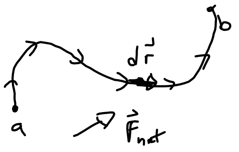
Work and Kinetic energy

Work is a certain kind of energy
The work done <u>on</u> an object that experiences a displacement and is acted on by a net force is

$$W_{net} = \int_a^b \vec{F}_{net} \cdot d\vec{r}$$



Now if $\vec{F}$ = constant, then

$$W_{net} = \vec{F} \cdot \int_a^b d\vec{r} = \vec{F} \cdot \vec{r}_{ab}$$

Example

(i) lift a book straight up to height = h

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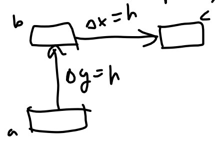
$\vec{F}_{av} = mg\hat{j}$
Displacement $\vec{r}_{ab} = h\hat{j}$
So $W_{net} = \vec{F}_{av} \cdot \vec{r}_{ab} = mgh$

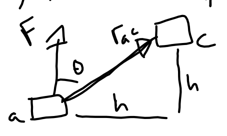
(ii) lift book up h, sideways a distance h



$W_{net} = W_{ac} = \underbrace{W_{ab}}_{mgh} + \underbrace{W_{bc}}_{\phi \text{ because } F \perp \Delta x}$

$W_{net} = mgh$

(iii) lift book up diagonally



$W_{net} = \vec{F} \cdot \vec{r}_{ac}$
$= F r_{ac} \cos\theta$

now $F = mg$

$r_{ac} = ?$ Well, we know $\cos\theta = \dfrac{h}{r_{ac}}$

So $r_{ac} = \dfrac{h}{\cos\theta}$

$W_{net} = mgh$

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The work done by a constant force is independent of path.

example

So for gravity,

$W_{net} = mgh$ net work done by me. (neglecting friction)

$W_{net} = -mgh$ net work done by gravity is negative

In case of a falling object, displacement is in $-y$ direction, so Work done by gravity should be negative.

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What about the normal force?

sliding box, work done by $\vec{N} = 0$

work done by friction

What about Friction?

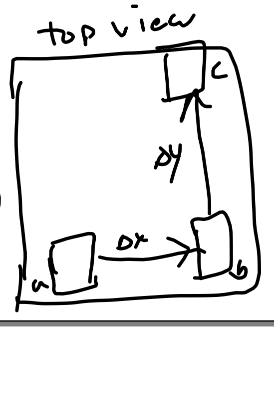
f_k is always opposite to the displacement, so $W \leq 0$

and W depends on path.

Example

slide a book from a to c on a table

$W_{ac}(\text{friction}) = -f_k \Delta x + -f_k \Delta y = -f_k(\Delta x + \Delta y)$



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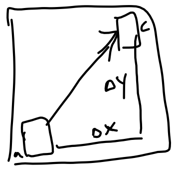
(ii) Suppose we slide the book diagonally



Displacement $= \sqrt{\Delta X^2 + \Delta Y^2}$

$W_{net}(\text{friction}) = -f_k \underbrace{\sqrt{\Delta X^2 + \Delta Y^2}}_{\text{smaller than } \Delta X + \Delta Y}$

So $|W_{net}(\text{friction})|$

is less for the shorter path

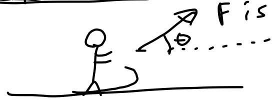
We have a few HW problems involving
W_{net}, friction, grav. force, (ETA has a spring)

Also in practice exam, but this is easier than #34

because

$\vec{F}$ is horizontal

In #34



$\vec{F}$ is angled upward

If $\vec{F}$ has an upward or downward component, then there is usually an effect on $\vec{N}$ (and thus, $\vec{f}_k$)

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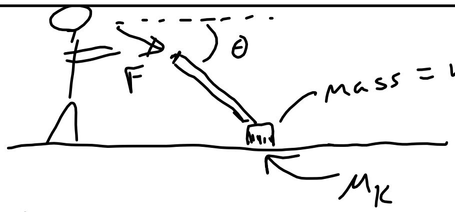
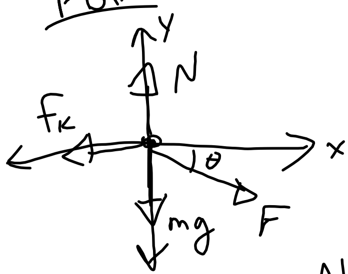


suppose we push at constant velocity

FBD



$\Sigma F_x = 0, \quad F\cos\theta - f_K = 0$

$\Sigma F_y = 0, \quad N - mg - F\sin\theta = 0$

$N = mg + F\sin\theta$

and $f_K = \mu_K N = \mu_K(mg + F\sin\theta)$

$F\cos\theta - \mu_K(mg + F\sin\theta) = 0$

algebra, $F = \dfrac{\mu_K mg}{\cos\theta - \mu_K\sin\theta}$ for this particular problem

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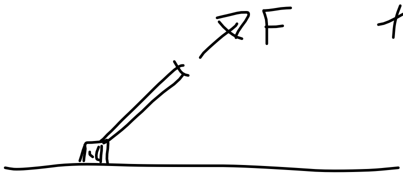
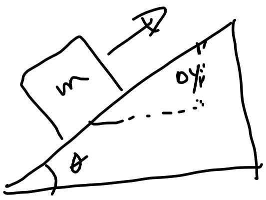
If we were pulling instead of pushing,



then F will be different

We can also think ramps



If there is no friction, then only grav. force and

$$W_{net} = mg\Delta y$$

If we include friction, then

$$W_{net} = \underbrace{W_{applied}}_{\text{must be larger than } mg\Delta y} + \underbrace{W_{friction}}_{\text{negative}} = mg\Delta y$$

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Work - Kinetic energy theorem

Define Kinetic energy $= K = \frac{1}{2}mv^2$

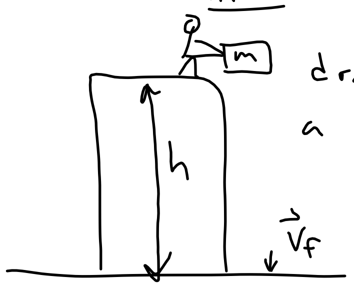
Then W_{net} = net work done on an object

$$= \Delta K = K_f - K_i$$

Final Kinetic energy after the work done

initial K, before work

Example



drop a mass from rest a distance h

$$K_f = \frac{1}{2}mv_f^2$$

$$K_i = 0$$

$$W(\text{gravity}) = W_{net} = mgh$$

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So $\cancel{m}gh = \frac{1}{2}\cancel{m}v_f^2$

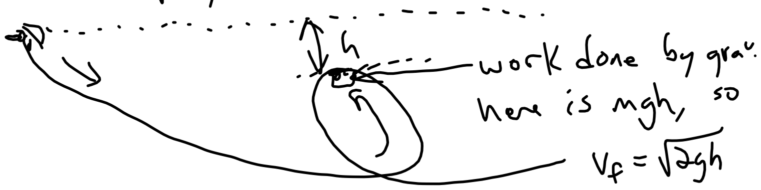
$$v_f = \sqrt{2gh}$$

Note from the Kinematic eq's,

$$v_y^2 = \underbrace{v_{oy}^2}_{0} - 2g\underbrace{(y-y_0)}_{h}$$

$$v_y = \sqrt{2gh}$$

For some problems, work - K thm is easier to apply



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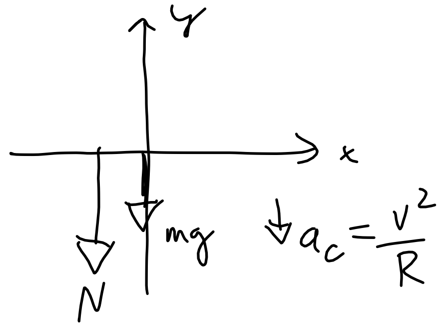
So FBD at top of loop



$$\Sigma F_y = ma_y = -ma_c = -\frac{mv^2}{R}$$

$$-N - mg = \frac{-mv^2}{R}$$

$$N = m\left(\frac{v^2}{R} - g\right)$$

check my algebra!

Oct 3-10:41 AM