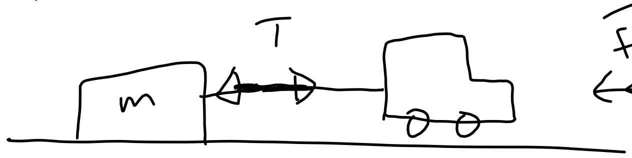
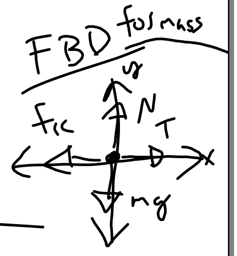
Units for work (and hence, energy) are Joules

$$W = \int \vec{F} \cdot d\vec{r} \quad \text{units of } N \cdot m = J = \frac{kg \cdot m^2}{s^2}$$

Example with friction





A large mass is pulled by a truck a distance Δx against frictional force f_K.

If $\vec{T}$ = constant, then

$$W_{truck} = \vec{T} \cdot \Delta x \hat{i} = T \Delta x$$

$$W_{Friction} = -f_K \Delta x$$

$$W_{net} = W_{truck} + W_{friction} = (T - f_K)\Delta x$$

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Work - KE Thm

$$W_{net} = \underbrace{\Delta K}_{K_f - K_i} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

If we know (or calculate) T and f_K, we can find v_f given v_i

Consider 2 situations:

(i) No net force on mass

$$\sum F_x = 0 \rightarrow T - f_K = 0 \rightarrow T = f_K$$

$$\sum F_y = 0 \rightarrow N - mg = 0 \rightarrow N = mg$$

$$W_{net} = (T - f_K)\Delta x = 0$$

$$v_f = v_i = \text{constant} \rightarrow v_x = \text{constant}$$

We already knew this because

$\vec{a} = 0$, $\vec{v}$ = constant

* If there is no net force, then there is no change in kinetic energy

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Suppose in case (i), we had $\Delta x = 100\,m$

$\mu_k = 0.25$, $m = 200\,kg$, $v_i = 3.2\,m/s$

Then $f_k = \mu_k N = \mu_k mg = 500\,N$

So $T = 500\,N$ too

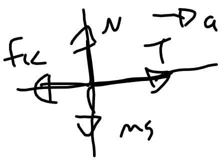
And, since $v =$ constant,

$$\Delta x = v\Delta t \rightarrow \Delta t = \frac{\Delta x}{v} = \frac{100\,m}{3.2\,m/s}$$

$$\Delta t = 32\,s$$

(ii) Suppose same frictional force, same initial velocity, same mass, but we pull with tension $T = 600\,N$

FBD for mass



$T - f_k = ma_x$

$a_x = \dfrac{100\,N}{200\,kg} = 0.5\,m/s^2$

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Now we have

$$W_{net} = (F - f_k)\Delta x$$
$$= (100\,N)(100\,m) = 10^4\,J$$

note from before, $v_i = 3.2\,m/s$

$$K_i = \frac{1}{2}mv_i^2 = \frac{200\,kg}{2}(3.2\,m/s)^2$$
$$= 10^3\,J$$

$$W_{net} = K_f - K_i \rightarrow K_f = W_{net} + K_i$$
$$\frac{1}{2}mv_f^2 = 1.1 \times 10^4\,J$$

$$v_f = \sqrt{\frac{2}{m}(1.1 \times 10^4\,J)} = \sqrt{110\,\frac{m^2}{s^2}} = 10.5\,m/s$$

Note from the Kinematic equations

$$v_x^2 = v_{x0}^2 + 2a_x\Delta x$$

$$v_x = \sqrt{(3.2\,m/s)^2 + 2(0.5\,m/s^2)(100\,m)}$$

$$= \sqrt{110\,m^2/s^2} = 10.5\,m/s$$

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Power $P = \frac{dW}{dt}$ = rate of doing work

In previous example, we could find the mean power delivered by the track

$\bar{P} = \frac{\Delta W_{track}}{\Delta t}$ not the same as W_{net}

in case (i), we had $W_{net} = 0$ on mass (net work done)

$\Delta W_{track} = (500\ N)(100\ m) = 5\times10^4\ J$

$\bar{P} = \frac{\Delta W_{track}}{\Delta t} = \frac{5\times10^4\ J}{32\ s} = 1.6\times10^3 \frac{J}{s}$

Watt

in case (ii)

$\Delta W_{track} = (600N)(100m) = 6\times10^4\ J$

$\bar{P} = \frac{6\times10^4\ J}{\Delta t}$. Need Δt for case (ii)

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$a_x = 0.5\ m/s^2$

$v_x = v_{x0} + a_x \Delta t$

$\Delta t = \frac{v_x - v_{x0}}{a_x} = \frac{10.5\ m/s - 3.2\ m/s}{0.5\ m/s^2}$

$= 14.6\ s$ (much shorter)

$\bar{P} = \frac{6\times10^4\ J}{14.6\ s} = 4.1\times10^3\ W$

(recall from case (i) $\bar{P} = 1.6\times10^3\ W$)

There is another expression for P that is sometimes useful (usually when $\vec{v}$ = const.)

$P = \frac{dW}{dt} = \frac{\vec{F}\cdot d\vec{r}}{dt} = \vec{F}\cdot\frac{d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$

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Previous example, for case (i),

$V = \text{constant} = 3.2\ m/s$

$F = \text{constant} = 500\ N$

$\bar{P} = (500\ N)(3.2\ m/s) = 1.6 \times 10^{3}\ \underbrace{\frac{N \cdot m}{s}}_{\frac{J}{s}} = \frac{J}{s} = W$

Potential Energy

For some forces (such as gravity, springs, electrostatic), we can define a potential energy

$$\underbrace{\Delta U_{AB}}_{} = U_B - U_A = -\underbrace{W_{AB}}_{\text{net work done on an object, with no dissipation}}$$

Important:

Only changes in U are physically relevant

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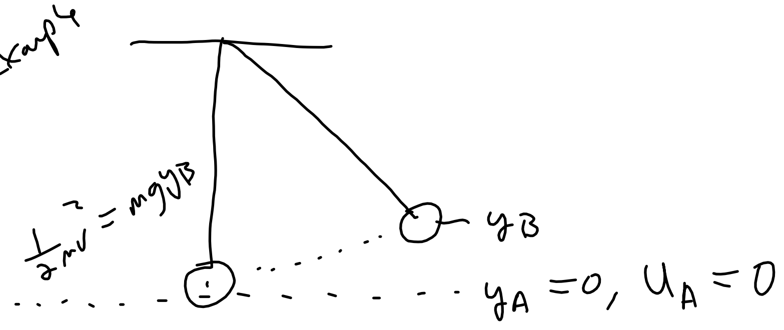
Types of PE

1. Gravitational

$$\Delta U_{grav} = -W_{AB} = mg(y_B - y_A)$$

example



$$\Delta U_{grav} = mgy_B$$

we will find in chapter 8,

$$\boxed{E = K + U} = \text{constant with no dissipation}$$

where E = total energy

(conservation of energy)

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