Gravity

Consider the moon's orbit

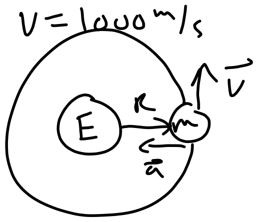
$R_{orbit} = 3.85 \cdot 10^8$ m

$T_{moon} = 27$ days

$= 2.4 \cdot 10^6$ sec

$$v = \frac{2\pi R}{T} = \frac{2\pi \cdot 3.85 \cdot 10^8 \text{ m}}{2.4 \cdot 10^6 \text{ sec}}$$

$v = 1000$ m/s



$$a = \frac{v^2}{r}$$

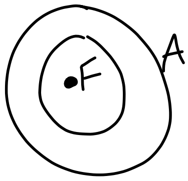
$g = 9.8$ m/s^2 $= a$?

$$\frac{v^2}{r} = \frac{(1000 \text{ m/s})^2}{3.85 \cdot 10^8 \text{ m}} = 0.0026 \text{ m/s}^2$$

≈ 3600 times less than g

the moon is about $60R_E$ away, and 3600 is 60^2

$$F_g \propto \frac{1}{r^2}$$



$A = 4\pi r^2$

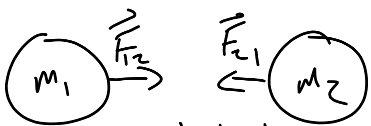
$$\frac{F}{4\pi r^2}$$

Which is greater?

a) The force of the earth on the moon

b) The force of the moon on the earth

c) They're equal



$|F_{12}| = |F_{21}|$

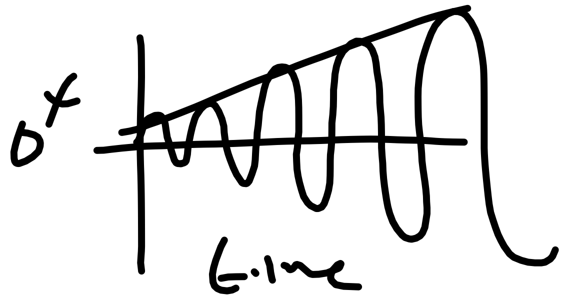
$$\vec{F}_{12} = \frac{G m_1 m_2}{r^2}\,\hat{r}_{12}$$

points from 1 to 2

The force on 1 from 2

$G = ?$

The Cavendish Experiment



$$G = 6.67 \cdot 10^{-11}\,\frac{Nm^2}{kg^2}$$

Ex 13.2) Attraction b/w Galaxies

Find the acceleration of the Milky Way due to the Andromeda galaxy. $M_{gal} \approx 800$ billion $M_\odot$ ($M_\odot = 2 \cdot 10^{30}$ kg) $d = 2.5 \cdot 10^6$ light years (1 lightyear $= 9.5 \cdot 10^{15}$ m)

$$F = \frac{G m_1 m_2}{r^2} \qquad m_1 = m_2$$

$$= \frac{6.67 \cdot 10^{-11} \frac{N\cancel{m^2}}{\cancel{kg^2}} (800 \cdot 10^9 \cdot 2 \cdot 10^{30}\ \cancel{kg})^2}{(2.5 \cdot 10^6 \cdot 9.5 \cdot 10^{15}\ \cancel{m})^2}$$

$$F = 3 \cdot 10^{29} N$$

$$a = \frac{F}{m} = \frac{3 \cdot 10^{29} N}{800 \cdot 10^9 \cdot 2 \cdot 10^{30} kg}$$

$$a = 1.9 \cdot 10^{-13}\ m/s^2$$

$\Rightarrow$ after 10^{13} s (300,000 yr)

$v = 1.9$ m/s

it will be $\sim 7 \cdot 10^9$ yrs until they collide

Ex) Attraction b/w 2 people

estimate the gravitational force b/w 2 people

$m_1 = 50$ kg $\quad m_2 = 80$ kg

$r = 0.5$ m

$$F = \frac{G m_1 m_2}{r^2}$$

$$F = \frac{6.67 \cdot 10^{-11} \frac{Nm^2}{kg^2} \cdot 50 kg \cdot 80 kg}{(.5 m)^2}$$

$$F = 1.1 \cdot 10^{-6} N$$

$$g = 9.8\,m/s^2$$

$$F = \frac{G m_1 m_2}{r^2}$$

$$W = mg$$

$$w = F \qquad \cancel{m}g = \frac{G \cancel{m_1} M_E}{R_E^2}$$

$$g = \frac{G \cdot M_E}{R_E^2} \simeq 9.8\,m/s^2$$

$$g = \frac{G \cdot M_E}{R_E^2}$$

$R_E \rightarrow 6.37 \cdot 10^6\,m$

$g \rightarrow 9.8\,m/s^2$

$$M_E = \frac{g R_E^2}{G}$$

$$= \frac{9.8\,m/s^2 (6.37 \cdot 10^6\,m)}{6.67 \cdot 10^{-11}\,\frac{Nm^2}{kg^2}}$$

$$M_E = 5.95 \cdot 10^{24}\,kg$$

$$M_E = 5.96 \cdot 10^{24}\,kg$$

Ex 13.4) Gravity above the surface

What is g 400km above the surface of the Earth?
(where the ISS orbits)

$$g = \frac{G M_E}{r^2} \quad M_E = 5.96 \cdot 10^{24} kg$$

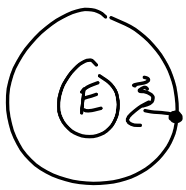
$$r = R_E + 400km$$

$$r = 6.37 \cdot 10^6 m + 4 \cdot 10^5 m$$

$$r = 6.77 \cdot 10^6 m$$

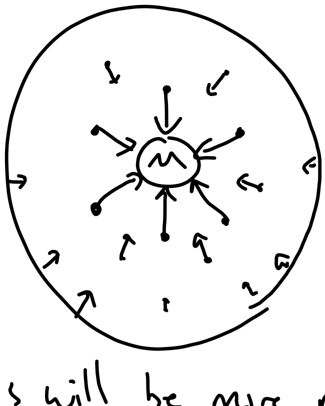
$$g = \frac{6.67 \cdot 10^{-11} \frac{Nm^2}{kg^2} \cdot 5.96 \cdot 10^{24} kg}{(6.77 \cdot 10^6 m)^2}$$

$$g = 8.67 \, m/s^2$$



$$\vec{g} = \frac{GM}{r^2} \hat{r}$$

$\vec{g}$ is a vector field



This will be more prevalent in Electromagnetics

Gauss's Law: only the mass within radius r contributes to F_{grav} and it can be considered at the center of mass

(mass outside of r does not contribute)

only for spherically symmetric

g inside a uniform ρ sphere

$$g = \frac{GM}{r^2} \quad M = \rho \frac{4}{3}\pi r^3$$

$$g = \frac{G\rho \frac{4}{3}\pi r^3}{r^2}$$

$$g = \frac{4}{3} G\rho\pi r$$

Gravitational Potential Energy

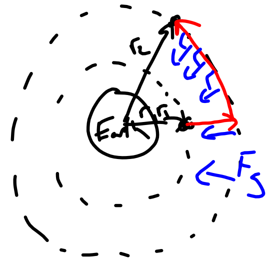
$U_g = mgy \quad \Delta U = \underline{mg\Delta y}$

$\Delta U = -W = -\int \vec{F} \cdot d\vec{x}$

if $F = mg$ (g = const)

$\Rightarrow -mg\int d\vec{x}$

Now g is not constant



$\Delta U = -W = -\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$

$\Delta U = \int_{r_1}^{r_2} F \cdot dr$

$\Delta U = \int_{r_1}^{r_2} \frac{Gm_1M_E}{r^2} dr$

$\Delta U = Gm_1M_E \int_{r_1}^{r_2} \frac{dr}{r^2} \quad (r^{-2}dr)$

$\Delta U = Gm_1M_E \left(\frac{-1}{r}\right)\Big|_{r_1}^{r_2}$

$\Delta U = -Gm_1M_E \left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$\Delta U = U_2 - U_1$

$$\boxed{U(r) = \frac{-Gm_1M_E}{r}}$$

$$U(r) = \frac{-Gm_1m_2}{r}$$

$U \to 0$ as $r \to \infty$

as objects separate, positive work is done against gravity, and U increases (becomes less negative)

Ex 13.6) How much Energy is required to lift the 9000 kg Soyuz vehicle from Earth's surface to the ISS?

$$\Delta U = U_{fin} - U_{init} = U_{ISS} - U_{Earth}$$

$$-\frac{GM_E m}{R_E + 400\,km} - \left(-\frac{GM_E m}{R_E}\right) =$$

$$-GM_E m\left(\frac{1}{R_E + 400\,km} - \frac{1}{R_E}\right)$$

$$-6.67\cdot10^{-11}\cdot5.96\cdot10^{24}\cdot9000\cdot\left(\frac{1}{6.77\cdot10^{6}} - \frac{1}{6.37\cdot10^{6}}\right)$$

$$\Delta U = 3.32\cdot10^{10}\,J$$

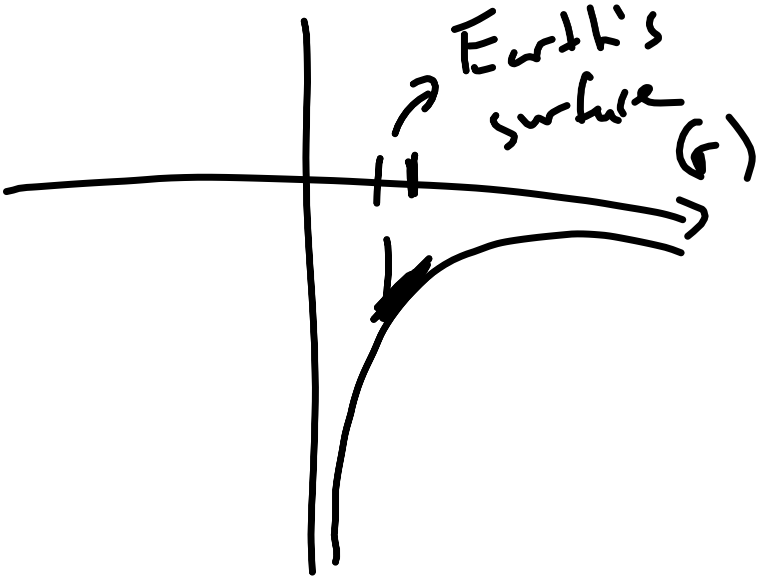
Conservation of Energy

$$E = \text{constant} = K + U$$

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2} m_1 v_1^2 - \frac{GMm}{r_1} = \frac{1}{2} m v_2^2 - \frac{GMm}{r_2}$$

Consider $U = \frac{-Gm_1m_2}{r}$



where does $U = mgy$ come from

Ch13 HW:

15, 24, 30, 33, 42, 48, 51