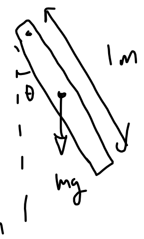
For a physical pendulum, we still have

$$\omega = \sqrt{\frac{mgL}{I}}$$

But now $I \neq mL^2$ (what we used for simple)

For the meter stick,



$L = 0.5\,m$,

$I = \frac{m\ell^2}{3}$ where ℓ = total length

from table 10.20. In this case $L = \frac{\ell}{2}$

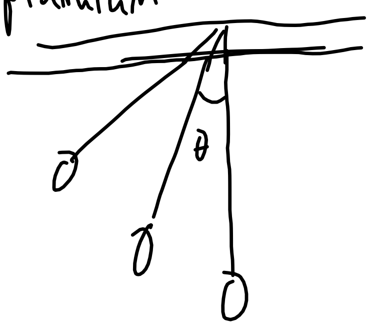
So $\omega = \sqrt{\frac{mgL}{\frac{1}{3}m(2L)^2}} = \sqrt{\frac{3g}{4L}} = 3.8\,s^{-1}$ or

$$T = \frac{2\pi}{\omega} = 1.64\,s$$

Nov 16-9:31 AM

Angular acceleration for a pendulum

$$\alpha = \frac{d^2\theta}{dt^2}$$



For a mass/spring, we have

$x = A\cos\omega t$ where $\omega = \sqrt{k/m}$.

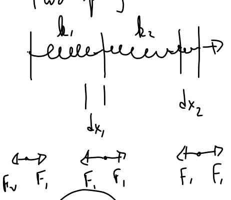
For pendulum

$\theta = \theta_{max}\cos(\omega t)$ $\quad \frac{d^2\theta}{dt^2} = -\theta_{max}\omega^2\cos(\omega t)$

where $\omega = \sqrt{\frac{g}{L}}$ for simple pendulum. Re-vote!

Nov 16-9:52 AM

Two springs in series



Total displacement

$dx_{tot} = dx_1 + dx_2$

Hooke's Law

$F_1 = -k_{eff}(dx_1 + dx_2)$

$F_1 = k_1 dx_1 = k_2 dx_2$

$dx_2 = \frac{k_1}{k_2} dx_1$

$F_1 = -k_{eff}(dx_1 + \frac{k_1}{k_2} dx_1)$

$F_1 = -k_1 dx_1$

$$F_1 = -k_{eff}\left(1 + \frac{k_1}{k_2}\right)\cancel{dx_1} = -k_1 \cancel{dx_1}$$

$$k_{eff} = \frac{k_1}{1 + \frac{k_1}{k_2}} = \frac{k_1 k_2}{k_1 + k_2}$$

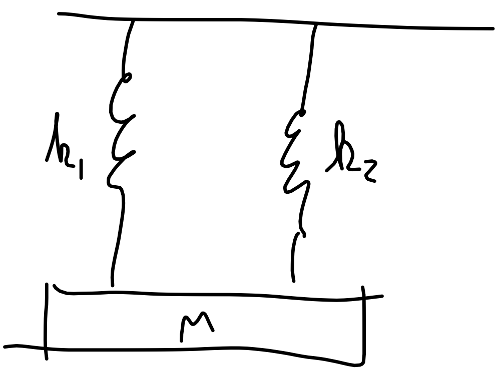
If $k_1 = k_2$ (ETA problem)

Then $k_{eff} = \frac{k}{2}$

This is similar to 2 capacitors in series (phy 122)

Nov 16-10:07 AM

Springs in parallel



$k_{eff} = k_1 + k_2$

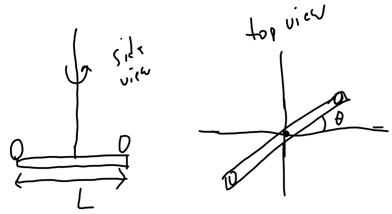
Nov 16-10:17 AM

Torsional oscillator

$\theta = \theta_{max} \cos(\omega t)$

where $\omega = \sqrt{\frac{K}{I}}$, K = torsional constant, I = moment of inertia

Cavendish:



$K = \omega^2 I$

We can find K by measuring ω, and

$I = I_{rod} + I_{masses}$

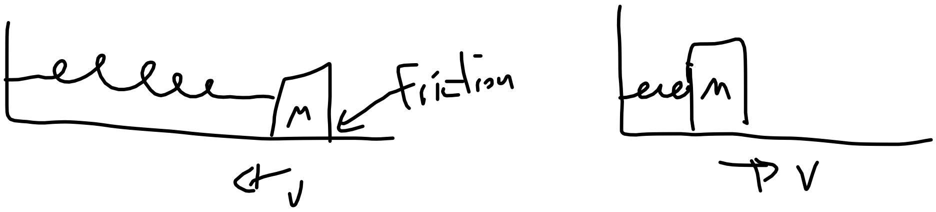
$= \frac{1}{12} M_{rod} L^2 + 2\left(m_{ball}\left(\frac{L}{2}\right)^2\right)$

measure M_{rod}, M_{ball}, L

Nov 16-10:23 AM

Damped oscillations

If we consider friction, (hard problem)



$\mu_k mg - kx = ma$ $\quad\quad -kx - \mu_k mg = ma$

Frictional force changes sign, depending on position

Nov 16-10:29 AM

But if damping depends on velocity, then we have a chance

$F_{damping} = -bv_x$ (viscous fluid, shock absorber)

$b \leftarrow$ damping constant

$$\sum F = -kx - bv_x = -kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{-k}{m}x - \frac{b}{m}\frac{dx}{dt}$$

Solution: $x(t) = Ae^{-\left(\frac{b}{2m}\right)t}\cos(\omega' t)$

where $\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

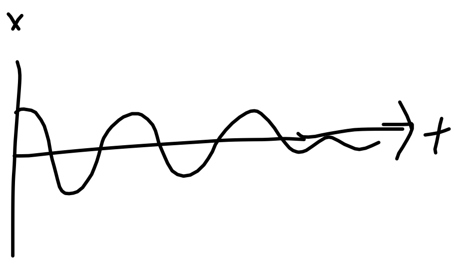
For ETA problem,

$E = \frac{1}{2}kA^2$

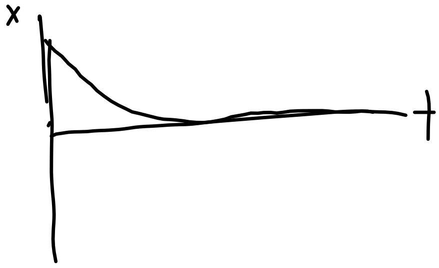
Nov 16-10:33 AM

3 cases for damped oscillators:

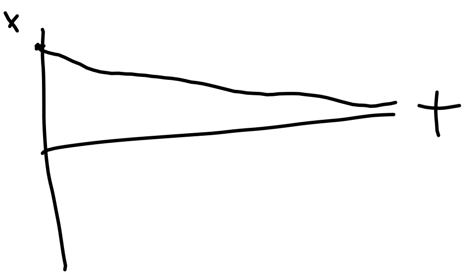
1. $b < 2\sqrt{km}$, underdamping



2. $b = 2\sqrt{km}$ critical damping



3. $b > 2\sqrt{km}$ overdamped



Nov 16-10:39 AM