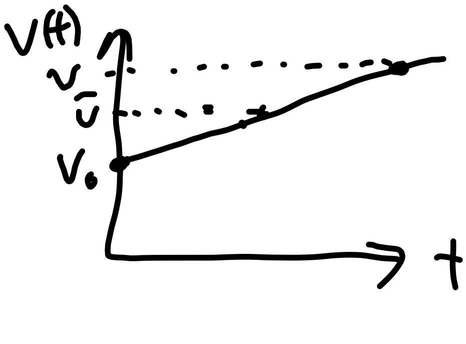
Now, for $x(t)$, first let $x = x_0$ at time $t_0 = 0$

$x = x_0 + \bar{v}t$

(by def'n of $\bar{v}$)

But we want this in terms of v not $\bar{v}$!

substitute



$$\bar{v} = \frac{v(t) + v_0}{2}$$

just the average

Sep 5-10:27 AM

$$x(t) = x_0 + \left(\frac{v(t) + v_0}{2}\right)t$$

$$= x_0 + \left(\frac{v_0 + at + v_0}{2}\right)t$$

$$\boxed{x(t) = x_0 + v_0 t + \frac{1}{2}at^2} \quad 3.13$$

~~(long)~~ "Sarge's" example

Finally, we can combine 3.12 and 3.13

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)} \quad 3.14$$

Sep 5-10:31 AM

We have 4 variables: x, t, v, a

Only have 2 indep equations

Usually, we specify one of variables,

then t is the indep variable

Sep 5-10:33 AM