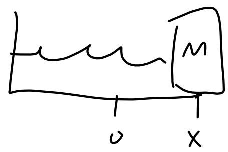
Nov 14^{th}, chap 15 Simple Harmonic Motion

stretch the spring



Let general displacement be x (x will be a function of time)

Let maximum displacement $= A =$ amplitude

Hooke's Law $F_x = -kx$

As mass is oscillating left and right,

define ω to be a measure of the frequency

$\omega = 2\pi f$, where $f =$ frequency in cycles per second

Calculate the period $T = \frac{1}{f} = \frac{2\pi}{\omega}$

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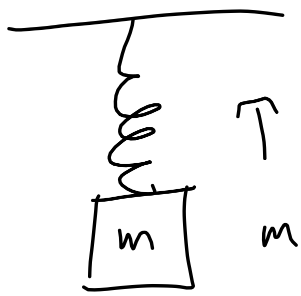
Example



$T = 0.4\ s$

$m = 2\ kg$

1. $\omega = \frac{2\pi}{T} = 15.7\ s^{-1}$

2. We will find that $\omega = \sqrt{\frac{k}{m}}$, so that

$k = m\omega^2 = 493\ N \cdot m^{-1}$

units are $kg\left(\frac{rad}{s}\right)^2 = kg/s^2 = \frac{kg\,m}{s^2}\,\frac{1}{m}$ N

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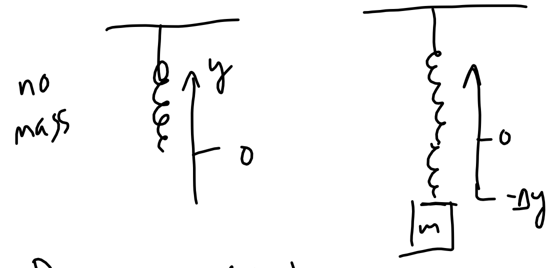
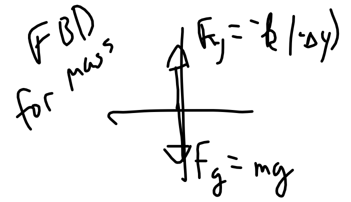
Is the hanging mass really the same as the horizontal spring/mass?





$\Delta y = \frac{mg}{k} \Rightarrow$ in previous example, $\Delta y = 0.04$ m

Now, the mass will oscillate about equilibrium position at $-\Delta y$.

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To get ω, note

$$F_x = -kx = ma = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

solution, guess that $x(t) = A\cos\sqrt{\frac{k}{m}}\,t$

check: $\frac{dx}{dt} = -A\sqrt{\frac{k}{m}}\sin\sqrt{\frac{k}{m}}\,t$

$$\frac{d^2x}{dt^2} = -A\left(\frac{k}{m}\right)\cos\sqrt{\frac{k}{m}}\,t = -\frac{k}{m}\left(A\cos\sqrt{\frac{k}{m}}\,t\right)$$

x!

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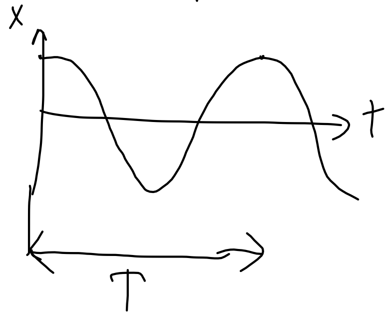
So $x(t) = A\cos\sqrt{\frac{k}{m}}\,t$

Let $\omega = \sqrt{\frac{k}{m}}$ = angular frequency

A = amplitude



Note: L+S includes a phase factor ϕ

$x(t) = A\cos(\omega t + \phi)$

(often set $\phi = 0$)

$x(t) = A\cos\omega t$

$v(t) = \frac{dx}{dt} = -(A\omega)\sin\omega t$, $|v_{max}| = A\omega$

$a(t) = \frac{d^2x}{dt^2} = -(A\omega^2)\cos\omega t$,

$|a_{max}| = A\omega^2$

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Energy in an oscillator.

$E_{tot} = K + U$ = constant with no dissipation

$$\left.\begin{aligned} K &= \tfrac{1}{2}mv^2 \\ U &= \tfrac{1}{2}kx^2 \end{aligned}\right\} \quad \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \text{constant} = E_{tot}$$

Note that at the turning points, $v = 0$, $x = \pm A$

so E_{tot} at turning pts, then $E_{tot} = \frac{1}{2}kA^2$

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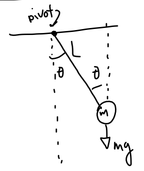
Pendulum using angular momentum



angular acceleration

$$\tau = I\alpha$$

$$-mgL\sin\theta = I\frac{d^2\theta}{dt^2}$$

for small θ, let $\sin\theta = \theta$

$$\frac{d^2\theta}{dt^2} = -\frac{mgL}{I}\theta$$

Just like $\frac{d^2x}{dt^2} = -\frac{k}{m}x$ (where $\omega = \sqrt{\frac{k}{m}}$)

For pendulum $\boxed{\omega = \sqrt{\frac{mgL}{I}}}$

For simple pendulum, all mass is located a distance L from pivot.

Then $I = mL^2$, so $\omega = \sqrt{\frac{g}{L}}$

Example: $L = 1m$. $\omega = \sqrt{\frac{9.8\ m/s^2}{1\ m}}$

$\omega \sim 3.2$ rad/s

$$T = \frac{2\pi}{\omega} \simeq 2\ s$$

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For meter stick,

$$f = \frac{10\ \text{cycles}}{17\ \text{sec}} \simeq 0.6\ \text{cycles/s}$$

$$T = \frac{1}{f} = 1.7\ s \quad (\text{less than } 2\ s)$$

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