Physics 121 – October 31, 2017

Announcements:

Interesting NRAO seminar on Friday, Nov 3.

Assignments:

This week:

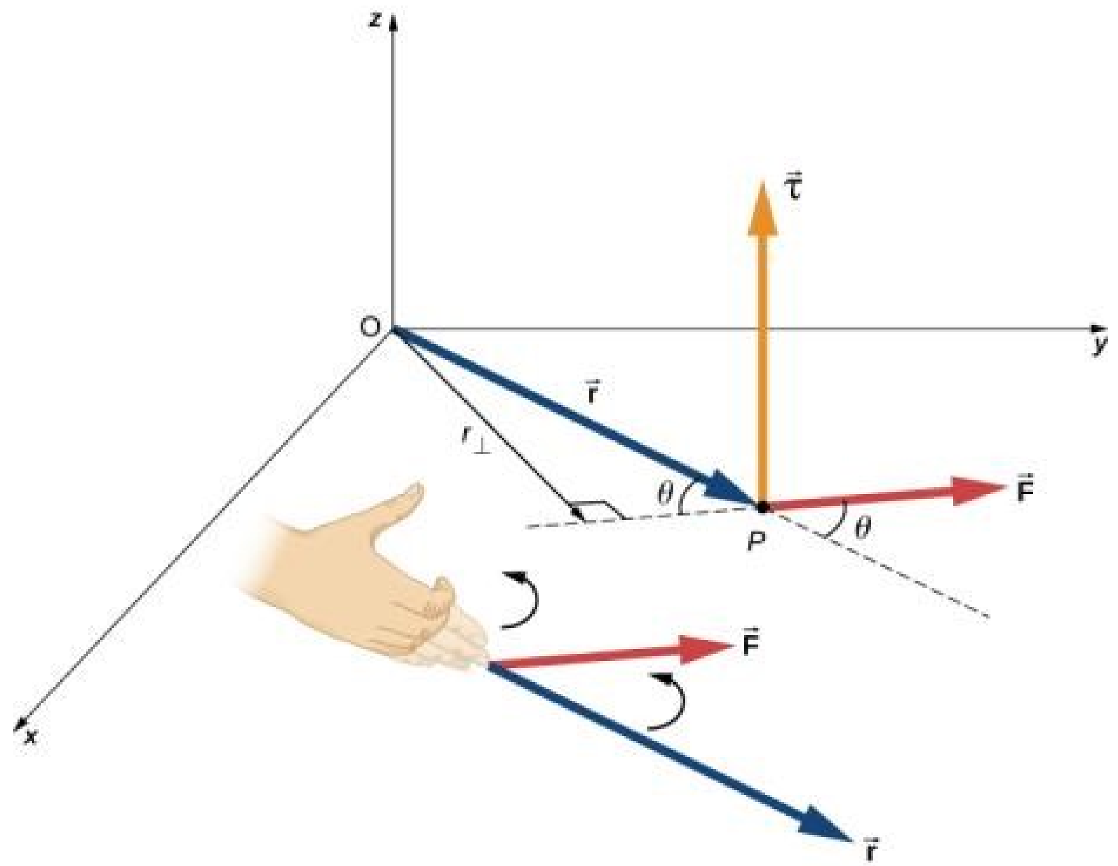
- Read Chapter 11.
- Complete ETA Problem Set #11 by Monday, Nov 6.

End-of-chapter problems: Ch 10: 71, 84, 104; Ch 11: 48, 53, 62. Due by 4 pm, Nov 6.

- Recitation: Practice problems on torque and angular momentum

Topics for today:

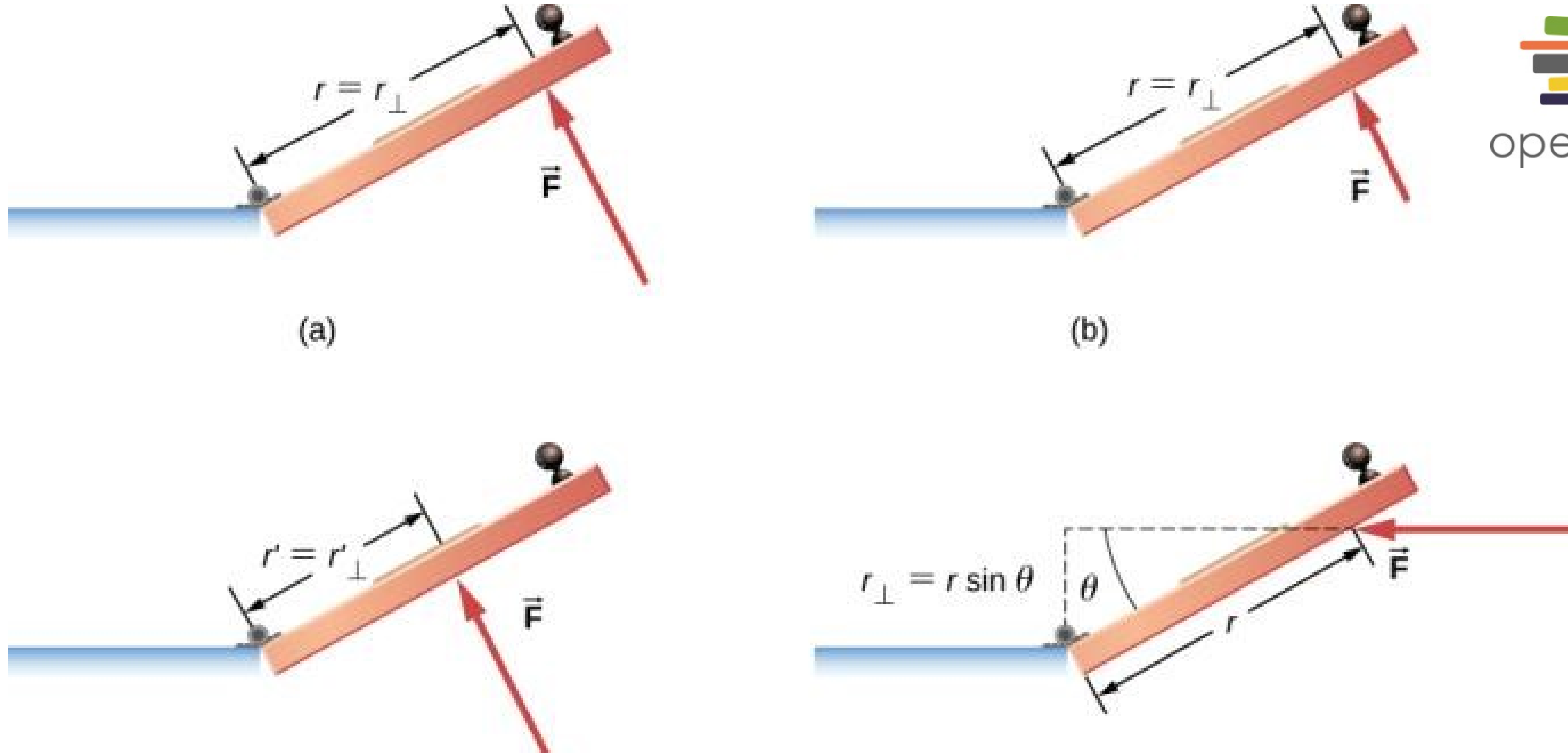
- Torque
- Center of mass and moment of inertia
- Newton's 2^{nd} law for rotation
- Work and power in rotational motion
- Angular momentum



Torque

When a force $\vec{\mathbf{F}}$ is applied to a point *P* whose position is $\vec{\mathbf{r}}$ relative to *O* (Figure 10.32), the torque $\vec{\tau}$ around *O* is

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}. \quad (10.22)$$



From the definition of the cross product, the torque $\vec{\tau}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$ and has magnitude

$$|\vec{\tau}| = |\vec{r} \times \vec{F}| = rF\sin\theta,$$

where θ is the angle between the vectors $\vec{r}$ and $\vec{F}$. The SI unit of torque is newtons times meters, usually written as $N \cdot m$. The quantity $r_\perp = r\sin\theta$ is the perpendicular distance from O to the line determined by the vector $\vec{F}$ and is called the **lever arm**. Note that the greater the lever arm, the greater the magnitude of the torque. In terms of the lever arm, the magnitude of the torque is

$$|\vec{\tau}| = r_\perp F. \quad (10.23)$$

$$\vec{\boldsymbol{\tau}}_{\text{net}} = \sum_i |\vec{\boldsymbol{\tau}}_i|. \tag{10.24}$$

Calculating Net Torque for Rigid Bodies on a Fixed Axis

In the following examples, we calculate the torque both abstractly and as applied to a rigid body.

We first introduce a problem-solving strategy.

Problem-Solving Strategy: Finding Net Torque

1. Choose a coordinate system with the pivot point or axis of rotation as the origin of the selected coordinate system.
2. Determine the angle between the lever arm $\vec{\mathbf{r}}$ and the force vector.
3. Take the cross product of $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$ to determine if the torque is positive or negative about the pivot point or axis.
4. Evaluate the magnitude of the torque using $r_{\perp} F$.
5. Assign the appropriate sign, positive or negative, to the magnitude.
6. Sum the torques to find the net torque.

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6. Sum the torques to find the net torque.

By definition, the lever arm is r(perpendicular), thus the angle is always 90°.
Either find the lever arm and multiply by the magnitude of the force (#4),
or find the angle theta between the vectors **r** and **F,** and calculate rFsin(theta).

10.6 Check Your Understanding A large ocean-going ship runs aground near the coastline, similar to the fate of the *Costa Concordia*, and lies at an angle as shown below. Salvage crews must apply a torque to right the ship in order to float the vessel for transport. A force of 5.0×10^5 N acting at point *A* must be applied to right the ship. What is the torque about the point of contact of the ship with the ground (**Figure 10.36**)?

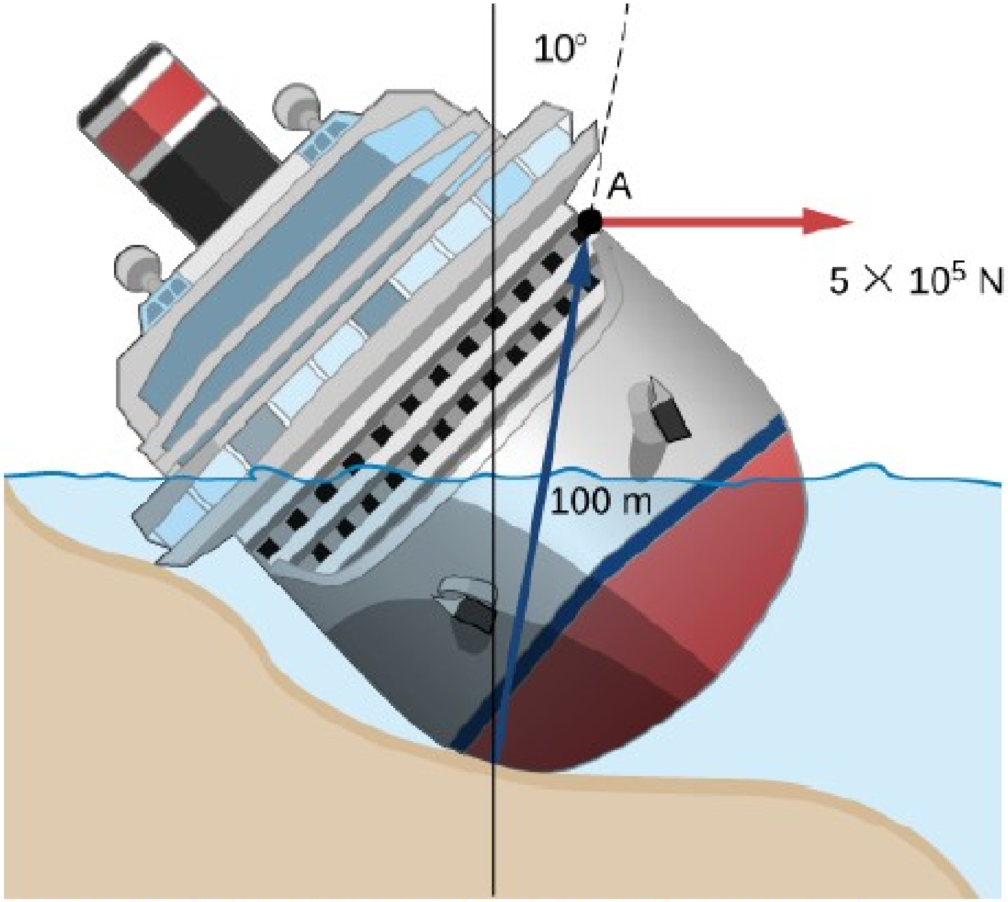
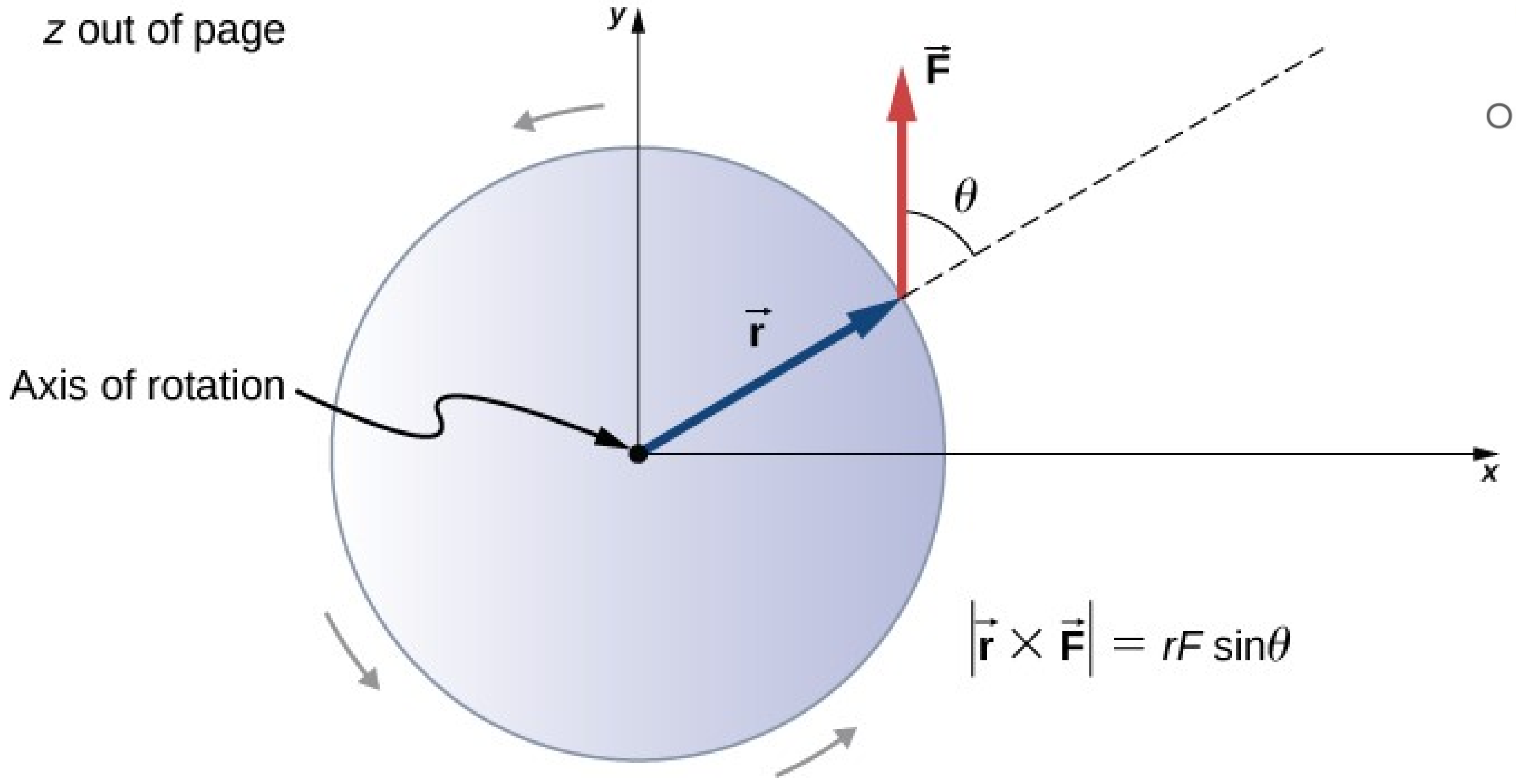


Figure 10.36 A ship runs aground and tilts, requiring torque to be applied to return the vessel to an upright position.



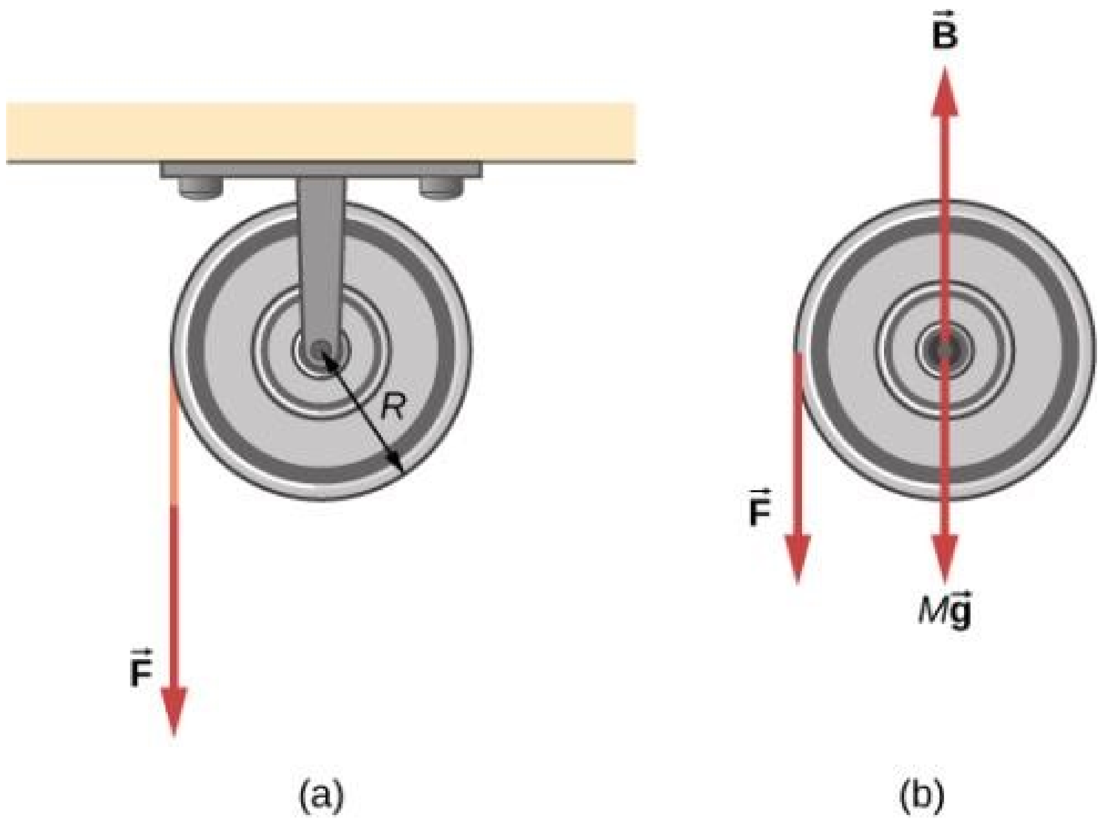
A disk is free to rotate about its axis through the center. The magnitude of the torque on the disk is $rF \sin\theta$. When $\theta = 0°$, the torque is zero and the disk does not rotate. When $\theta = 90°$, the torque is maximum and the disk rotates with maximum angular acceleration.

Newton's Second Law for Rotation

If more than one torque acts on a rigid body about a fixed axis, then the sum of the torques equals the moment of inertia times the angular acceleration:

$$\sum_i \tau_i = I\alpha. \quad (10.25)$$

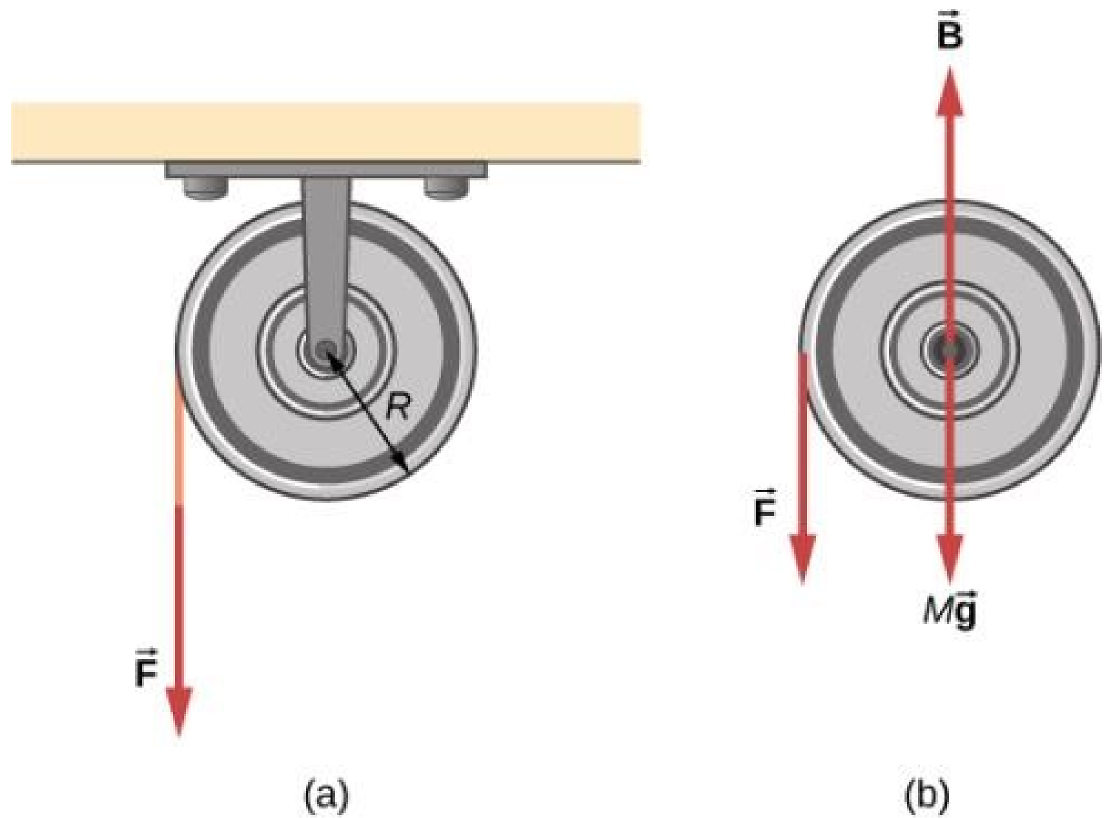
Figure 10.40



(a) A string is wrapped around a pulley of radius *R*.

(b) The free-body diagram. Not! It is a cartoon. Why isn't this a true FBD?

Figure 10.40



(a) A string is wrapped around a pulley of radius *R*.

(b) The free-body diagram. Not! It is a cartoon. Why isn't this a true FBD?

No coordinate axes. Tails of force vectors not at origin. Pulley should be represented as a dot. Scale of vector B is obviously wrong (too short).

Example: Suppose the above pulley has a mass of 1 kg and a radius of 0.1 m, and F is due to a tension force of 10 N from the string wrapped around the pulley.

What is the force B?

What is the net torque on the pulley?

What would be the angular acceleration of the pulley, assuming frictionless bearings?

Some hints for solving problems:

Draw FBD's if necessary in order to set up Newton's 2nd law for translational motion ($\Sigma\mathbf{F}=m\mathbf{a}$)

Draw cartoons for setting up Newton's 2nd law for rotational motion ($\Sigma\tau = I\alpha$). Include an outline sketch of the object, all forces, moment arms, and direction of rotation (if known).

You may need to consider the center of mass in order to account for any weight forces.

Particularly for static problems, remember that you can place your axis of rotations at strategic locations for solving the problem.

Example: Consider the pipe problem from the midterm, except now we have a worker sitting on the end.

The pipe's mass is 125 kg, and the worker's mass is 75 kg.

What are the tensions in the cables if the pipe is 16 m long and the cables are separated by 8 m?

(Note this solution is similar to what is needed for the duck problem in ETA)