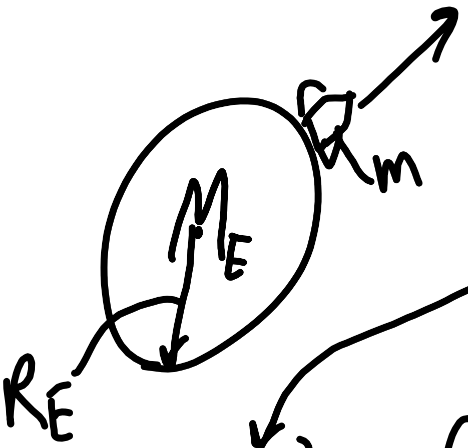
11/9

end of 13.3

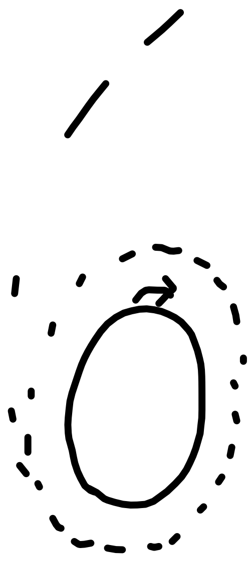
escape velocity:



$$\frac{1}{2}mv_i^2 - \frac{GM_Em}{R_E} = \frac{1}{2}m\underbrace{v_f^2}_{=0} - \frac{GM_Em}{\underbrace{r}_{\infty}}$$

$$\frac{1}{2}mv_{esc}^2 = \frac{GM_Em}{R_E}$$

$$\boxed{v_{esc} = \sqrt{\frac{2GM_E}{R_E}}}$$



13.4 Satellite orbits & energy

$$F_c = \frac{mV^2}{r}$$

$$F_g = \frac{GMm}{r^2}$$

for circular orbit

$$F_c = F_g$$

$$\frac{\cancel{m}V^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$$V_{tan}^{2} = \sqrt{\frac{GM}{r}}$$

$$\cancel{\left(V_{esc} = \sqrt{\frac{2GM_E}{R_E}}\right)}$$

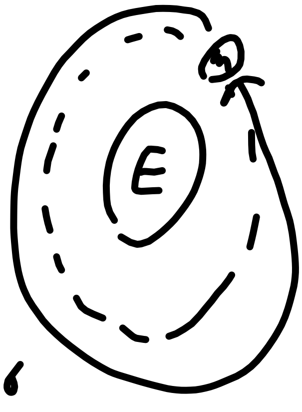
v for circular orbit

$$V_{tan}^2 = \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

period for circular orbit

$$T^2 = \frac{(2\pi)^2 r^3}{GM} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{r^3}{GM}}}$$

ex 1 Earth Mass = ?



period (T) = 27.3 days = $2.36 \cdot 10^{6}$ s

orbital radius = $3.84 \cdot 10^{8}$ m
(average)

$$\frac{(2\pi)^2 r^3}{G M_E} = T^2$$

$$M_E = \frac{G T^2}{4\pi^2 r^3} = \frac{(6.67 \cdot 10^{-11})(2.36 \cdot 10^{6}\,\mathrm{s})^2}{4\pi^2 (3.84 \cdot 10^{8}\,\mathrm{m})}$$

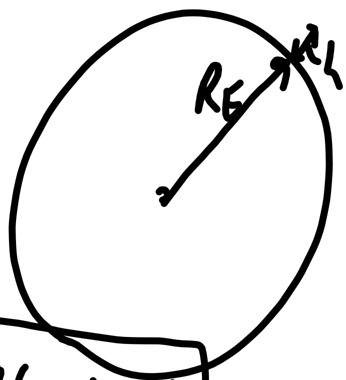
$$M_E \cong 6 \cdot 10^{24}\ \mathrm{kg}$$

text says $M_E = 5.96 \cdot 10^{24}$ kg

ex 2

escape velocities
radial & tangential

$$V_{esc_r} = \sqrt{\frac{2GM_E}{R_E}}, \quad V_{esc_t} = \sqrt{\frac{GM_E}{R_E+h}} \approx \sqrt{\frac{GM_E}{R_E}}$$



$R_E + h \approx R_E$

$$V_{esc_r} = \sqrt{\frac{2(6.67\cdot10^{-11})(5.96\cdot10^{24}\,kg)}{6.36\cdot10^{6}\,m}}$$

$$= 11,181 \frac{m}{s}$$

$$V_{esc_t} = \sqrt{\frac{GM_E}{R_E}} = \frac{V_{esc_r}}{\sqrt{2}} = 7,906 \frac{m}{s}$$

energy in circular orbits

K = kinetic energy $= \frac{1}{2}mv^2$

U = potentia " $= -\frac{GMm}{r}$

$v^2 = \frac{GM}{r}$

$E = K + U = \frac{1}{2}mv^2 - \frac{GMm}{r}$,

verial thrm.

$\boxed{K = \frac{1}{2}U}$

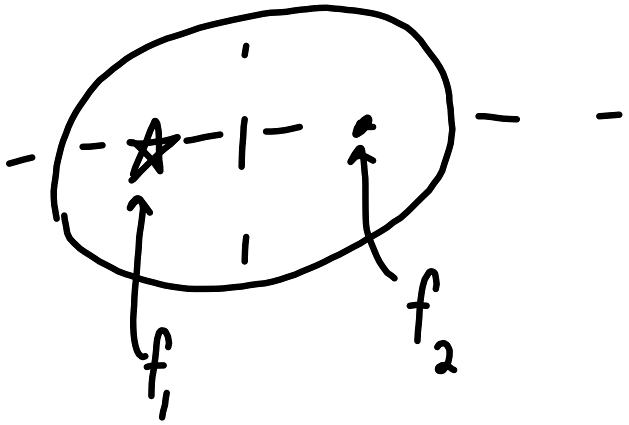
$= \frac{1}{2}m\frac{GM}{r} - \frac{GMm}{r}$

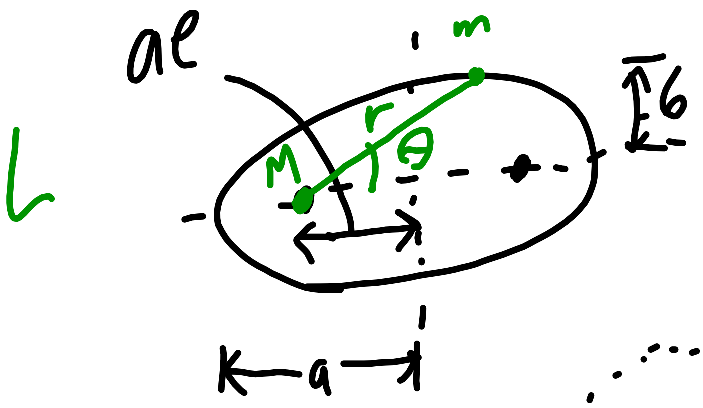
$$\boxed{E_{tot} = -\frac{GMm}{2r}}$$

$E_{tot} < 0$ for m to be gravitationally band.

13.5 Keplars Law's of Planetary motion

- 1st Law: all Planets follow elliptical orbits with the sun @ one of the focus points.





a = semimajor axis

b = semiminor axis

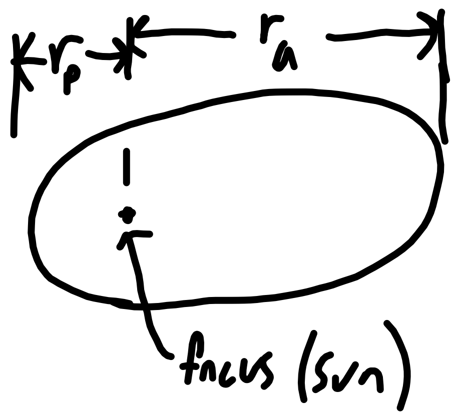
α = semi latus rectum, e = eccentricity

$$\frac{\alpha}{r} = 1 + e\cos(\theta) \quad (\text{polar eq}^{\underline{n}})$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (\text{cartesion eq}^{\underline{n}})$$

for elliptical + circular ($b=a$)

$$e=\sqrt{1-\frac{b^2}{a^2}}\,,\quad \alpha=\frac{b^2}{a}$$



r_p = perihelion

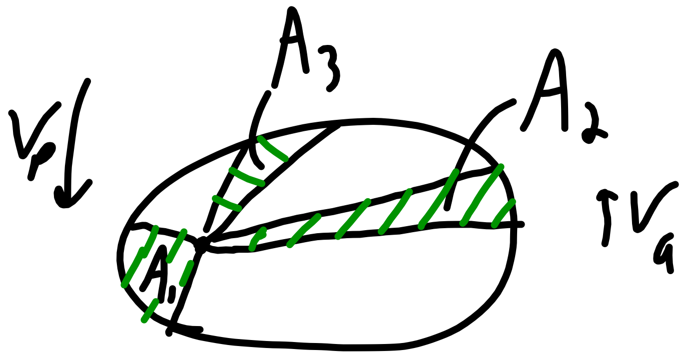
r_a = aphelion

$$r_p = a-ae = a(1-e)$$
$$r_a = a+ae = a(1+e)$$

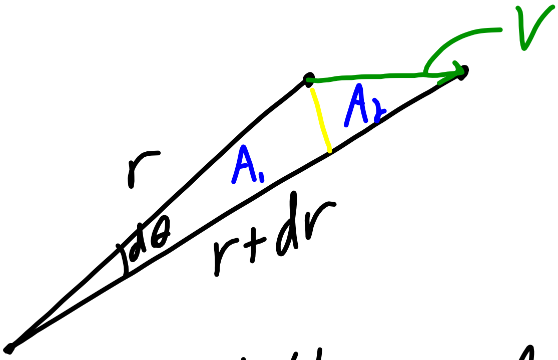
☆

- **2nd Law**

$$\frac{dA}{dt}=\text{const.}$$



in time t, $A_1 = A_2 = A_3$



$$dA = \frac{1}{2}bh = A_1 + A_2$$

$$dA = \frac{1}{2}r^2 d\theta\, k + \underbrace{\cancel{\frac{1}{2}r\,d\theta\,dr}}_{\to 0}$$

$$\frac{dA}{dt} = \frac{1}{2}r^2 \underbrace{\frac{d\theta}{dt}}_{\omega} = \frac{1}{2m}\underbrace{(mr^2)}_{I_{point}}\omega = \frac{I\omega}{2m}$$

$$\frac{dA}{dt} = \frac{L}{2m} = \text{const.} \checkmark$$

by conservation of angular momentum
($L = \text{const.}$)

3rd law:

$$T^2 \propto a^3$$

(period squared is proportional to semi major axis cubed.)

$$T^2 = \frac{4\pi}{GM}a^3$$

$(a = r)$